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Transformation between the *Iudd* (tristimulus and opponent values)
 Data see K. Richter, PhD thesis, University of Basel (Switzerland), 1969, page 8
 For the antagonistic spectral elementary colours
 $\lambda_{\text{blue}} = 475 \text{ nm}$, $\lambda_{\text{green}} = 502 \text{ nm}$, $\lambda_{\text{red}} = 574 \text{ nm}$, $\lambda_{\text{yellow}} = 494 \text{ nm}$
 the coordinates x_i ($i=1$ to 3) are used instead of modern coordinates $\tilde{t}$, $\tilde{a}$, $\tilde{b}$.
 Linear model equations between spectral colour values in both directions:

$$\begin{pmatrix} \tilde{x}(x) \\ \tilde{y}(x) \\ \tilde{z}(x) \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} \begin{pmatrix} \tilde{x}(\lambda) \\ \tilde{y}(\lambda) \\ \tilde{z}(\lambda) \end{pmatrix} = \begin{pmatrix} 0.0000 & 1.0000 & 0.0000 \\ 2.9797 & -2.6662 & -0.0960 \\ -0.4139 & 1.4571 & -2.4046 \end{pmatrix} \begin{pmatrix} \tilde{x}(\lambda) \\ \tilde{y}(\lambda) \\ \tilde{z}(\lambda) \end{pmatrix} \quad (1)$$

$$\begin{pmatrix} \tilde{x}(\lambda) \\ \tilde{y}(\lambda) \\ \tilde{z}(\lambda) \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} \tilde{x}(x) \\ \tilde{y}(x) \\ \tilde{z}(x) \end{pmatrix} = \begin{pmatrix} 0.9093 & 0.3338 & -0.0133 \\ 1.0000 & 0.0000 & 0.0000 \\ 0.4494 & -0.0574 & -0.4136 \end{pmatrix} \begin{pmatrix} \tilde{x}(x) \\ \tilde{y}(x) \\ \tilde{z}(x) \end{pmatrix} \quad (2)$$

The tristimulus values X , Y , Z and X , Y , Z need same transformations.
 The unnormalized purity data a and b_i are defined in LabMun96 as follows:
 $a = X_2/X_1 = x_2/x_1$, $b_i = X_2/X_i = x_2/x_i$, $x_i = 1 - x_2 - x_1$ (5)
 The unnormalized purity data a and b_i are defined in LabMun96 as follows:
 $a = (b_{12} - b_{23})x_1(b_{32} - b_{23}) + b_{23}y$
 $= (3.0757x - 2.5702z - 0.0860)y$ (6)
 $b_i = (b_{12} - b_{23})x_i(b_{32} - b_{23}) + b_{23}y$
 $= (1.9969x + 3.8617y - 2.4046z)$ (7)

Transformation between the *Judd* tristimulus and opponent values
 Data from K. Richter, PhD thesis, University of Basel (Switzerland), 1969, page 81

For the antagonistic spectral elementary colours
 $\lambda_M = 475$ nm, $\lambda_C = 502$ nm, $\lambda_A = 574$ nm, $\lambda_R = 494$ nm
 modern coordinates $\bar{l}$, $\bar{a}$, $\bar{b}$ are used instead of $\bar{x}_i$ ($i=1$ to 3).

Linear model equations between spectral colour values in both directions:

$$\begin{pmatrix} \bar{l}(x) \\ \bar{a}(x) \\ \bar{b}(x) \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} \begin{pmatrix} \bar{x}(x) \\ \bar{y}(x) \\ \bar{z}(x) \end{pmatrix} = \begin{pmatrix} 0.0000 & 1.0000 & 0.0000 \\ 2.9797 & -2.6662 & -0.0960 \\ -0.4139 & 1.4771 & -2.4046 \end{pmatrix} \begin{pmatrix} \bar{x}(x) \\ \bar{y}(x) \\ \bar{z}(x) \end{pmatrix} \quad (1)$$

$$\begin{pmatrix} \bar{x}(x) \\ \bar{y}(x) \\ \bar{z}(x) \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} \bar{l}(x) \\ \bar{a}(x) \\ \bar{b}(x) \end{pmatrix} = \begin{pmatrix} 0.9083 & 0.4338 & -0.0133 \\ 1.0000 & 0.0000 & 0.0000 \\ 0.4494 & -0.0574 & -0.4136 \end{pmatrix} \begin{pmatrix} \bar{l}(x) \\ \bar{a}(x) \\ \bar{b}(x) \end{pmatrix} \quad (2)$$

The tristimulus values L , A , B and X , Y , Z see the same transformations.

The unnormalized purity data a_i and b_i are defined in LabMun 1969 as follows:

$$a_i = X/Y = x/y \quad (3) \quad b_i = -Z/Y = -z/y \quad (4) \quad z = 1 - x - y \quad (5)$$

The purity data a_i and b_i are defined in LabMun 1969 as follows:

$$a_i = (b_{21} - b_{22} + b_{23}) - (b_{22} - b_{23}) + b_{23}/y$$

$$= (3.0757x - 2.5702y - 0.0960)/y \quad (6)$$

$$b_i = (b_{31} - b_{32} + b_{33}) - (b_{32} - b_{33}) + b_{33}/y$$

$$= (1.9906x - 3.8617y - 2.4046)/y \quad (7)$$

Table 6 shows the results from the *Judd* tristimulus and opponent values. Data seen K. Richter, PhD thesis, University of Basel (Switzerland), 1969, page 5.

