Relation of *Weber* law and *Stevens* law at achromatic threshold

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Abstract

The results of this paper are intended as a basis to test the relation between the *Weber* and *Stevens* law for achromatic adjacent and separated stimuli at threshold

According to Valberg (2005) the *Weber* law is valid for achromatic stimuli at photopic light levels in the range between about 50 to 10 000 cd/m^2. The *Weber* law is usually described as *Weber* contrast ratio $C_{\mathrm{Web}} = \Delta L_{\mathrm{w}} / L$ and this is a constant (approximately 1/100). Therefore the *Weber* luminance threshold ΔL_{w} is a **linear** function of the luminance L of the stimulus.

The *Stevens* law describes the scaling of a sensory magnitude that follows a power law of the stimulus magnitude, Here the lightness L^* is a function of the luminance L. The relation is described by the equation $L^* = \mathrm{const}\ L^{1/2}$. The *Stevens* luminance threshold ΔL_{s} can be calculated by derivation of the *Stevens* law and leads to the equation $\Delta L_{\mathrm{s}} = \mathrm{const}\ L^{1/2}$. Therefore the *Stevens* luminance threshold ΔL_{s} is a **power** function of the luminance L of the stimulus.

The different viewing situations of the stimuli at threshold seem to be the main reason for the different laws. The *Weber* threshold ΔL_{w} is valid for **adjacent** achromatic stimuli. This is the usual viewing situation for colour differences in industrial applications. This viewing situation with **adjacent** stimuli was used as basis for the CIEDE2000 colour difference formula.

The *Stevens* threshold ΔL_{s} is valid for **separated** achromatic stimuli, for example for separated samples on a grey background. This is the typical viewing situation of the samples used in colour order systems (Munsell, OSA, NCS). This viewing situation with **separated** stimuli was used as basis for the definition of the CIELAB 1976 colour space.

1. Introduction

CIE 15 (2004) includes the definitions of the CIE 1976 $L^*a^*b^*$ (CIELAB) colour space with lightness L^*. CIE TC1-57 works at present to produce for this space a joint CIE/ISO standard. The proposed standard does not cover the present colour difference formulae based on CIELAB, for example the actual CIEDE2000 colour difference formula (CIE, 2001).

The CIELAB lightness equation

$$L^* = 100\ (Y/Y_{\mathrm{n}})^{1/3}\ -16 \qquad (1)$$

may be approximated by the power function using the luminance factor Y or the luminance L of the stimuli

$$L^* = 100\ (Y/Y_{\mathrm{n}})^{1/2,4} \qquad (2a)$$

$$= \mathrm{const}\ L^{0,42} \qquad (2b)$$

This equation uses the *Stevens* law for the scaling of lightness. The *Stevens* threshold luminance ΔL_{s} is calculated by derivation of this *Stevens* law

$$\Delta L_{\mathrm{s}} = \mathrm{const}\ L^{0,58} \qquad (3)$$

The *Weber* threshold luminance ΔL_{w}

$$\Delta L_{\mathrm{w}} = \mathrm{const}\ L \qquad (4)$$

is very different compared to the *Stevens* threshold luminance ΔL_{s}. This paper presents a model which shows the relationship of both equations at threshold. This relationship may be an important basis for the work of the following two CIE committees:

CIE TC1-55 works at present to test and change the CIEDE2000 colour difference formula for the important range of industrial colour differences between about 1 and 5 CIELAB. The colour difference data at threshold (between about

0,3 to 0,5 CIELAB) and colour differences larger than 5 CIELAB which both seem to be of minor importance for industrial applications may not be used in TC1-55

CIE TC1-63 works at present to test the validity of CIEDE2000 in the range between threshold (about 0.3 to 0.5 CIELAB) up to color differences larger than 5 CIELAB. CIE TC1-63 has produced test charts in reflective mode with large colour differences (about 40 CIELAB). The visual results of about 7 international groups are expected in 2007. In CIE TC1-63 the visual evaluations for large colour differences and at threshold will be used to test the validity of CIEDE2000.

