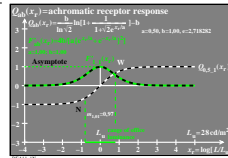
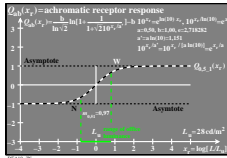


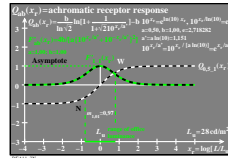
**Achromatic receptor-response function**  
 $Q_{ab}(x_r/a)$  for  $a=0.5$  and  $b=1.0$   
 with  $x_r = \log [L/L_u]$  ( $L =$  test luminance)  
 $L_u =$  surround luminance  
 $Q_{ab}(x_r/a) = \frac{b}{\ln \sqrt{2}} \ln \left[ \frac{1}{1 + \sqrt{2} e^{(x_r/a)}} \right] - b$   
**function values for  $b=1$  and any  $a>0$ :**  
 $Q_{a1}(x_r/a \rightarrow -\infty) = -1$   $x = \log L, u = \log L_u$   
 $Q_{a1}(x_r/a = 0) = 0$   $x_r = \log [L/L_u]$   
 $Q_{a1}(x_r/a \rightarrow +\infty) = +1$   $-x = u$



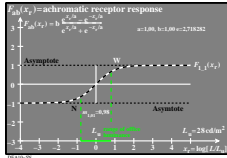
**Mathematical equations of hyperbel functions**  
 See: Papula, L., (2003), *Mathematische Formelsammlung, Vieweg*  
 $F(x) = \tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{u(x)}{v(x)}$  [1]  
 $F'(x) = \frac{u'(x)v(x) - u(x)v'(x)}{v^2(x)} = \frac{v^2(x) - u^2(x)}{v^2(x)}$  [2]  
 $F'(x) = \frac{[e^x + e^{-x}][e^x + e^{-x}] - [e^x - e^{-x}][e^x - e^{-x}]}{[e^x + e^{-x}]^2}$  [3]  
 $F'(x) = \frac{4}{[e^x + e^{-x}]^2} = \frac{1}{\cosh^2(x)}$  [4]



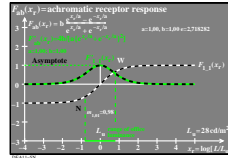
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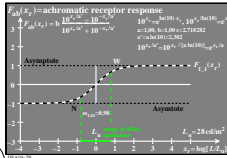
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 $F'(x/a) = \frac{u'(x/a)v(x/a) - u(x/a)v'(x/a)}{v^2(x/a)}$  [2]  
 $F'(x/a) = \frac{v^2(x/a) - u^2(x/a)}{a^2 v^2(x/a)}$  [3]  
 $F'(x/a) = \frac{4}{a^2 [e^{x/a} + e^{-x/a}]^2} = \frac{1}{a^2 \cosh^2(x/a)}$  [4]



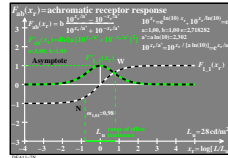
**Mathematical equations of hyperbel functions**  
 See: Papula, L., (2003), *Mathematische Formelsammlung, Vieweg*  
 $\sinh(x) = \frac{e^x - e^{-x}}{2}$  [1],  $\cosh(x) = \frac{e^x + e^{-x}}{2}$  [2]  
 $\tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$  [3]  
 $\tanh(x/2) = \frac{\sinh(x)}{\cosh(x)+1} = \frac{e^{x/2} - e^{-x/2}}{e^{x/2} + e^{-x/2}}$  [4]  
 $\sinh^2(x) + \cosh^2(x) = 1$  [5]



**Mathematical equations of hyperbel functions**  
 See: Papula, L., (2003), *Mathematische Formelsammlung, Vieweg*  
 $F_{1b}(x) = b \tanh(x) = b \frac{e^x - e^{-x}}{e^x + e^{-x}} = b \frac{u(x)}{v(x)}$  [1]  
 $F'_{1b}(x) = b \frac{u'(x)v(x) - u(x)v'(x)}{v^2(x)}$  [2]  
 $F'_{1b}(x) = b \frac{v^2(x) - u^2(x)}{a^2 v^2(x)}$  [3]  
 $F'_{1b}(x) = \frac{4b}{[e^x + e^{-x}]^2} = \frac{b}{\cosh^2(x)}$  [4]



**Mathematical equations of hyperbel functions**  
 See: Papula, L., (2003), *Mathematische Formelsammlung, Vieweg*  
 $\sinh(x) = \frac{10^{x/a} - 10^{-x/a}}{2}$  [1],  $\cosh(x) = \frac{10^{x/a} + 10^{-x/a}}{2}$  [2]  
 $\tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{10^{x/a} - 10^{-x/a}}{10^{x/a} + 10^{-x/a}}$  [3]  
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**Mathematical equations of hyperbel functions**  
 See: Papula, L., (2003), *Mathematische Formelsammlung, Vieweg*  
 $F_{ab}(x/a) = b \tanh(x/a) = b \frac{e^{x/a} - e^{-x/a}}{e^{x/a} + e^{-x/a}} = b \frac{u(x/a)}{v(x/a)}$  [1]  
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 $F'_{ab}(x/a) = b \frac{v^2(x/a) - u^2(x/a)}{a^2 v^2(x/a)}$  [3]  
 $F'_{ab}(x/a) = \frac{4b}{a^2 [e^{x/a} + e^{-x/a}]^2} = \frac{b}{a^2 \cosh^2(x/a)}$  [4]

TUB-test chart DEA1; Model of two normalized response functions  $F_{ab}(x_r)$  &  $Q_{ab}(x_r)$  and derivation Tangens hyperbolicus  $\tanh(x_r)$  and modified functions with  $e^{x_r}$  and  $10^{x_r}$ ;  $a^n = a^{1.0}$

see similar files: http://farbe.li.tu-berlin.de/DEA1/DEA1LON1.TXT /PS  
 technical information: http://farbe.li.tu-berlin.de or http://color.li.tu-berlin.de

TUB registration: 20210901-DEA1/DEA1LON1.TXT /PS  
 application for evaluation and measurement of display or print output

TUB material code=thadta