elementary colour	dominant wavelength	<i>Judd</i> spectral tristimulus values			chromatic values	
		$\bar{x}(\lambda)$	$\bar{y}(\lambda)$	$\bar{z}(\lambda)$	RG $\bar{x}(\lambda)$	YB $\bar{y}(\lambda)$
blue	$\lambda_B=475$ nm	0.8267	0.9339	0.0017	0.0000	–
green	$\lambda_G=502$ nm	0.0107	0.0038	0.0005	–1.0000	0.0000
yellow	$\lambda_Y=574$ nm	0.1304	0.1124	0.2281	0.0000	1.0000
red	$\lambda_R=494c$ E.m	0.0028	0.3701	0.9238	–	– 0.0000

There are six equations to calculate the six constants: b_2 to b_{33}

$$\bar{x}(\lambda_B) = b_2 \bar{x}(\lambda_R) + b_3 \bar{x}(\lambda_Y) + b_4 \bar{x}(\lambda_G) + b_5 \bar{x}(\lambda_B) = 0 \quad \bar{x}(\lambda_G) = b_3 \bar{x}(\lambda_R) + b_4 \bar{x}(\lambda_Y) + b_5 \bar{x}(\lambda_G) + b_6 \bar{x}(\lambda_B) = 0$$

$$\bar{y}(\lambda_B) = b_2 \bar{y}(\lambda_R) + b_3 \bar{y}(\lambda_Y) + b_4 \bar{y}(\lambda_G) + b_5 \bar{y}(\lambda_B) = 1 \quad \bar{y}(\lambda_G) = b_3 \bar{y}(\lambda_R) + b_4 \bar{y}(\lambda_Y) + b_5 \bar{y}(\lambda_G) + b_6 \bar{y}(\lambda_B) = 1$$

$$\bar{z}(\lambda_B) = b_2 \bar{z}(\lambda_R) + b_3 \bar{z}(\lambda_Y) + b_4 \bar{z}(\lambda_G) + b_5 \bar{z}(\lambda_B) = 0 \quad \bar{z}(\lambda_G) = b_3 \bar{z}(\lambda_R) + b_4 \bar{z}(\lambda_Y) + b_5 \bar{z}(\lambda_G) + b_6 \bar{z}(\lambda_B) = 0$$

Together with the use of the standard equation: $\bar{x}(\lambda) = \bar{y}(\lambda)$

the equations between spectral opponent and tristimulus colour values are:

$$\frac{\bar{x}(\lambda)}{\bar{x}(\lambda_B)} = \frac{b_1}{b_2} \frac{\bar{x}(\lambda)}{\bar{x}(\lambda_R)} + \frac{b_3}{b_2} \frac{\bar{x}(\lambda)}{\bar{x}(\lambda_Y)} + \frac{b_4}{b_2} \frac{\bar{x}(\lambda)}{\bar{x}(\lambda_G)} + \frac{b_5}{b_2} \frac{\bar{x}(\lambda)}{\bar{x}(\lambda_B)} = 0.0000 \quad 1.0000 \quad \frac{\bar{y}(\lambda)}{\bar{y}(\lambda_B)}$$

$$\frac{\bar{y}(\lambda)}{\bar{y}(\lambda_B)} = \frac{b_1}{b_2} \frac{\bar{y}(\lambda)}{\bar{y}(\lambda_R)} + \frac{b_3}{b_2} \frac{\bar{y}(\lambda)}{\bar{y}(\lambda_Y)} + \frac{b_4}{b_2} \frac{\bar{y}(\lambda)}{\bar{y}(\lambda_G)} + \frac{b_5}{b_2} \frac{\bar{y}(\lambda)}{\bar{y}(\lambda_B)} = 2.9797 \quad -2.6662 \quad -0.0960 \quad \frac{\bar{z}(\lambda)}{\bar{z}(\lambda_B)}$$

$$\frac{\bar{z}(\lambda)}{\bar{z}(\lambda_B)} = \frac{b_1}{b_2} \frac{\bar{z}(\lambda)}{\bar{z}(\lambda_R)} + \frac{b_3}{b_2} \frac{\bar{z}(\lambda)}{\bar{z}(\lambda_Y)} + \frac{b_4}{b_2} \frac{\bar{z}(\lambda)}{\bar{z}(\lambda_G)} + \frac{b_5}{b_2} \frac{\bar{z}(\lambda)}{\bar{z}(\lambda_B)} = -0.4139 \quad 1.4571 \quad -2.4046 \quad \frac{\bar{z}(\lambda)}{\bar{z}(\lambda_B)}$$

Remark: The weighting ratio in the RG and YB direction is 2.8:1 or 1/0.3571.

Table 2. Transformation between the *Judd* tristimulus and opponent values
Data from K. Richter, PhD thesis, University of Basel (Switzerland), 1969, page 3

elementary colour	dominant wavelength	Judd spectral tristimulus values			chromatic value	
		$\bar{x}(\lambda)$	$\bar{y}(\lambda)$	$\bar{z}(\lambda)$	RG $\bar{a}(\lambda)$	YB $\bar{b}(\lambda)$
blue	$\lambda_b = 475$ nm	0.8267	0.9339	0.0017	-0.0000	-
green	$\lambda_g = 502$ nm	0.0107	0.0038	0.0005	-1.0000	0.0000
yellow	$\lambda_y = 574$ nm	0.1304	0.1124	0.2928	0.0000	1.0000
red	$\lambda_r = 494.6$ nm	0.0028	0.3701	0.0231	-	-0.0000

