CIELAB modifications of the colour space and the colour-difference formula by line elements for different applications

Example line element in lightness direction of *Stiles (1946)*, who uses the luminance L instead of the tristimulus value Y.

$$L^*(Y) = s \ln[1+9Y] \quad s=\text{scaling factor} \qquad [1a]$$

$$d(L^*(Y)) / dY = 9s / [1+9Y] \qquad [2a]$$

For this derivation, and for the lightness threshold $d(L^*(Y))=1$ it is valid:

$$dY = [1+9Y] / 9s \qquad [3a]$$

For the normalization with the surround values $Y_u=18$, dY_u and $L^*(Y_u)$:

$$dY/dY_u = [1+9Y] / [1+9Y_u] \qquad [4a]$$

$$L^*(Y) / L^*(Y_u) = \ln[1+9Y] / \ln[1+9Y_u] \qquad [5a]$$

For the CIELAB colour-space according to ISO/CIE 11664-4:2019

$$L^*(Y) = s\,[Y/Y_n]^k - 16 = s_u[Y/Y_u]^k - 16 \quad s=116,\ k=1/3,\ Y_n=100,\ Y_u=18 \qquad [1b]$$

$$d(L^*(Y)) / dY = s_u k\,[Y/Y_n]^{k-1} \quad \text{with } s_u=116[Y_u/Y_n]^k=65{,}50,\ 1<=Y<=100 \qquad [2b]$$

For this derivation, and for the lightness threshold $d(L^*(Y))=1$ it is valid:

$$dY = [Y/Y_u]^{1-k} / s_u k \qquad [3b]$$

$$dY/dY_u = [Y/Y_u]^{1-k} \qquad [4b]$$

$$L^*(Y) / L^*(Y_u) = \{s_u[Y/Y_u]^k - 16\} / \{s_u - 16\} \qquad [5b]$$

Line elements for the CIELAB-colour space are included in ISO/CIE 11664-4.