

**Line-element examples for grey samples (0,2≤x≤5)**

F(x) is called the line-element function of f(x).

The following relations are valid for x=Y/Y<sub>u</sub>=Y/18:

$$\frac{d[F(x)]}{dx} = f(x) \quad [1]$$

$$F(x) = \int \frac{f'(x)}{f(x)} dx \quad [2]$$

Example for the normalized tristimulus value x=Y/Y<sub>u</sub>:

$$\frac{d[\ln(1+b)x]}{dx} = \frac{ab}{1+b} \quad [3]$$

$$a \ln(1+b)x = \int \frac{ab}{1+b} dx \quad [4]$$

CEA00-1N

**Line-element examples for grey samples (0,2≤x≤5)**

F<sub>u</sub>(x) is called the line-element function of f<sub>u</sub>(x).

Both functions are normalized to the surround value:

$$\frac{d[F_u(x)]}{dx} = f_u(x) \quad [1]$$

$$F_u(x) = \int \frac{f'_u(x)}{f_u(x)} dx = \int \frac{b}{1+b} dx \quad [2]$$

Example for L\*(x) & ΔY with x=Y/Y<sub>u</sub>, x<sub>u</sub>=1, b=6,141:

$$L^*_u(x) = \frac{L^*(x)}{L^*(x_u)} = \frac{\ln(1+b)x}{\ln(1+b)} \quad [3]$$

$$f_u(x) = \frac{\Delta Y}{\Delta Y_u} = \frac{1+b}{1+b} \quad [4]$$

CEA00-3N

**Line-element equations according to CIE 230:2019**

Colour-discrimination function f(x) = ΔY = Δx Y<sub>u</sub> [0]

ΔY=(A<sub>1</sub>+A<sub>2</sub>Y)/A<sub>0</sub> A<sub>0</sub>=1,5, A<sub>1</sub>=0,0170, A<sub>2</sub>=0,0058

$$f_u(x) = \frac{\Delta Y}{\Delta Y_u} = \frac{1+b}{1+b} \quad b=A_2 Y_u / A_1 \quad x=Y/Y_u \quad [1]$$

$$F_u(x) = \int \frac{f'_u(x)}{f_u(x)} dx = \int \frac{b}{1+b} dx \quad [2]$$

Example for L\*(x) & ΔY with x=Y/Y<sub>u</sub>, x<sub>u</sub>=1, b=6,141:

$$L^*_u(x) = \frac{L^*(x)}{L^*(x_u)} = \frac{\ln(1+b)x}{\ln(1+b)} \quad [3]$$

$$f_u(x) = \frac{\Delta Y}{\Delta Y_u} = \frac{1+b}{1+b} \quad [4]$$

CEA00-5N

**Line-element equations for thresholds and scaling**

Colour-discrimination function f(x) = ΔY = Δx Y<sub>u</sub> [0]

ΔY=1/[(1+x)(2+x)]=1/[1+x]-1/[2+x] x=√2 e<sup>k(u-u<sub>0</sub>)</sup>

$$f_u(x) = \frac{\Delta Y}{\Delta Y_u} = \frac{1+x}{2} - \frac{2}{3} \quad x=Y/Y_u \quad [1]$$

$$F_u(x) = \int \frac{f'_u(x)}{f_u(x)} dx = \int \frac{1}{1+x} dx - \int \frac{1}{2+x} dx \quad [2]$$

Example for L\*(x) & ΔY with x=Y/Y<sub>u</sub>, x<sub>u</sub>=1:

$$L^*_u(x) = \frac{L^*(x)}{L^*(x_u)} = \frac{\ln(1+x)}{\ln(2)} - \frac{\ln(1+0,5x)}{\ln(1,5)} \quad [3]$$

$$f_u(x) = \frac{\Delta Y}{\Delta Y_u} = \frac{1+x}{2} - \frac{1+0,5x}{1,5} \quad [4]$$

see K. Richter (1985), Computer Graphic and Colorimetry, p. 113-127  
<http://color.li.tu-berlin.de/BUA4BF.PDF>

CEA00-7N

**Line-element examples for grey samples (0,2≤x≤5)**

F<sub>u</sub>(x) is called the line-element function of f<sub>u</sub>(x).