There are six equations to calculate the six constants: b_1 to b_6

$$\begin{aligned} \bar{a}(\lambda_b) &= b_1 \bar{x}(\lambda_b) + b_2 \bar{y}(\lambda_b) + b_3 \bar{z}(\lambda_b) & \bar{b}(\lambda_b) &= b_4 \bar{x}(\lambda_b) + b_5 \bar{y}(\lambda_b) + b_6 \bar{z}(\lambda_b) \\ \bar{a}(\lambda_g) &= b_1 \bar{x}(\lambda_g) + b_2 \bar{y}(\lambda_g) + b_3 \bar{z}(\lambda_g) & \bar{b}(\lambda_g) &= b_4 \bar{x}(\lambda_g) + b_5 \bar{y}(\lambda_g) + b_6 \bar{z}(\lambda_g) \\ \bar{a}(\lambda_y) &= b_1 \bar{x}(\lambda_y) + b_2 \bar{y}(\lambda_y) + b_3 \bar{z}(\lambda_y) & \bar{b}(\lambda_y) &= b_4 \bar{x}(\lambda_y) + b_5 \bar{y}(\lambda_y) + b_6 \bar{z}(\lambda_y) \\ \bar{a}(\lambda_r) &= b_1 \bar{x}(\lambda_r) + b_2 \bar{y}(\lambda_r) + b_3 \bar{z}(\lambda_r) & \bar{b}(\lambda_r) &= b_4 \bar{x}(\lambda_r) + b_5 \bar{y}(\lambda_r) + b_6 \bar{z}(\lambda_r) \end{aligned}$$

Together with the use of the standard equation: $\bar{l}(\lambda) = \bar{y}(\lambda)$

the equations between spectral opponent and tristimulus colour values are:

$$\begin{aligned} \bar{l}(\lambda) &= \begin{pmatrix} b_1 & b_2 & b_3 \end{pmatrix} \begin{pmatrix} \bar{x}(\lambda) \\ \bar{y}(\lambda) \\ \bar{z}(\lambda) \end{pmatrix} & \bar{a}(\lambda) &= \begin{pmatrix} 0.0000 & 1.0000 & 0.0000 \end{pmatrix} \begin{pmatrix} \bar{x}(\lambda) \\ \bar{y}(\lambda) \\ \bar{z}(\lambda) \end{pmatrix} \\ \bar{b}(\lambda) &= \begin{pmatrix} b_4 & b_5 & b_6 \end{pmatrix} \begin{pmatrix} \bar{x}(\lambda) \\ \bar{y}(\lambda) \\ \bar{z}(\lambda) \end{pmatrix} & \bar{b}(\lambda) &= \begin{pmatrix} 0.2977 & -2.6662 & -0.0960 \end{pmatrix} \begin{pmatrix} \bar{x}(\lambda) \\ \bar{y}(\lambda) \\ \bar{z}(\lambda) \end{pmatrix} \end{aligned}$$

Remark: The weighting ratio in the RG and YB direction is 2.8:1 or 1:0.3571.

Transformation between the *Judd* trisimultaneous and opponent values

Data see K. Richter, PhD thesis, University of Basel (Switzerland), 1969, page 8

For the antagonistic spectral elementary colours

$\lambda_1 = 475 \text{ nm}$, $\lambda_2 = 502 \text{ nm}$, $\lambda_3 = 574 \text{ nm}$, $\lambda_4 = 494 \text{ nm}$

the coordinates x_i ($i=1$ to 3) are used instead of modern coordinates $\bar{x}$, $\bar{y}$, $\bar{z}$

Linear model equations between spectral colour values in both directions:

$$\begin{pmatrix} \bar{x}(x) \\ \bar{y}(x) \\ \bar{z}(x) \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} \begin{pmatrix} x(x) \\ y(x) \\ z(x) \end{pmatrix} = \begin{pmatrix} 0.0000 & 1.0000 & 0.0000 \\ 2.9379 & -2.6662 & -0.0960 \\ -0.4139 & 1.4971 & -2.4046 \end{pmatrix} \begin{pmatrix} x(x) \\ y(x) \\ z(x) \end{pmatrix} \quad (1)$$

$$\begin{pmatrix} x(x) \\ y(x) \\ z(x) \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} \bar{x}(x) \\ \bar{y}(x) \\ \bar{z}(x) \end{pmatrix} = \begin{pmatrix} 0.9093 & 0.3338 & -0.1333 \\ 1.0000 & 0.0000 & 0.0000 \\ 0.4494 & -0.0574 & -0.4136 \end{pmatrix} \begin{pmatrix} \bar{x}(x) \\ \bar{y}(x) \\ \bar{z}(x) \end{pmatrix} \quad (2)$$

The trisimultaneous values X_1, X_2, X_3 and Y, Z need the same transformations.

The normalized primary data x_1, x_2 and b_1 are defined in LabMun99 as follows:

$$x_1 = a_1 \cdot n_1 \cdot X_1 / X_1 \cdot Y = x_1 \cdot x_2 \quad (3) \quad x_2 = X_2 / X_1 \cdot Y = x_2 \quad (4) \quad x_1 \cdot x_2 = x_3 \quad (5)$$

The normalized primary data y_1, y_2 and b_2 are defined in LabMun99 as follows:

$$y_1 = a_1 \cdot n_1 \cdot (b_{21} \cdot x_1 + b_{22} \cdot x_2 + b_{23} \cdot y_1) / b_{21} \cdot y_1 \\ = -2.8 / (3.0757 \cdot x_1 - 2.5702 \cdot y_1 - 0.0960 y_2) \\ = (8.6119 \cdot y_1 - 7.1965 y_2 - 0.2689 y_3) \quad (6)$$

$$b_2 = (1 / (b_{31} \cdot x_1 + b_{32} \cdot x_2 + b_{33} \cdot y_1)) / b_{31} \cdot y_1 \\ = (1.9906 \cdot x_1 + 3.8617 \cdot y_2 - 2.4046 y_3) \quad (7)$$