2. The two main viewing situations with adjacent and separated greys

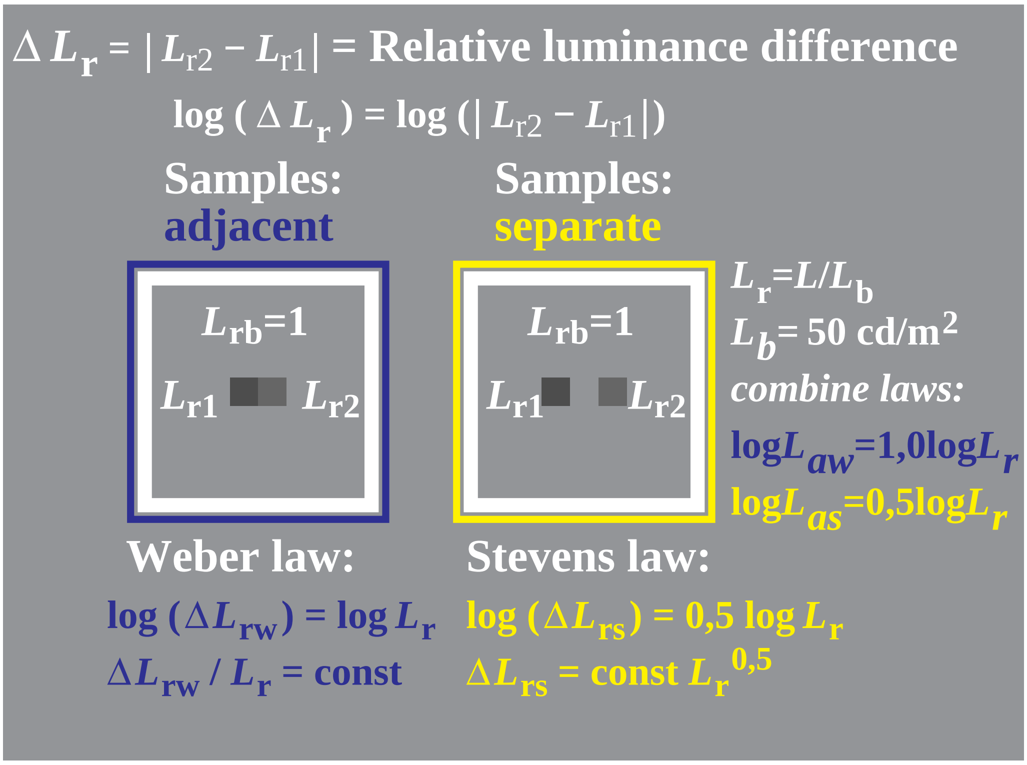

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Figure 1: Relationship of *Weber* threshold ΔL_{rw} and *Stevens* threshold ΔL_{rs} using adaptation luminance L_a

Fig 1 shows the relationship of the *Weber* threshold ΔL_{rw} and the *Stevens* threshold ΔL_{rs}. The relative luminance L_r is used and the definition is given in Fig. 1. Additionally a *special local* adaptation luminance L_a (equation on right side) is used. The adaptation luminance at the border of either the two stimuli L_{r1} and L_{r2} (left side) or the stimuli L_{r1} (or L_{r2}) and the grey background L_{rb} (right side) is different and calculated in the following.

Many results of vision research experiments show that the **log luminance** (log L) is appropriate to calculate the mean. Therefore the adaptation luminance on the left side is calculated by the equation

$$\log L_{aw} = 0{,}5 \, [\log L_{r1} + \log L_{r2}] \qquad (5a)$$

$$\sim \log L_r \qquad (5b)$$

and on the right side by the following equation with the background luminance L_{rb} ($\log L_{rb} = \log 1 = 0$) at the border

$$\log L_{as} = 0{,}5 \, [\log L_{r1} + \log L_{rb}] \qquad (6a)$$

$$\sim 0{,}5 \log L_r \qquad (6b)$$

It is allowed to delete the small amounts ΔL_{rw} or ΔL_{rs} in both cases because the values are 100 times smaller compared to L_r (or L_{r1} or L_{r2}). The "~" sign indicates the approximation, if ΔL_{rw} or ΔL_{rs} is neglected.

Therefore both the *Weber* law and the *Stevens* law can be rewritten at threshold in the simple form

$$\log \Delta L_r = \log L_a \qquad (7a)$$

or

$$\Delta L_r / L_a = \mathrm{const} \qquad (7b)$$

In the following for both viewing situations simplified relations are used between the relative threshold luminance ΔL_{rw} or ΔL_{rs} and the relative luminance L_r. In a first step we assume slopes of 1,0 and 0,5 for the relation on log scales. The *Weber* slope with the value 1,0 is modified later by a small *Weber* slope correction (value = –1/6) to consider the local adaptation to the “absolute” luminance level of the background L_b. Similar the *Stevens* slope with the value 0,5 is modified later by a small *Stevens* slope correction (value = –1/12). The value of the *Stevens* slope correction is half of the slope of the *Weber* slope correction.

Therefore instead of only a fixed background luminance L_b.= 50 cd/m^2 we will consider a large range between L_b.= 0,5 cd/m^2 and L_b.= 5000 cd/m^2. But then there is the problem that the lower value is outside the photopic luminance level. Therefore the model produced here should be taken with care for a field or background luminance below about L_r = 5 cd/m^2. This value is outside the office application range and therefore of minor value for application.

According to *Valberg* (2005) the *Weber* law is valid in the range between 50 cd/m^2 and 10 000 cd/m^2. The *Valberg* data are assumed for a white background, so in our case the lower value 50 cd/m^2 must be reduced to the value 10 cd/m^2 for the grey background.

For very low and very large luminance there is a change of the slope of the *Weber* law from the value 1 to the value zero. Within plus and minus 1 log units compared to the background the local luminance adaptation leads to the resulting slopes and corrected *Weber* and *Stevens* laws. Instead of the value 1,0 for the *Weber* slope the value 0,84 and instead of 0,5 for the *Stevens* slope the value 0,42 is appropriate. The last value is the slope for the approximation of CIELAB (see equation 2b).