Both functions are normalized to the surround value:

$$\frac{d[F_u(x)]}{dx} = f_u(x) \quad [1]$$

$$F_u(x) = \int \frac{f'_u(x)}{f_u(x)} dx = \int \frac{b}{1+b} dx \quad [2]$$

Example for the normalized functions with x<sub>u</sub>=1:

$$F_u(x) = \frac{F(x)}{F(x_u)} = \frac{\ln(1+b)x}{\ln(1+b)} \quad [3]$$

$$f_u(x) = \frac{f(x)}{f(x_u)} = \frac{1+b}{1+b} \quad [4]$$

CEA00-2N

**Line-element equations according to CIE 230:2019**

Colour-threshold (t) function f<sub>t</sub>(x) = ΔY<sub>t</sub> = Δx Y<sub>u</sub> [0]

ΔY<sub>t</sub>=(A<sub>1</sub>+A<sub>2</sub>Y)/A<sub>0</sub> A<sub>0</sub>=1,5, A<sub>1</sub>=0,0170, A<sub>2</sub>=0,0058

$$f_{tu}(x) = \frac{\Delta Y_t}{\Delta Y_{tu}} = \frac{1+b}{1+b} \quad b=A_2 Y_u / A_1 \quad x=Y/Y_u \quad [1]$$

$$F_{tu}(x) = \int \frac{f'_{tu}(x)}{f_{tu}(x)} dx = \int \frac{b}{1+b} dx \quad [2]$$

Example for L\*(x) & ΔY<sub>t</sub> with x=Y/Y<sub>u</sub>, x<sub>u</sub>=1, b=6,141:

$$L^*_{tu}(x) = \frac{L^*(x)}{L^*(x_u)} = \frac{\ln(1+b)x}{\ln(1+b)} \quad [3]$$

$$f_{tu}(x) = \frac{\Delta Y_t}{\Delta Y_{tu}} = \frac{1+b}{1+b} \quad [4]$$

CEA00-4N

**Line-element equations for thresholds and scaling**

Colour-discrimination function f(x) = ΔY = Δx Y<sub>u</sub> [0]

ΔY=1/[(1+x)(2+x)]=1/[1+x]-1/[2+x] x=√2 e<sup>k(u-u<sub>0</sub>)</sup>

$$f_u(x) = \frac{\Delta Y}{\Delta Y_u} = \frac{1+b}{1+b} - \frac{1+0,5b}{1+0,5b} \quad b=1, x=Y/Y_u \quad [1]$$

$$F_u(x) = \int \frac{f'_u(x)}{f_u(x)} dx = \int \frac{b}{1+b} dx - \int \frac{0,5b}{1+0,5b} dx \quad [2]$$

Example for L\*(x) & ΔY with x=Y/Y<sub>u</sub>, x<sub>u</sub>=1, b=1:

$$L^*_u(x) = \frac{L^*(x)}{L^*(x_u)} = \frac{\ln(1+b)x}{\ln(1+b)} - \frac{\ln(1+0,5b)x}{\ln(1+0,5b)} \quad [3]$$

$$f_u(x) = \frac{\Delta Y}{\Delta Y_u} = \frac{1+b}{1+b} - \frac{1+0,5b}{1+0,5b} \quad [4]$$

see K. Richter (1985), Computer Graphic and Colorimetry, p. 113-127  
<http://color.li.tu-berlin.de/BUA4BF.PDF>

CEA00-6N

**Line-element equations for thresholds and scaling**

Colour-discrimination function f(y) = ΔY = Δy Y<sub>u</sub> [0]

ΔY=1/[y(1+y)]=1/y-1/(1+y) y=(1+√2) e<sup>k(u-u<sub>0</sub>)</sup>

$$f_u(y) = \frac{\Delta Y}{\Delta Y_u} = \frac{y}{2} - \frac{1+y}{3} \quad y=1+Y/Y_u, dy=dx \quad [1]$$

$$F_u(y) = \int \frac{f'_u(y)}{f_u(y)} dy = \int \frac{1}{y} dy - \int \frac{1}{1+y} dy \quad [2]$$

Example for L\*(y) & ΔY with y=1+Y/Y<sub>u</sub>, y<sub>u</sub>=2:

$$L^*_u(y) = \frac{L^*(y)}{L^*(y_u)} = \frac{\ln(y)}{\ln(2)} - \frac{\ln(1+y)}{\ln(3)} \quad [3]$$