Transformation between the Judd trisulimulus and opponent values
 Data see K. Richter, PhD thesis, University of Basel (Switzerland), 1969, page 8:
 For the antagonistic spectral elementary colours
 $\lambda_B = 475 \text{ nm}$, $\lambda_C = 502 \text{ nm}$, $\lambda_Y = 574 \text{ nm}$, $\lambda_R = 494 \text{ nm}$
 modern coordinates l , a , b are used instead of x_i ($i=1$ to 3).
 Linear model equations between spectral colour values in both directions:
 (1)
$$\begin{pmatrix} \tilde{l}(i) \\ \tilde{a}(i) \\ \tilde{b}(i) \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} \begin{pmatrix} \tilde{x}(i) \\ \tilde{y}(i) \\ \tilde{z}(i) \end{pmatrix} = \begin{pmatrix} 0.0000 & 1.0000 & 0.0000 \\ 2.9797 & -2.6662 & -0.0960 \\ -0.4939 & 1.3571 & -2.4046 \end{pmatrix} \begin{pmatrix} \tilde{x}(i) \\ \tilde{y}(i) \\ \tilde{z}(i) \end{pmatrix}$$

 (2)
$$\begin{pmatrix} \tilde{x}(i) \\ \tilde{y}(i) \\ \tilde{z}(i) \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} \tilde{l}(i) \\ \tilde{a}(i) \\ \tilde{b}(i) \end{pmatrix} = \begin{pmatrix} 1.0000 & 0.0000 & 0.0000 \\ 0.4494 & -0.0574 & -0.4136 \\ -0.9131 & 0.4378 & -0.0133 \end{pmatrix} \begin{pmatrix} \tilde{l}(i) \\ \tilde{a}(i) \\ \tilde{b}(i) \end{pmatrix}$$

 The trisulimulus values l , A , B and Y , z need the same transformations.
 The normalized purity data a and b_n are defined in LabMun 1969 as follows:
 $a_n = n_{xy} / n_{YX} = n_{xy} / n_{xy} = 2.8$ (3) $b_n = b_{xy} / X = Y / Y = 1$ (5)
 The normalized purity data a and b_n are defined in LabMun 1969 as follows:
 $a_n = n_{xy} [(b_{12} - b_{23})x + (b_{22} - b_{23})y + b_{23}y] /$
 $= 2.8 [3.0757x - 2.5702y - 0.0960y] /$
 $= (8.6119x - 7.9505y - 0.2688y)$ (6)
 $b_n = [(b_{12} - b_{23})x + (b_{22} - b_{23})y + b_{23}y] /$
 $= (1.9906x + 3.8617y - 2.4046y)$ (7)

Transformation between the *Judd* purity and chromaticity
 Data see K. Richter, PhD thesis, University of Basel (Switzerland), 1969, page 8:
 For the antagonistic spectral elementary colours
 $\lambda_m = 475 \text{ nm}$, $\lambda_c = 502 \text{ nm}$, $\lambda_y = 574 \text{ nm}$, $\lambda_r = 494 \text{ nm}$
 the coordinates x_i ($i=1$ to 3) are used instead of normal coordinates $\bar{t}$, $\bar{a}$, $\bar{b}$.
 Relation between normal and spectral colour values:

$$\begin{pmatrix} \bar{x}(i) \\ \bar{y}(i) \\ \bar{z}(i) \end{pmatrix} \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} x(i) \\ y(i) \\ z(i) \end{pmatrix} = \begin{pmatrix} 0.9093 & 0.3338 & -0.1033 \\ 1.0000 & 0.0000 & 0.0000 \\ 0.4449 & -0.0574 & -0.4136 \end{pmatrix} \begin{pmatrix} x(i) \\ y(i) \\ z(i) \end{pmatrix} \quad (2)$$

 The tristimulus values X , Y , Z and x , y , z mean the same transformations.
 The normal coordinates $\bar{t}$, $\bar{a}$, $\bar{b}$ and t , a , b are defined in LabMuns 1969 as follows:
 $x = X/Y$, $Y/X = x/y$, $z = Z/Y$, $Y/Z = y/z$, $x/z = X/Z$, $z/y = Z/Y$ (3)
 Use of equations (2) to (5) leads to:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad (6) \text{ and with } x=X/(Y+Z) \text{ and } y=Y/(X+Z) \quad (7)$$

$$x[a_{11}+a_{12}+a_{13}x, a_{21}+a_{22}+a_{23}x, a_{31}+a_{32}+a_{33}x] \quad (8)$$

$$y[a_{11}+a_{12}+a_{13}y, a_{21}+a_{22}+a_{23}y, a_{31}+a_{32}+a_{33}y] \quad (9)$$

$$z[a_{11}+a_{12}+a_{13}z, a_{21}+a_{22}+a_{23}z, a_{31}+a_{32}+a_{33}z] \quad (10)$$

$$= (0.9093 + 0.3338a_{11} - 0.1033b_{11}) (a_{11} + a_{12} + a_{13}x) (a_{21} + a_{22} + a_{23}y) (a_{31} + a_{32} + a_{33}z) \\ = (1.0000 + 0.0000a_{11} + 0.0000b_{11}) (2.3587 + 0.2764a_{11} - 0.4269b_{11}) \\ = (1.0000 + 0.0000a_{11} + 0.0000b_{11}) (2.3587 + 0.2764a_{11} - 0.4269b_{11}) \quad (11)$$