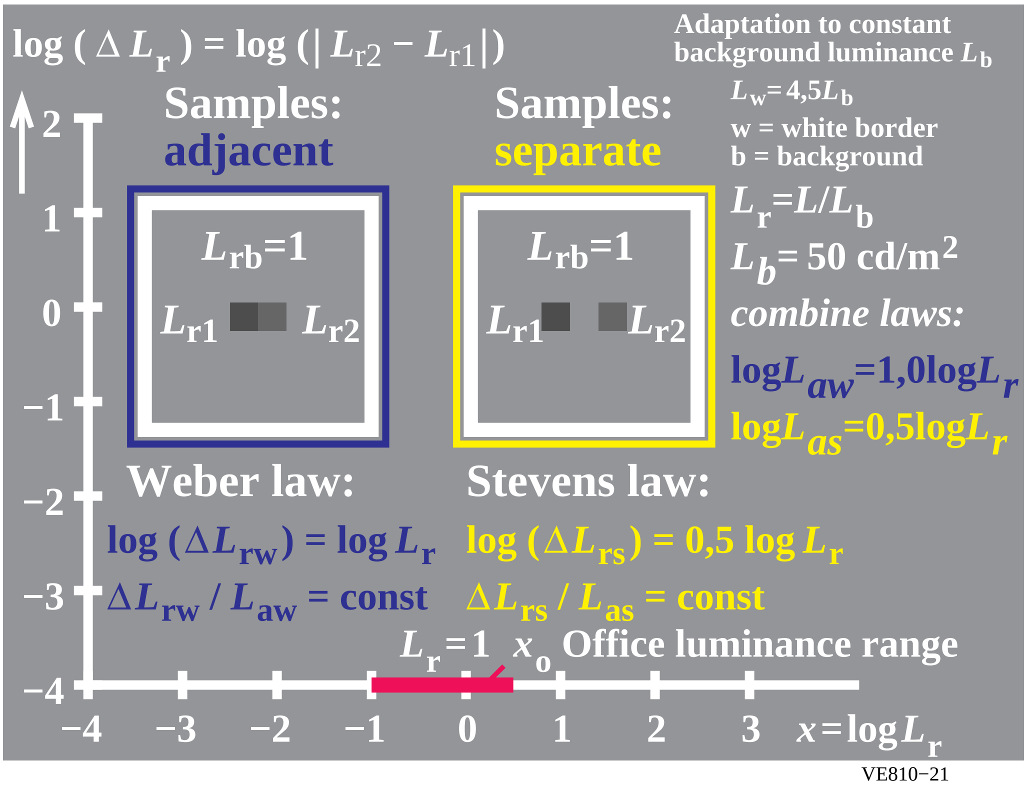


Figure 2: Threshold and luminance range to be considered compared to the office luminance range

Fig. 2 adds the relative luminance range between below L=0,5 cd/m^2 and above L=5000 cd/m^2 (range of 4 log units). It is important to know that the application range in offices is only a small part (1,5 log units) of this range.

Of main interest are results in this office application range. The following figures include the wider range which is drawn by dashed lines (within 8 log units) and solid lines (within 4 log units). The threshold in the wide range is unknown. Of main interest are results in the range 0,5 log units above and 1,0 log unit below the grey surround.

This paper does not propose that the relation is exactly as indicated. But this paper produces an appropriate relation of both laws with the intention to test the new equation with the mean adaptation luminance L_a calculated by a log equation. The exponent n = 0.5 is to be determined. Experimental viewing situations with sample separation and sample size of 2 degrees are preferred.

At threshold the experimental results for **separated** samples may fail to be described by the *Stevens* equation. Then probably the ***Stevens*** law fails at threshold and a linear instead of the log luminance L_r summation may be used to calculate the mean adaptation luminance L_a.

3. Threshold and scaling data for constant background luminance

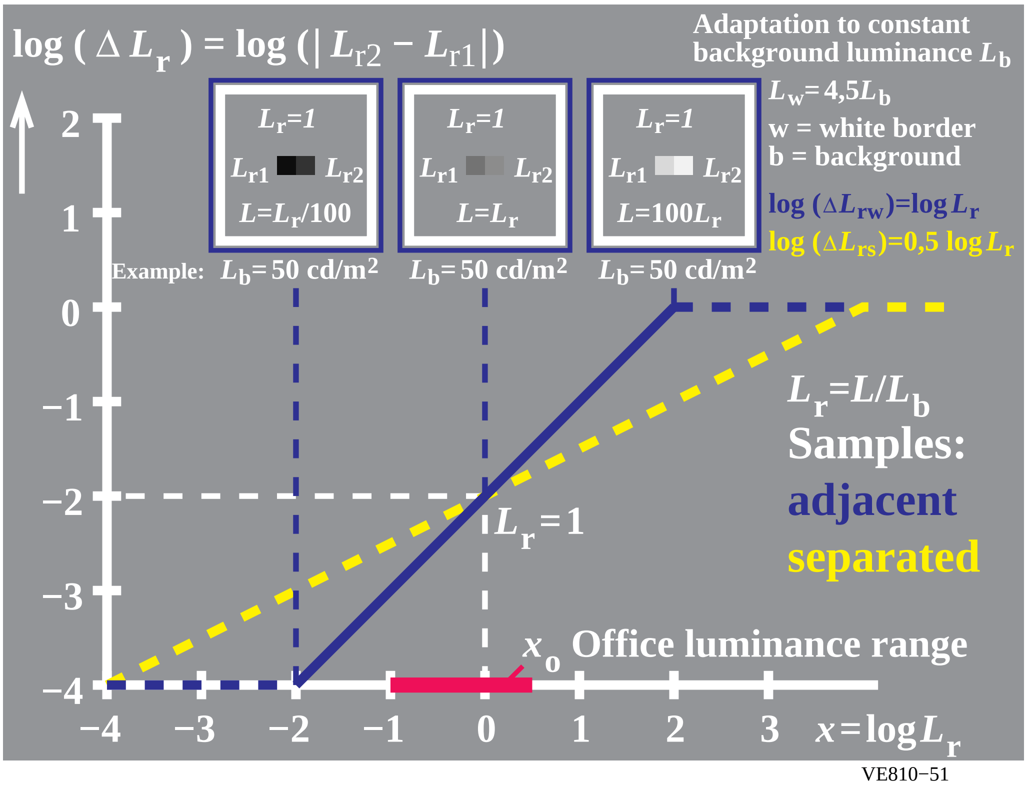


Figure 3: ***Weber*** **threshold ΔL_{rw} as function of relative luminance L_r for constant background luminance L_b**

