

### Hyperbolic response function of achromatic vision space $T^*_{HYP3X}$

| nonlinear color terms                                           | name and relationship with test field luminance $L$                                                                                                                                                                                                                                                                                                                                          | notes                                                                                                                    |
|-----------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------|
| <b>threshold sum</b><br>$T^*_{HYP3X}$<br>$T^*_{HYP3}$           | $T^*_{HYP3X} = A / (1 + A_2 / L^t); X=1/L^t=L^{-t}$<br>$= A_1 / (1 + A_2 \cdot X); dX/dL = -t \cdot L^{-t-1}$<br>$= A_1 \cdot [1 + A_2 \cdot X]^{-1}$                                                                                                                                                                                                                                        | $T$ . Seim 2014:<br>exponent: $t = 0,8$<br>for presentation time:<br>$t_p = 0,1s$<br>of Avramopoulos<br>experiments 1989 |
| <b>CIE luminance contrast sensitivity threshold</b><br>$L / dL$ | $dT^*_{HYP3X} / dX = -A_1 \cdot A_2 \cdot [1 + A_2 \cdot X]^{-2}$<br>$dT^*_{HYP3X} / dL = dT^*_{HYP3X} / dX \cdot dX / dL$<br>$dT^*_{HYP3X} / dL = A_1 \cdot A_2 [1 + A_2 \cdot X]^{-2} \cdot L^{-t-1}$<br>for $dT^*_{HYP3X}=1$ , and multiplication with $L$ :<br>$L / dL = A_1 \cdot A_2 [1 + A_2 \cdot X]^{-2} \cdot L^{-t}$<br>$= A_1 \cdot A_2 \cdot t \cdot / (L^t [1 + A_2 / L^t])^2$ | Hyperbolic function:<br>$T^*_{max} = A_1$<br>$T^*_{average} = 0,5 \cdot A_1$<br>$A_{2x} = A_2^{1/t}$                     |
| <b>CIE luminance difference threshold <math>dL</math></b>       | $dL = L \cdot (L^t [1 + A_2 / L^t]) / (A_1 \cdot A_2 \cdot t)$<br>$= (L^t + A_2)^2 / (A_1 \cdot A_2 \cdot t \cdot L^{t-1})$                                                                                                                                                                                                                                                                  |                                                                                                                          |

1-000030-L0

UE150-3N

### Exponential response function of achromatic vision space $T^*_{EXP3X}$

| nonlinear color terms                                           | name and relationship with test field luminance $L$                                                                                                                                                                                                                                                                                                 | notes                                                                                                                      |
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| <b>threshold sum</b><br>$T^*_{EXP3X}$                           | $T^*_{EXP3X} = A_1 \cdot \log(1 + A_2 \cdot 10^{X-X_u}); X = \log L$<br>$= A_1 \cdot \log(1 + A_2 \cdot 10^X / 10^{X_u}); L = 10^X$<br>$= A_1 \cdot \log(1 + A_4 \cdot L); dL/dX = \ln(10) \cdot 10^X$                                                                                                                                              | $L/dL$<br>$= A_1 \cdot A_4 \cdot L / (1 + A_4 \cdot L)$<br>for large $L$ it is valid:<br>$1 \ll A_4 \cdot L$<br>therefore: |
| <b>CIE luminance contrast sensitivity threshold</b><br>$L / dL$ | $dT^*_{EXP3X} / dL = A_1 \cdot A_4 / (1 + A_4 \cdot L)$<br>$= A_1 \cdot A_2 / [L_u \cdot (1 + A_2 \cdot (L/L_u))]$<br>for $dT^*_{EXP3X}=1$ , and multiplication with $L$ :<br>$L / dL = A_1 \cdot A_4 \cdot L / (1 + A_4 \cdot L)$<br>$= A_1 \cdot A_2 \cdot (L/L_u) \cdot [1 + A_2 \cdot (L/L_u)]$<br>The ratio $L / dL$ is constant for large $L$ | $L/dL = A_1$<br>$= \text{constant}$<br>$= \text{Weber-Fechner law}$<br>$A_4 = A_2 / L_u$                                   |
| <b>CIE luminance difference threshold <math>dL</math></b>       | $dL = (1 + A_4 \cdot L) / (A_1 \cdot A_4)$<br>$= [1 + A_2 \cdot (L/L_u)] / (A_1 \cdot A_2 / L_u)$                                                                                                                                                                                                                                                   |                                                                                                                            |