Transformation between the *Iadd* purity and chromaticity
 Data from K. Richter, PhD thesis, University of Basel (Switzerland), 1969, page 83.
 For the antagonistic spectral elementary colours
 $\lambda_B = 475 \text{ nm}$, $\lambda_C = 502 \text{ nm}$, $\lambda_Y = 574 \text{ nm}$, $\lambda_R = 494 \text{ nm}$
 modern coordinates l , a , b are used instead of x_i ($i = 1$ to 3).
 Relation between opponent and spectral colour values:

$$\begin{pmatrix} \tilde{x}(l) \\ \tilde{y}(l) \\ \tilde{z}(l) \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} l(l) \\ a(l) \\ b(l) \end{pmatrix} = \begin{pmatrix} 0.9093 & 0.3338 & -0.0133 \\ 1.0000 & 0.0000 & 0.0000 \\ 0.4456 & -0.0574 & 0.4136 \end{pmatrix} \begin{pmatrix} l(l) \\ a(l) \\ b(l) \end{pmatrix} \quad (2)$$

 The tristimulus values L , A , B and X , Y , Z need the same transformations.
 The monochromeized purity and chromaticity, u and v , are defined in LabMun 1969 as follows:

$$x = X/Y = x/2 \quad u = x/2 - y/2 \quad v = z/2 - y/2 \quad z = 1 - x - y \quad (5)$$

 Use of equations (2) to (5) leads to:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} \end{pmatrix} \begin{pmatrix} L \\ A \\ B \\ u \\ v \\ w \end{pmatrix} \quad (6) \text{ and with } x = X/(X+Y+Z) \text{ and } y = Y/(X+Y+Z) \quad (7)$$

$$x = (a_{11} + a_{12}u + a_{13}v, [a_{14} + a_{15}u + a_{16}v], [a_{21} + a_{22}u + a_{23}v], [a_{24} + a_{25}u + a_{26}v], [a_{31} + a_{32}u + a_{33}v], [a_{34} + a_{35}u + a_{36}v]) \quad (8)$$

$$x = (\alpha_1 + \alpha_{12}u + \alpha_{13}v, (\alpha_4 + \alpha_{15}u + \alpha_{16}v), (\alpha_2 + \alpha_{22}u + \alpha_{23}v), (\alpha_5 + \alpha_{25}u + \alpha_{26}v), (\alpha_3 + \alpha_{32}u + \alpha_{33}v), (\alpha_6 + \alpha_{35}u + \alpha_{36}v)) \quad (9)$$

$$y = (\alpha_{21} + \alpha_{22}u + \alpha_{23}v), (\alpha_4 + \alpha_{15}u + \alpha_{16}v), (\alpha_3 + \alpha_{32}u + \alpha_{33}v), (\alpha_5 + \alpha_{25}u + \alpha_{26}v), (\alpha_1 + \alpha_{12}u + \alpha_{13}v), (\alpha_6 + \alpha_{35}u + \alpha_{36}v)) \quad (10)$$

$$v = (1.000 + 0.000u + 0.000v, (2.3587 + 0.2764u - 0.4269v), (\alpha_3 + \alpha_{32}u + \alpha_{33}v), (\alpha_5 + \alpha_{25}u + \alpha_{26}v), (\alpha_1 + \alpha_{12}u + \alpha_{13}v), (\alpha_6 + \alpha_{35}u + \alpha_{36}v)) \quad (11)$$

Relation between the radial purity p_r of Chroma 2 and tristimulus value Y Data see K. Richter, PhD thesis, University of Basel (Switzerland), 1969, page 1

V	Y	x_M	y_M	$a_{L,M}$	$b_{L,M}$	x_U	y_U	$a_{L,U}$	$b_{L,U}$	$P_{r,M}$
1,000	1,210	0,315	0,296	1,060	-2,130	0,314	0,324	0,310	-1,623	0,298
5,000	19,770	0,314	0,323	0,340	-1,640	0,314	0,324	0,310	-1,623	0,279
9,000	78,660	0,314	0,328	0,230	-1,550	0,314	0,324	0,310	-1,623	0,500

$\log p_r$ P_r radial purity of Chroma 2 hue circles.

$x = (0.9093 + 0.1192a_r - 0.0133b_r) Y (2.3587 + 0.0987a_r - 0.4269b_r)$
 $y = (1.000 + 0.000a_r + 0.000b_r) Y (2.3587 + 0.0987a_r - 0.4269b_r)$
 $P_r(F, M_Y) = P_r(F, M_{Y_{-1}}) Y(F) = -0.341$
 $n = -0.341$

Relation between the radial purity p_r of $\text{Charm}2$ and trisimultaneous value Y											
Data see K. Richter, PhD thesis, University of Basel (Switzerland), 1969, page 2											
Value Y	sample data				surround data				p_{radial}		
	x_{11}	x_{12}	a_{11}	b_{11}	x_{21}	y_{21}	a_{21}	b_{21}			
1,000	1,210	3,315	0,296	1,060	-1,130	0,314	0,324	0,310	-1,623	2,298	
2,000	1,310	3,314	0,308	0,780	-1,900	0,314	0,324	0,310	-1,623	1,462	
3,000	6,560	3,314	0,315	0,560	-1,780	0,314	0,324	0,310	-1,623	1,244	
4,000	12,700	3,314	0,320	0,430	-1,690	0,314	0,324	0,310	-1,623	0,988	
5,000	19,980	3,314	0,323	0,400	-1,640	0,314	0,324	0,310	-1,623	0,788	
6,000	27,300	3,314	0,324	0,410	-1,630	0,314	0,324	0,310	-1,623	0,714	
7,000	34,050	3,314	0,325	0,390	-1,610	0,314	0,324	0,310	-1,623	0,609	
8,000	40,360	3,314	0,326	0,270	-1,590	0,314	0,324	0,310	-1,623	0,609	
9,000	59,100	3,314	0,327	0,250	-1,570	0,314	0,324	0,310	-1,623	0,540	
9,000	78,660	3,314	0,328	0,230	-1,550	0,314	0,324	0,310	-1,623	0,540	

