LABJND modifications of the colour space and the colour-difference formula by line elements for different applications

Example line element in lightness direction of *Stiles (1946)*, who uses the luminance *L* instead of the tristimulus value *Y*.

$L^*(Y)$ = s ln[1+9Y] s=scaling factor [1a]

$d(L^*(Y)) / dY$ = 9s / [1+9Y] [2a]

For this derivation, and for the lightness threshold $d(L^*(Y))$=1 it is valid:

dY = [1+9Y] / 9s [3a]

For the normalization with the surround values Y_u=18, dY_u and $L^*(Y_u)$:

dY/dY_u = [1+9Y] / [1+9Y_u] [4a]

$L^*(Y) / L^*(Y_u)$ = ln[1+9Y] / ln[1+9Y_u] [5a]

For the LABJND colour-difference formula according to CIE 230:2019:

dY/dY_u = [A_1+A_2Y] / [A_1+A_2Y_u] A_1=0,0170, A_2=0,0058 [1b]

It is valid with the definition $A_{2u}=A_2Y_u/A_1$=6,141

dY/dY_u= [1+$A_{2u}(Y/Y_u)$] / [1+A_{2u}] [2b]

$L^*(Y) / L^*(Y_u)$ = ln[1+$A_{2u}(Y/Y_u)$] / ln[1+A_{2u}] [3b]

Line elements for the LABJND-colour space are NOT included in CIE 230. The value of A_{2u} is known for many applications, for example as function of the distance, the presentation time and the luminance of the samples.

CEA5-3N

CIELAB modifications of the colour space and the colour-difference formula by line elements for different applications

Example line element in lightness direction of *Stiles (1946)*, who uses the luminance *L* instead of the tristimulus value *Y*.

$L^*(Y)$ = s ln[1+9Y] s=scaling factor [1a]

$d(L^*(Y)) / dY$ = 9s / [1+9Y] [2a]

For this derivation, and for the lightness threshold $d(L^*(Y))$=1 it is valid:

dY = [1+9Y] / 9s [3a]

For the normalization with the surround values Y_u=18, dY_u and $L^*(Y_u)$:

dY/dY_u = [1+9Y] / [1+9Y_u] [4a]

$L^*(Y) / L^*(Y_u)$ = ln[1+9Y] / ln[1+9Y_u] [5a]

For the CIELAB colour-space according to ISO/CIE 11664-4:2019

$L^*(Y) = s\,[Y/Y_u]^k - 16 = s_u[Y/Y_u]^k - 16$ s=116, k=1/3, Y_u=100, Y_u=18 [1b]

$d(L^*(Y)) / dY = s_u k\,[Y/Y_u]^{k-1}$ with s_u=116$[Y_u/Y_u]^k$=65,50, 1<=Y<=100 [2b]

For this derivation, and for the lightness threshold $d(L^*(Y))$=1 it is valid:

$dY = [Y/Y_u]^{1-k} / s_u k$ [3b]

$dY/dY_u = [Y/Y_u]^{1-k}$ [4b]

$L^*(Y) / L^*(Y_u) = \{s_u[Y/Y_u]^k - 16\} / \{s_u - 16\}$ [5b]

Line elements for the CIELAB-colour space are included in ISO/CIE 11664-4.

CEA5-3N

Colour-difference formula LABJND 1985 (JND=just noticeable difference)

$\Delta E^*_{JND} = \Delta E^*_{85} = A_0\,[\,(\Delta Y)^2 + (A_3\,\Delta a''\cdot Y)^2 + (A_4\,\Delta b''\cdot Y)^2\,]^{1/2} / (A_1 + A_2\cdot Y)$ [1]

$a = x/y \quad a_n = x_n/y_n \quad b = -0{,}4\,z/Y \quad b_n = -0{,}4\,z_n/y_n$ [2]

$a'' = a_n + (a - a_n) / (1 + 0{,}5\,|\,a - a_n\,|\,) \qquad$ n = D65 or A (background) [3]

$b'' = b_n + (b - b_n) / (1 + 0{,}5\,|\,b - b_n\,|\,)$ [4]

$Y = (Y_1 + Y_2)/2 \quad \Delta Y = Y_1 - Y_2 \quad \Delta a'' = a_1'' - a_2'' \quad \Delta b'' = b_1'' - b_2''$ [5]

A_1 = 0,0170 A_2 = 0,0058 [6]

A_3 = 1,0 A_4 = 1,8 A_0 = 1,5 background D65 [7]

A_3 = 1,0 A_4 = 1,7 A_0 = 1,0 background A [8]

Just noticeable difference (JND) in four colour directions

ΔY = const $(A_1 + A_2\cdot Y) / A_0$ in luminance direction *WN* [1a]

$\Delta a''\cdot Y$ = const $(A_1 + A_2\cdot Y) / (A_0\cdot A_3)$ in chromaticity direction *RG* [2a]

$\Delta b''\cdot Y$ = const $(A_1 + A_2\cdot Y) / (A_0\cdot A_4)$ in chromaticity direction *YB* [3a]

$\Delta c_{ab}''\cdot Y$ = const $(A_1 + A_2\cdot Y) / (A_0\cdot [A_3^2 + A_4^2]^{1/2})$ in chromaticity direction c_{ab} [4a]

CEA2J-3N

Colour-difference formula LABJND 1985 for near achromatic colours

$\Delta E^*_{JND} = \Delta E^*_{85} = A_0\,[\,(\Delta Y)^2 + (A_3\,\Delta a\cdot Y)^2 + (A_4\,\Delta b\cdot Y)^2\,]^{1/2} / (A_1 + A_2\cdot Y)$ [1]

$a = x/y \quad b = -0{,}4\,z/y$ [2]

$Y = (Y_1 + Y_2)/2 \quad \Delta Y = Y_1 - Y_2 \quad \Delta a = a_1 - a_2 \quad \Delta b = b_1 - b_2$ [3]

A_1 = 0,0170 A_2 = 0,0058 [4]

A_3 = 1,0 A_4 = 1,8 A_0 = 1,5 background D65 [5]

A_3 = 1,0 A_4 = 1,7 A_0 = 1,0 background A [6]

Just noticeable difference (JND) in three colour directions and line elements

$A_0\cdot\Delta Y = (A_1 + A_2\cdot Y)$ in luminance direction *WN* [1a]

$A_0\cdot\Delta a\cdot A_3\cdot Y = (A_1 + A_2\cdot Y)$ in chromaticity direction *RG* [2a]

$A_0\cdot\Delta b\cdot A_4\cdot Y = (A_1 + A_2\cdot Y)$ in chromaticity direction *YB* [3a]

$dE^*_{85,L} = \frac{\delta}{\delta Y} L^*_{85} = \frac{\delta}{\delta Y}[(A_0 / A_2)\cdot ln\,(A_1 + A_2\cdot Y)] = A_0\cdot dY/(A_1 + A_2\cdot Y)$ [4a]

$dE^*_{85,a} = \frac{\delta}{\delta a} a^*_{85} = \frac{\delta}{\delta a}[(A_0\cdot A_3\cdot Y\cdot a)/(A_1 + A_2\cdot Y)] = A_0\cdot da\cdot A_3\cdot Y/(A_1 + A_2\cdot Y)$ [5a]

$dE^*_{85,b} = \frac{\delta}{\delta b} b^*_{85} = \frac{\delta}{\delta b}[(A_0\cdot A_4\cdot Y\cdot b)/(A_1 + A_2\cdot Y)] = A_0\cdot db\cdot A_4\cdot Y/(A_1 + A_2\cdot Y)$ [6a]

CEA2J-3N