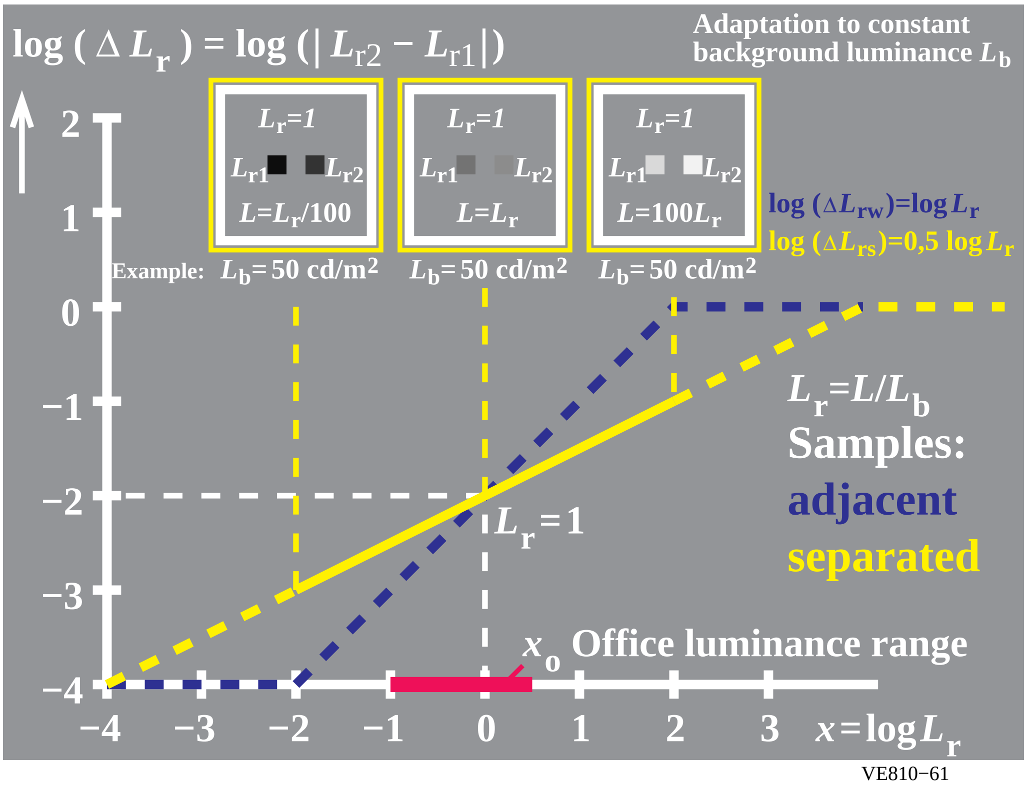


Figure 4: ***Stevens*** **threshold ΔL_{rs} as function of relative luminance L_r for constant background luminance L_b**

The figures 3 and 4 show the *Weber* threshold ΔL_{rw} and the *Stevens* threshold ΔL_{rs} as function of relative luminance

L_r for constant grey background luminance L_b.

Fig. 3 shows a **linear** relation for the *Weber* luminance difference ΔL_{rw}

$$\Delta L_{rw} = \text{const } L_r \tag{8a}$$

or with the log scales

$$\log \Delta L_{rw} = \log L_r \tag{8b}$$

Fig. 4 shows a **power** function for the *Stevens* luminance difference ΔL_{rs}

$$\Delta L_{rs} = \text{const } L_r^{0,5} \tag{9a}$$

or with the log scales

$$\log \Delta L_{rs} = 0{,}5 \log L_r \tag{9b}$$

> Remark: The power function exponent with the value 0,5 needs later a modification because usually there is some additional local adaptation to the “absolute” sample luminance L_r.

For the different viewing situations with adjacent and separate grey samples the colours blue and yellow are used. The blue colour is used for the adjacent samples and the corresponding equations. The yellow colour is used for the separated samples and the corresponding equations. The white colour is used for equations which are valid for both viewing situations.

The office relative luminance range is indicated in red colour. The range covers about 1,5 log units (factor 40:1). This office range is only a small part of the luminance range. The discussion in the following is limited to the luminance range between -2 and +2 log units compared to the grey background. At both ends of the luminance range there is a saturation effect. The start and shape of the saturation is unknown and indicated by dashed lines.

For a grey surround with a luminance of 50 cd/m^2 (CIE luminance factor Y=20) the luminance for printed paper output is in the range between 6 cd/m^2 (Y=2,5) and 225 cd/m^2 (Y=90).

One can compare the luminance of 50 cd/m^2 with the luminance produced by the recommended office illuminance which is between 500 lux (standard condition) and 1000 lux (special condition recommended for color comparison). Both recommendations lead for grey paper (Y=20) to a luminance range between 32 cd/m^2 (=500/3.14*0,2 cm) and 64 cd/m^2. The value 50 cd/m^2 used in the two figures is within this luminance range.

There may be a vertical shift of the *Stevens* curve to higher values by a factor two at the value L_r = 1 for the grey background. This is because we need at least either one *Weber* decrement *or* one increment for the *Weber* threshold to see a difference between the left and right grey. For the *Stevens* threshold we need at least both one *Weber* decrement *and* one *Weber* increment at the background to see a difference between the left and right grey.