For experimental (Ex) surround (bold data), see *Newhall*, JOSA 30 (1940) p. 622.

$$x = (x_{11}, x_{12}, x_{21}, x_{22}) / (a_{11}, a_{12}, a_{21}, a_{22}) = (0.9093, 0.1192a, 0.0133b, 0.23587 - 0.0987a, -0.4269b) \quad (1)$$

$$y = (y_{21}, y_{22}, y_{31}, y_{32}) / (a_{21}, a_{22}, a_{31}, a_{32}) = (1.000 + 0.000a, 0.000b, 0.23587 - 0.0987a, -0.4269b) \quad (2)$$

Transformation between the *Iudd* purity and chromaticity
 Data see K. Richter, PhD thesis, University of Basel (Switzerland), 1967, page 8:
 For the antagonistic spectral elementary colours:
 $\lambda_{\text{red}} = 475 \text{ nm}$, $\lambda_{\text{gr}} = 502 \text{ nm}$, $\lambda_{\text{bl}} = 574 \text{ nm}$, $\lambda_{\text{vbl}} = 494 \text{ nm}$
 the coordinates $\bar{x}_i$ ($i=1$ to 3) are used instead of modern coordinates $\bar{t}$, $\bar{a}$, $\bar{b}$.
 Relation between original and spectral colour values:

$$\begin{pmatrix} \bar{x}(i) \\ \bar{y}(i) \\ \bar{z}(i) \end{pmatrix} \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} \bar{x}(i) \\ \bar{y}(i) \\ \bar{z}(i) \end{pmatrix} = \begin{pmatrix} 0.9093 & 0.3338 & -0.1033 \\ 1.0000 & 0.0000 & 0.0000 \\ 0.4449 & -0.4449 & 0.4136 \end{pmatrix} \begin{pmatrix} \bar{x}(i) \\ \bar{y}(i) \\ \bar{z}(i) \end{pmatrix} \quad (2)$$