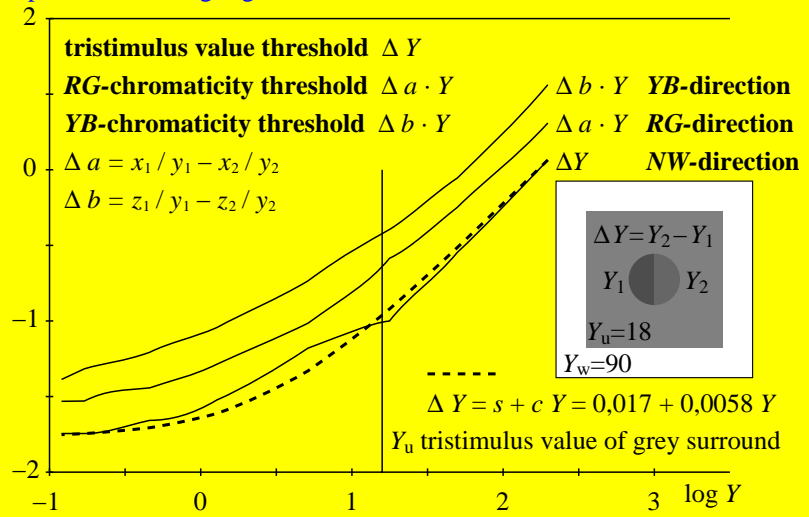
$$f_u(x) = \frac{\Delta Y}{\Delta Y_u} = \frac{1+x}{2} - \frac{1+0,5x}{1,5} \quad [4]$$

see K. Richter (1985), Computer Graphic and Colorimetry, p. 113-127  
<http://color.li.tu-berlin.de/BUA4BF.PDF>

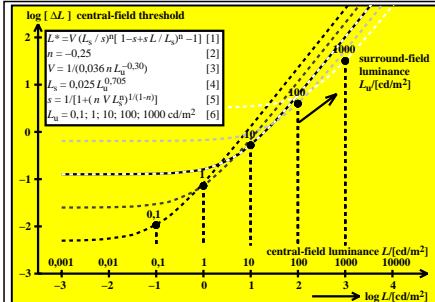
CEA00-8N

**NW-achromatic, and RG- and YB-chromatic thresholds as function of Y**

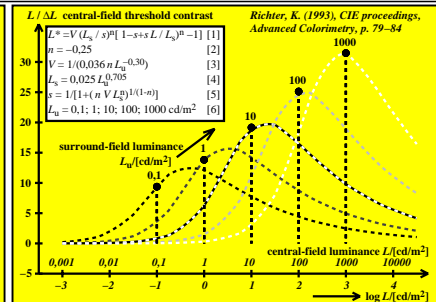
experiments and data: BAM-research report no. 115 (1985), page 72, see  
<https://nbn-resolving.org/urn:nbn:de:kobv:b43-3350>



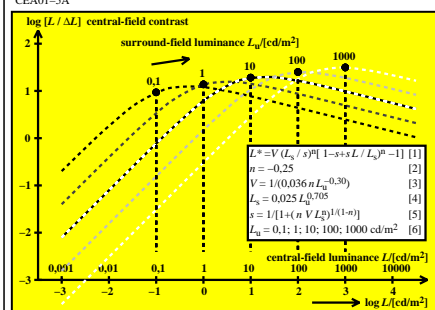
CEA01-3N



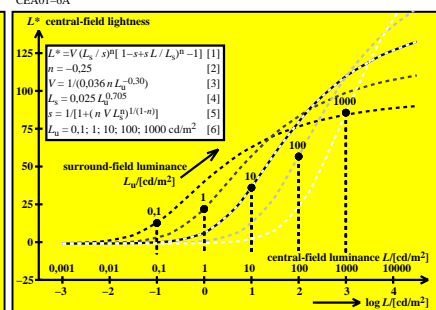
CEA01-5A



CEA01-6A



CEA01-7A



CEA01-8A

see similar files: <http://farbe.li.tu-berlin.de/CEA0/CEA0LONP.PDF> / .PS  
 technical information: <http://farbe.li.tu-berlin.de> or <http://130.149.60.45/~farbmetrik>

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 application for evaluation and measurement of display or print output  
 TUB material: code=rha4ta