4. Threshold and scaling data considering local luminance adaptation

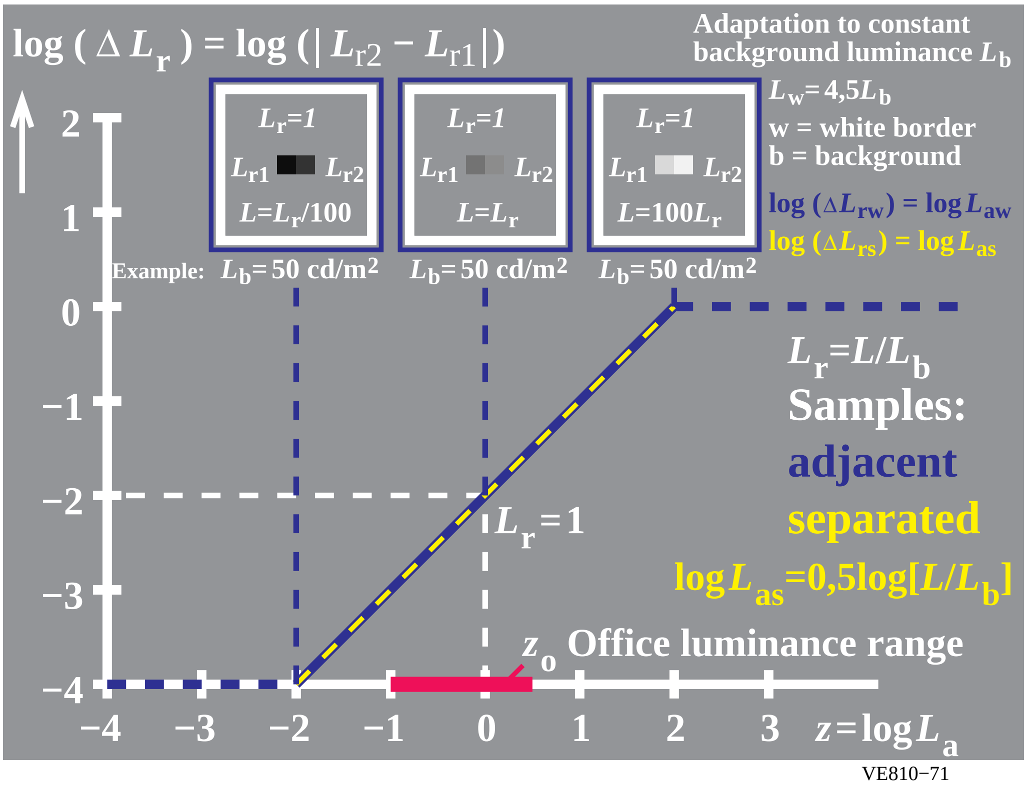


Figure 5: ***Weber*** **threshold ΔL_{rw} as function of adaptation luminance L_a at the border of adjacent samples**

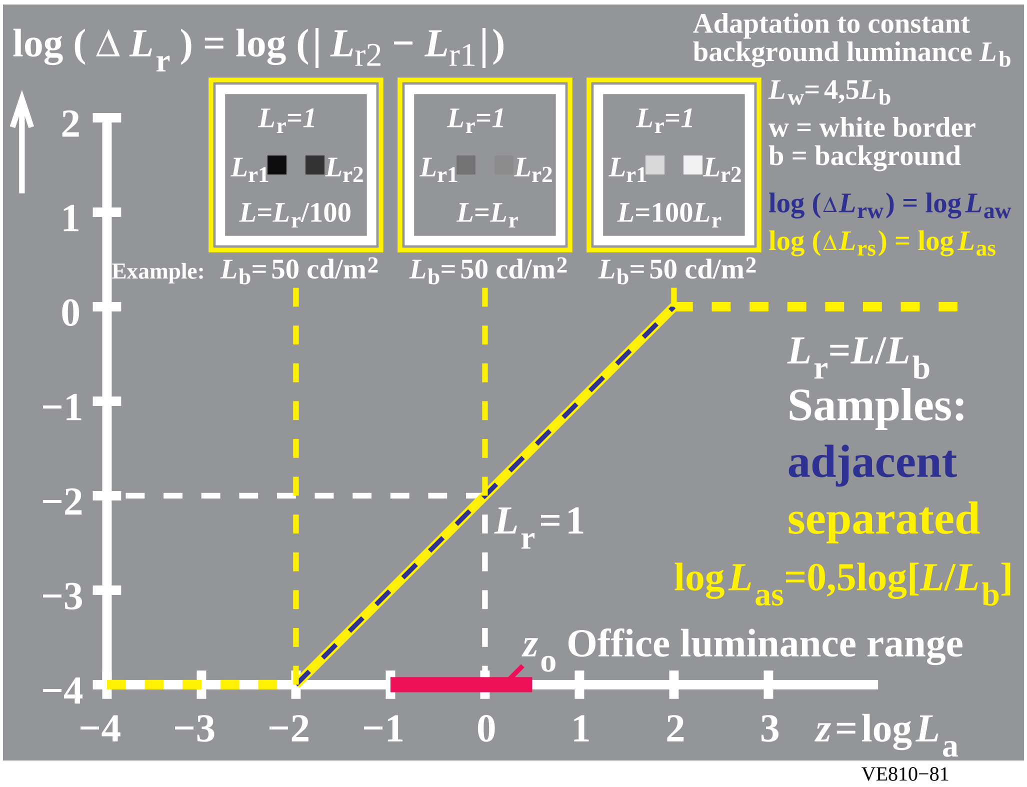


Figure 6: ***Stevens*** **threshold ΔL_{rs} as function of adaptation luminance L_a at the border of separated samples**

The figures 5 and 6 show the *Weber* threshold ΔL_{rw} and the *Stevens* threshold ΔL_{rs} as function of adaptation luminance L_a for constant grey background luminance L_b. Again the local adaptation at the border of the samples is considered.

In the case of **adjacent** grey samples the mean luminance at the border of the two samples is approximately equal to the luminance of either one of the two samples. In vision in many cases the mean is given if the log luminance is used at the border. Therefore:

$$\log L_{aw} = 0.5 (\log L_r + \log L_r + \Delta L_{rw}) \quad (10a)$$

$$\sim 0.5 (\log L_r + \log L_r) \quad (10b)$$

$$\sim \log L_r \quad (10c)$$

The two samples differ by a small amount in luminance $\Delta L_{rw.}$ This amount is usually 1/100 of the luminance L_r and can be neglected. For this approximation the sign "~" has been used.

Fig. 5 shows a **linear** relation for the *Weber* luminance threshold ΔL_{rw}

$$\Delta L_{rw} = \text{const } L_r = \text{const } L_{aw} \quad (11)$$

This indicates no change compared to Fig. 4.

In the case of **separated** grey samples the mean luminance at the border of each of the two samples is approximately equal to the log relative luminance of sample and the grey surround.

In vision in many cases the mean is given if the log luminance is used at the border. Therefore:

$$\log L_{as} = 0.5 \log L_r \text{ (sample)} + 0.5 \log L_r \text{ (grey background)} \quad (12)$$

$$\sim 0.5 \log L_r$$

The relative luminance of the grey background is 1 and therefore the log value is zero. Similar as in equation (10a) the small value ΔL_{rs} compared to L_r can be neglected.