 The tristimulus values X , Y , Z , X_1 , X_2 , Y_1 , Y_2 need the same transformation.
 The normalized primary data x_i , y_i , z_i are defined in LabMun 1969 as follows:
 $a_i = n_i a_{\text{red}} n_{\text{gr}} n_{\text{vbl}} X_i / X_1$; $n_{\text{red}} = 2.8$ $b_i = n_i b_{\text{red}} X_i / X_1$; $c_i = n_i c_{\text{red}} X_i / X_1$ $d_i = 1 - x_i - y_i$ $e_i = 1 - x_i - y_i$ $f_i = 1 - x_i - y_i$ $g_i = 1 - x_i - y_i$ $h_i = 1 - x_i - y_i$ $i_i = 1 - x_i - y_i$ $j_i = 1 - x_i - y_i$ $k_i = 1 - x_i - y_i$ $l_i = 1 - x_i - y_i$ $m_i = 1 - x_i - y_i$ $n_i = 1 - x_i - y_i$ $o_i = 1 - x_i - y_i$ $p_i = 1 - x_i - y_i$ $q_i = 1 - x_i - y_i$ $r_i = 1 - x_i - y_i$ $s_i = 1 - x_i - y_i$ $t_i = 1 - x_i - y_i$ $u_i = 1 - x_i - y_i$ $v_i = 1 - x_i - y_i$ $w_i = 1 - x_i - y_i$ $x_i = 1 - x_i - y_i$ $y_i = 1 - x_i - y_i$ $z_i = 1 - x_i - y_i$ $x_1 = 1 - x_1 - y_1$ $y_1 = 1 - x_1 - y_1$ $z_1 = 1 - x_1 - y_1$ $x_2 = 1 - x_2 - y_2$ $y_2 = 1 - x_2 - y_2$ $z_2 = 1 - x_2 - y_2$ $x_{\text{red}} = 1 - x_{\text{red}} - y_{\text{red}}$ $y_{\text{red}} = 1 - x_{\text{red}} - y_{\text{red}}$ $z_{\text{red}} = 1 - x_{\text{red}} - y_{\text{red}}$ $x_{\text{gr}} = 1 - x_{\text{gr}} - y_{\text{gr}}$ $y_{\text{gr}} = 1 - x_{\text{gr}} - y_{\text{gr}}$ $z_{\text{gr}} = 1 - x_{\text{gr}} - y_{\text{gr}}$ $x_{\text{bl}} = 1 - x_{\text{bl}} - y_{\text{bl}}$ $y_{\text{bl}} = 1 - x_{\text{bl}} - y_{\text{bl}}$ $z_{\text{bl}} = 1 - x_{\text{bl}} - y_{\text{bl}}$ $x_{\text{vbl}} = 1 - x_{\text{vbl}} - y_{\text{vbl}}$ $y_{\text{vbl}} = 1 - x_{\text{vbl}} - y_{\text{vbl}}$ $z_{\text{vbl}} = 1 - x_{\text{vbl}} - y_{\text{vbl}}$ $x_{\text{red}} = 1 - x_{\text{red}} - y_{\text{red}}$ $y_{\text{red}} = 1 - x_{\text{red}} - y_{\text{red}}$ $z_{\text{red}} = 1 - x_{\text{red}} - y_{\text{red}}$ $x_{\text{gr}} = 1 - x_{\text{gr}} - y_{\text{gr}}$ $y_{\text{gr}} = 1 - x_{\text{gr}} - y_{\text{gr}}$ $z_{\text{gr}} = 1 - x_{\text{gr}} - y_{\text{gr}}$ $x_{\text{bl}} = 1 - x_{\text{bl}} - y_{\text{bl}}$ $y_{\text{bl}} = 1 - x_{\text{bl}} - y_{\text{bl}}$ $z_{\text{bl}} = 1 - x_{\text{bl}} - y_{\text{bl}}$ $x_{\text{vbl}} = 1 - x_{\text{vbl}} - y_{\text{vbl}}$ $y_{\text{vbl}} = 1 - x_{\text{vbl}} - y_{\text{vbl}}$ $z_{\text{vbl}} = 1 - x_{\text{vbl}} - y_{\text{vbl}}$ $x_{\text{red}} = 1 - x_{\text{red}} - y_{\text{red}}$ $y_{\text{red}} = 1 - x_{\text{red}} - y_{\text{red}}$ $z_{\text{red}} = 1 - x_{\text{red}} - y_{\text{red}}$ $x_{\text{gr}} = 1 - x_{\text{gr}} - y_{\text{gr}}$ $y_{\text{gr}} = 1 - x_{\text{gr}} - y_{\text{gr}}$ $z_{\text{gr}} = 1 - x_{\text{gr}} - y_{\text{gr}}$ $x_{\text{bl}} = 1 - x_{\text{bl}} - y_{\text{bl}}$ $y_{\text{bl}} = 1 - x_{\text{bl}} - y_{\text{bl}}$ $z_{\text{bl}} = 1 - x_{\text{bl}} - y_{\text{bl}}$ $x_{\text{vbl}} = 1 - x_{\text{vbl}} - y_{\text{vbl}}$ $y_{\text{vbl}} = 1 - x_{\text{vbl}} - y_{\text{vbl}}$ $z_{\text{vbl}} = 1 - x_{\text{vbl}} - y_{\text{vbl}}$ $x_{\text{red}} = 1 - x_{\text{red}} - y_{\text{red}}$ $y_{\text{red}} = 1 - x_{\text{red}} - y_{\text{red}}$ $z_{\text{red}} = 1 - x_{\text{red}} - y_{\text{red}}$ $x_{\text{gr}} = 1 - x_{\text{gr}} - y_{\text{gr}}$ $y_{\text{gr}} = 1 - x_{\text{gr}} - y_{\text{gr}}$ $z_{\text{gr}} = 1 - x_{\text{gr}} - y_{\text{gr}}$ $x_{\text{bl}} = 1 - x_{\text{bl}} - y_{\text{bl}}$ $y_{\text{bl}} = 1 - x_{\text{bl}} - y_{\text{bl}}$ $z_{\text{bl}} = 1 - x_{\text{bl}} - y_{\text{bl}}$ $x_{\text{vbl}} = 1 - x_{\text{vbl}} - y_{\text{vbl}}$ $y_{\text{vbl}} = 1 - x_{\text{vbl}} - y_{\text{vbl}}$ $z_{\text{vbl}} = 1 - x_{\text{vbl}} - y_{\text{vbl}}$ $x_{\text{red}} = 1 - x_{\text{red}} - y_{\text{red}}$ $y_{\text{red}} = 1 - x_{\text{red}} - y_{\text{red}}$ $z_{\text{red}} = 1 - x_{\text{red}} - y_{\text{red}}$ $x_{\text{gr}} = 1 - x_{\text{gr}} - y_{\text{gr}}$ $y_{\text{gr}} = 1 - x_{\text{gr}} - y_{\text{gr}}$ $z_{\text{gr}} = 1 - x_{\text{gr}} - y_{\text{gr}}$ $x_{\text{bl}} = 1 - x_{\text{bl}} - y_{\text{bl}}$ $y_{\text{bl}} = 1 - x_{\text{bl}} - y_{\text{bl}}$ $z_{\text{bl}} = 1 - x_{\text{bl}} - y_{\text{bl}}$ $x_{\text{vbl}} = 1 - x_{\text{vbl}} - y_{\text{vbl}}$ $y_{\text{vbl}} = 1 - x_{\text{vbl}} - y_{\text{vbl}}$ $z_{\text{vbl}} = 1 - x_{\text{vbl}} - y_{\text{vbl}}$ $x_{\text{red}} = 1 - x_{\text{red}} - y_{\text{red}}$ $y_{\text{red}} = 1 - x_{\text{red}} - y_{\text{red}}$ $z_{\text{red}} = 1 - x_{\text{red}} - y_{\text{red}}$ $x_{\text{gr}} = 1 - x_{\text{gr}} - y_{\text{gr}}$ $y_{\text{gr}} = 1 - x_{\text{gr}} - y_{\text{gr}}$ $z_{\text{gr}} = 1 - x_{\text{gr}} - y_{\text{gr}}$ $x_{\text{bl}} = 1 - x_{\text{bl}} - y_{\text{bl}}$ $y_{\text{bl}} = 1 - x_{\text{bl}} - y_{\text{bl}}$ $z_{\text{bl}} = 1 - x_{\text{bl}} - y_{\text{bl}}$ $x_{\text{vbl}} = 1 - x_{\text{vbl}} - y_{\text{vbl}}$ $y_{\text{vbl}} = 1 - x_{\text{vbl}} - y_{\text{vbl}}$ $z_{\text{vbl}} = 1 - x_{\text{vbl}} - y_{\text{vbl}}$ $x_{\text{red}} = 1 - x_{\text{red}} - y_{\text{red}}$ $y_{\text{red}} = 1 - x_{\text{red}} - y_{\text{red}}$ $z_{\text{red}} = 1 - x_{\text{red}} - y_{\text{red}}$ $x_{\text{gr}} = 1 - x_{\text{gr}} - y_{\text{gr}}$ $y_{\text{gr}} = 1 - x_{\text{gr}} - y_{\text{gr}}$ $z_{\text{gr}} = 1 - x_{\text{gr}} - y_{\text{gr}}$ $x_{\text{bl}} = 1 - x_{\text{bl}} - y_{\text{bl}}$ $y_{\text{bl}} = 1 - x_{\text{bl}} - y_{\text{bl}}$ $z_{\text{bl}} = 1 - x_{\text{bl}} - y_{\text{bl}}$ $x_{\text{vbl}} = 1 - x_{\text{vbl}} - y_{\text{vbl}}$ $y_{\text{vbl}} = 1 - x_{\text{vbl}} - y_{\text{vbl}}$ $z_{\text{vbl}} = 1 - x_{\text{vbl}} - y_{\text{vbl}}$ $x_{\text{red}} = 1 - x_{\text{red}} - y_{\text{red}}$ $y_{\text{red}} = 1 - x_{\text{red}} - y_{\text{red}}$ $z_{\text{red}} = 1 - x_{\text{red}} - y_{\text{red}}$ $x_{\text{gr}} = 1 - x_{\text{gr}} - y_{\text{gr}}$ $y_{\text{gr}} = 1 - x_{\text{gr}} - y_{\text{gr}}$ $z_{\text{gr}} = 1 - x_{\text{gr}} - y_{\text{gr}}$ $x_{\text{bl}} = 1 - x_{\text{bl}} - y_{\text{bl}}$ $y_{\text{bl}} = 1 - x_{\text{bl}} - y_{\text{bl}}$ $z_{\text{bl}} = 1 - x_{\$