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UE150-7N

### Logarithmic response function of achromatic vision space $T^*_{LOG3}$

| nonlinear color terms                                           | name and relationship with test field luminance $L$                                                                                                                                                                                                                                                                                               | notes                                                                                                                               |
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| <b>threshold sum</b><br>$T^*_{LOG3}$                            | $T^*_{LOG3} = A_1 \cdot \log(1 + A_3 \cdot L)^t$<br>$= A_1 \cdot t \cdot \log(X)$<br>$X = 1 + A_3 \cdot L; dX/dL = A_3$                                                                                                                                                                                                                           | exponent: $t = A_2$                                                                                                                 |
| <b>CIE luminance contrast sensitivity threshold</b><br>$L / dL$ | $dT^*_{LOG3} / dX = A_1 \cdot t \cdot X^{-1}$<br>$dT^*_{LOG3} / dL = dT^*_{LOG3} / dX \cdot dX / dL$<br>$dT^*_{LOG3} / dL = A_1 \cdot A_3 \cdot t \cdot X^{-1}$<br>for $dT^*_{LOG3}=1$ , and multiplication with $L$ :<br>$L / dL = L \cdot A_1 \cdot A_3 \cdot t \cdot X^{-1}$<br>$= L \cdot A_1 \cdot A_3 \cdot t \cdot (1 + A_3 \cdot L)^{-1}$ | for large $L$ :<br>$T^*_{LOG3} = A_1 \cdot t \cdot \log(A_3 \cdot L)$<br><br>for least square fit:<br>$dX/dA3 = 1$<br>$dX/dL = A_3$ |
| <b>CIE luminance difference threshold <math>dL</math></b>       | $dL = X / [A_1 \cdot A_3 \cdot t]$<br>$= [1 + A_3 \cdot L] / [A_1 \cdot A_3 \cdot t]$                                                                                                                                                                                                                                                             |                                                                                                                                     |

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UE151-3N

### Logarithmic response function of achromatic vision space $T^*_{LOG3}$

| nonlinear color terms                                           | name and relationship with test field luminance $L$                                                                                                                                                                                                                                                                                                          | notes                                                                                   |
|-----------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------|
| <b>threshold sum</b><br>$T^*_{LOG3}$                            | $T^*_{LOG3} = A_1 \cdot \log(1 + A_2 \cdot L + A_3 \cdot L^2)$<br>$= A_1 \log(X)$<br>$X = 1 + A_2 \cdot L + A_3 \cdot L^2; dX/dL = A_2 + A_3 \cdot L$                                                                                                                                                                                                        | for large $L$ :<br>$T^*_{LOG3} = A_1 \cdot \log(A_3 \cdot L^2)$                         |
| <b>CIE luminance contrast sensitivity threshold</b><br>$L / dL$ | $dT^*_{LOG3} / dX = A_1 \cdot X^{-1}$<br>$dT^*_{LOG3} / dL = dT^*_{LOG3} / dX \cdot dX / dL$<br>$dT^*_{LOG3} / dL = A_1 \cdot (A_2 + A_3 \cdot L) \cdot X^{-1}$<br>for $dT^*_{LOG3}=1$ , and multiplication with $L$ :<br>$L / dL = L \cdot A_1 \cdot (A_2 + A_3 \cdot L) \cdot X^{-1}$<br>$= L \cdot A_1 (A_2 + A_3 \cdot L) / (1 + A_2 L + A_3 \cdot L^2)$ | for least square fit:<br>$dX/dA2 = L$<br>$dX/dA3 = L^2$<br>$dX/dL = A_2 + 2A_3 \cdot L$ |
| <b>CIE luminance difference threshold <math>dL</math></b>       | $dL = X / [A_1 \cdot (A_2 + A_3 \cdot L)]$<br>$= (1 + A_2 \cdot L + A_3 \cdot L^2) / [A_1 (A_2 + A_3 \cdot L)]$                                                                                                                                                                                                                                              |                                                                                         |

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UE151-7N

TUB-test chart UE15; Colour thresholds spaces  
Hyperbolic and logarithmic luminance sensitivity

input: w/rgb/cmyk -> w/rgb/cmyk-  
output: no change