Equation (12) can therefore be rewritten in the form

$$L_{as} = L_r^{0,5} \quad (13)$$

Fig. 4 has shown a **power** function for the *Stevens* luminance threshold ΔL_{rs}

$$\Delta L_{rs} = \text{const } L_r^{0,5} \quad (14)$$

If equation (13) is used then again a **linear** equation (instead of the **power** equation used in Fig. 3) describes the threshold ΔL_{rs} of the separated colours.

$$\Delta L_{rs} = \text{const } L_{as} \quad (15)$$

or for both log scales used in Fig. 5 and Fig. 6

$$\log \Delta L_r = \log L_a \quad (16)$$

Remark: The power function exponent with the value 0,5 needs later a modification because usually there is local adaptation to the sample luminance L_r which may vary over four log units in natural scenes.

5. .Threshold and scaling data for increasing background luminance

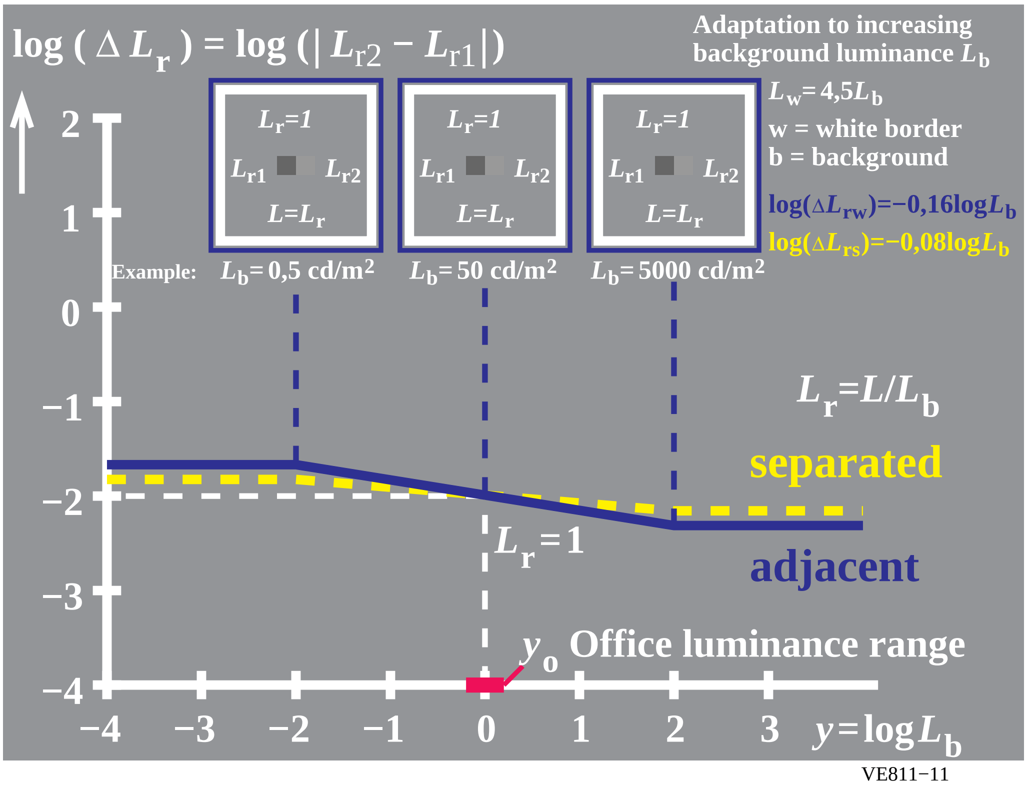


Figure 7: ***Weber*** **threshold ΔL_{rw} as function of background luminance L_b**

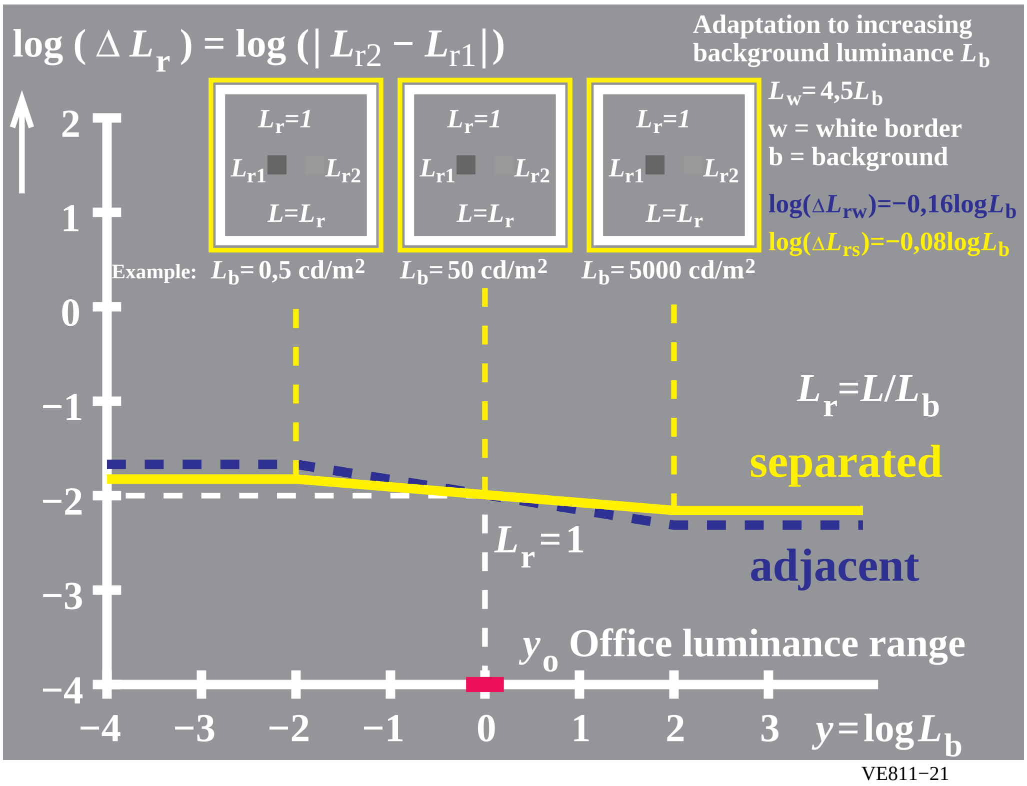


Figure 8: ***Stevens*** **threshold ΔL_{rs} as function of background luminance L_b**