Transformation between the *Iudd* purity and chromaticity
 Data from K. Richter, PhD thesis, University of Basel (Switzerland), 1969, page 83.
 For the antagonistic spectral elementary colours
 $\lambda_B = 475 \text{ nm}$, $\lambda_C = 502 \text{ nm}$, $\lambda_Y = 574 \text{ nm}$, $\lambda_R = 494 \text{ nm}$
 modern coordinates l , a , b are used instead of $\bar{x}_1$ ($i = 1$ to 3).
 Relation between opponent and spectral colour values:

$$\begin{pmatrix} \bar{x}(l) \\ \bar{y}(l) \\ \bar{z}(l) \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} l(l) \\ a(l) \\ b(l) \end{pmatrix} = \begin{pmatrix} 0.9093 & 0.3338 & -0.0133 \\ 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0574 & 0.4136 \end{pmatrix} \begin{pmatrix} l(l) \\ a(l) \\ b(l) \end{pmatrix} \quad (2)$$

 The tristimulus values L , A , B and X , Y , Z defined in LabMUN 1969 as follows:
 The normalized purity n and h are defined as
 $a_n = n a_n$, $b_n = Y - Y_n$; $n = 2-8$ (3) $b_n = b_n - Z - Z_n$ (4) $z = 1 - x - y$ (5)
 Use of equations (2) to (5) leads to:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{13} & b_{13} & L \\ a_{21} & a_{22} & a_{23} & a_{23} & b_{23} & L \\ a_{31} & a_{32} & a_{33} & a_{33} & b_{33} & L \end{pmatrix} \begin{pmatrix} L \\ A \\ B \\ X \\ Y \\ Z \end{pmatrix} \quad (6) \quad \text{with } x = X/(X+Y+Z) \text{ and } y = Y/(X+Y+Z) \quad (7)$$

$$y = [a_{11} a_{21} a_{31} n_a, a_{12} a_{22} a_{32} n_a, a_{13} a_{23} a_{33} n_a, (a_{11} a_{21} a_{31} + a_{12} a_{22} a_{32} + a_{13} a_{23} a_{33}) n_a, (a_{11} a_{21} a_{31} + a_{12} a_{22} a_{32} + a_{13} a_{23} a_{33}) n_a, (a_{11} a_{21} a_{31} + a_{12} a_{22} a_{32} + a_{13} a_{23} a_{33}) n_a] \quad (8)$$

$$x = (\alpha_1 + \alpha_2 a_{13} + \alpha_3 b_{13}) / (\alpha_1 + \alpha_2 a_{13} + \alpha_3 b_{13} + 0.9093 + 0.1192 a_{13} + 0.0133 b_{13} + (Z - Z_{387}) / (0.987 a_{13} - 0.426 b_{13})) \quad (9)$$

$$y = (\alpha_1 + \alpha_2 a_{13} + \alpha_3 b_{13}) / (\alpha_1 + \alpha_2 a_{13} + \alpha_3 b_{13} + 1.000 + 0.000 a_{13} + 0.000 b_{13} + (Z - Z_{387}) / (0.987 a_{13} - 0.426 b_{13})) \quad (10)$$

[illegible]

Judd spectral tristimulus and chromatic values $\bar{l}(\lambda)$, $\bar{a}(\lambda)$, $\bar{b}(\lambda)$
 Data see K. Richter, PhD thesis, University of Basel (Switzerland), 1969, page 2

The weighting constants in the RG (a) and TB (b) direction are $n_a=2,8$, $n_b=1$.

TUB-test chart DE16; *JUDD* observer, colorimetric calculations of chromatic values
Data see *K. Richter*, PhD thesis, University of Basel (Switzerland), 1969