The figures 7 and 8 show the *Weber* threshold ΔL_{rw} and the *Stevens* threshold ΔL_{rs} as function of relative increasing background luminance L_b (instead of constant grey background luminance L_b, compare Fig. 3 to 6).

Fig. 7 shows a **nonlinear** relation for the *Weber* luminance difference ΔL_{rw}

$$\Delta L_{rw} = L_b^{-0,16} \qquad (17a)$$

or for the log scales used in Fig. 6

$$\log \Delta L_{rw} = -0,16 \log L_b \qquad (17b)$$

Fig. 8 shows a **power** function for the *Stevens* luminance difference ΔL_{rs}

$$\Delta L_{rs} = \text{const } L_b^{-0,08} \qquad (18a)$$

or for the log scales used in Fig. 7

$$\log \Delta L_{rs} = -0,08 \log L_b \qquad (18b)$$

The following figures 9 and 10 on the next page show the *Weber* threshold ΔL_{rw} and the *Stevens* threshold ΔL_{rs} as function of the adapted sample field luminance L_a for the constant grey background luminance L_b.

Fig. 9 shows a **nonlinear** (power) relation for the *Weber* luminance difference ΔL_{rw}

$$\Delta L_{rw} = L_r^{0,84} \qquad (19a)$$

or for the log scales used in Fig. 9

$$\log \Delta L_{rw} = 0,84 \log L_r \qquad (19b)$$

Fig. 10 shows a **power** function for the *Stevens* luminance difference ΔL_{rs}

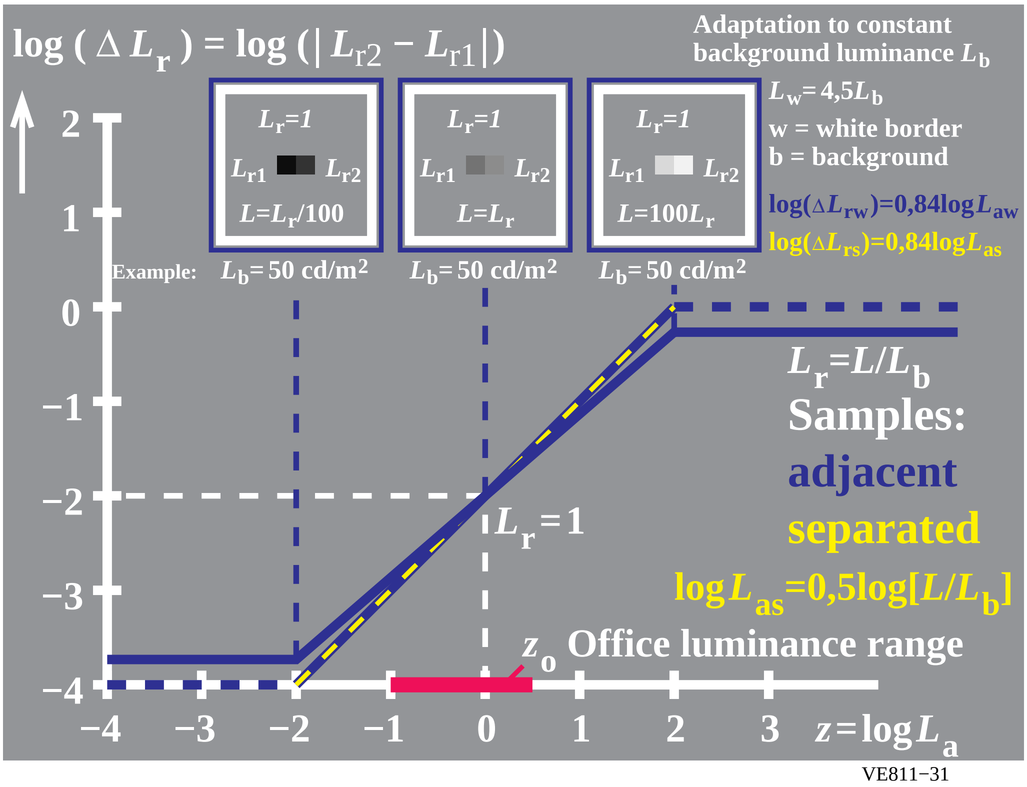
$$\Delta L_{rs} = \text{const } L_r^{0,42} \qquad (20a)$$

or for the log scales used in Fig. 10

$$\log \Delta L_{rs} = 0,42 \log L_r \qquad (20b)$$

Remark: The exponent 0,42 is used in the CIELAB approximation (see equations 2a and 2b) and in the IPT colour space of *Fairchild* (2005), which is one space to be included in the work of CIE TC1-55.

6. Threshold and scaling data considering both luminance adaptations



Relation of Weber law and Stevens law at achromatic threshold

Figure 9: *Weber* threshold ΔL_{rw} as function of adaptation luminance L_a of adjacent samples

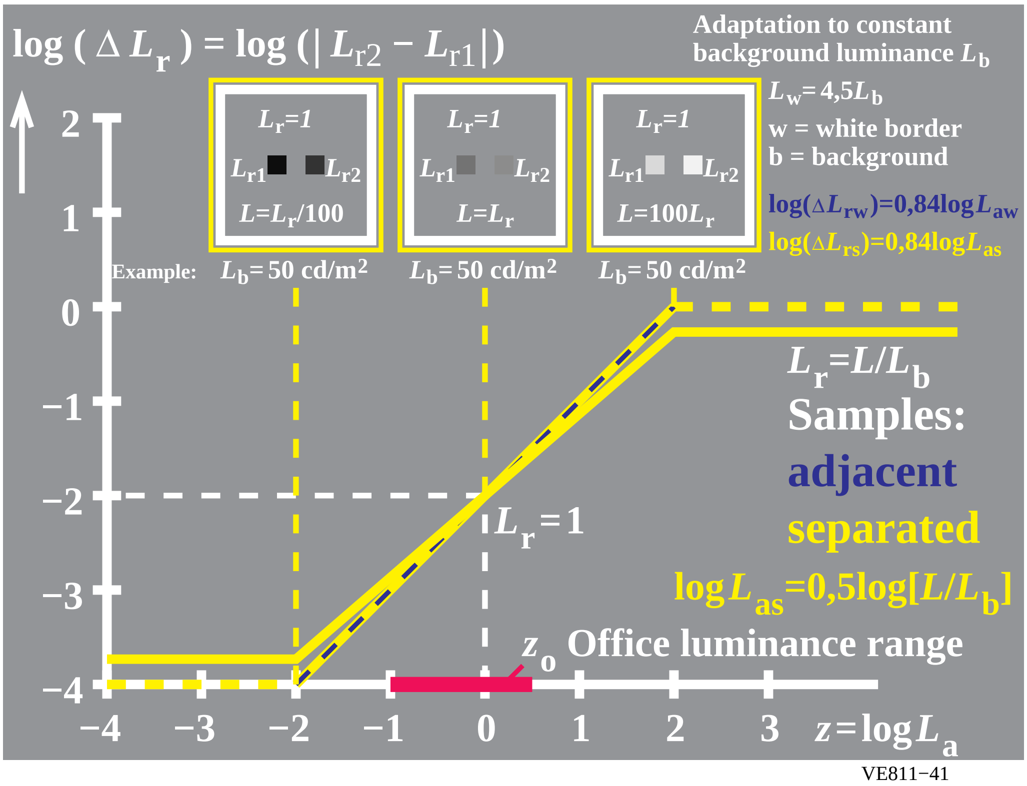


Figure 10: *Stevens* threshold ΔL_{rs} as function of adaptation luminance L_a of separated samples

7. Conclusion

One of the goals of CIE TC1-55 is to produce new visual data and use existent data as the basis for a better uniform color space which may replace CIELAB and CIEDE2000. This uniform colour space is intended to be used for color difference calculations in industrial applications and may replace CIEDE2000.

For industrial color tolerance applications there seem to be no need to use the data outside the range of about 1 to 5 CIELAB which is the range used for CIEDE2000.

The goal of CIE TC1-63 “Validity of CIEDE2000” is to produce new visual data and use existent data as the basis to check the validity of the formula CIEDE200 between threshold (0.3 to 0.5 CIELAB) up to colour differences larger than 5 CIELAB.

In this paper for achromatic colours a relation at threshold between the *Weber* law and the *Stevens* law is developed. The log of the adaptation luminance (log L_a) is used at the sample border. This is either the log relative luminance (log L_r) for the adjacent samples (*Weber* viewing situation on grey background) or half the log relative luminance (0,5 log L_r) for the separated samples (*Stevens* viewing situation). With this local adaptation luminance (log L_a) at the luminance borders both laws are combined to the new law

$$\log \Delta L_r = \log L_a \qquad (21)$$

or

$$\Delta L_r / L_a = \text{const} \qquad (22)$$

If the “absolute” adaptation luminance of the grey background is changed, for example between 0,5 cd/m^2 and 5000 cd/m^2, then there is a decrease of the threshold in both cases according to the equation

$$\log \Delta L_r = -\,0{,}16 \log L_a \qquad (23)$$

In application cases the local luminance adaptation will follow the additional luminance adaptation property of equation (23). Therefore the combined *Weber* law and *Stevens* law together with the local “absolute” luminance adaptation will lead to only one equation

$$\log \Delta L_r = 0{,}84 \log L_a \qquad (23)$$

with the adaptation luminance defined by either

$$\log L_a = \log L_r$$

for the adjacent sample viewing situation and

$$\log L_a = 0{,}5 \log L_r$$

for the separate sample viewing situation.

8. References

Valberg, Arne (2005), Light Vision Color, John Wiley & Sons, 462 pages.

Remark: This book includes the *Weber* and *Stevens* law. The *Weber* law is mentioned to be valid between about 50 and 10 000 cd/m^2, see page 436.

Fairchild, Mark (2005), Color Appearance Models, Second edition, John Wiley & Sons, 385 pages

Remark: This book lists many colour appearance spaces. For example the IPT colour space uses the exponent n=0,43 for the transformation of the L, M, and S cone sensitivities, see page 344.

Stevens, S. S. (1961), To honor *Fechner* and repeal his law, *Science* 133, 80-88

Richter, K. (2005), Linear relationship between CIELAB and device coordinates for Colorimetric Image Technology (CIT), see the URL (140 kByte, 6 pages)

http://www.ps.bam.de/CIE05.PDF

Richter, K. (2006), Device dependent linear relative CIELAB data *lab** and colorimetric data for corresponding colour input and output on monitors and printers, CIE Symposium, 75 years colorimetric observer, Ottawa/Canada, 2006, to be published.

Remark: For further publications of the author and analog and digital BAM-, DIN-, CEN- and ISO/IEC-test charts, see the following URL under “Publications” and “Technical Reports”

http://www.ps.bam.de