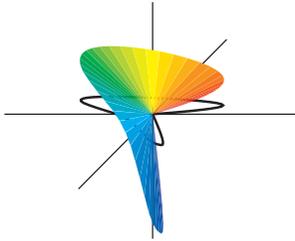


Colour, Colour Vision and Elementary Colours in Colour Information Technology



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Special print for the exhibition **Colour and Colour Vision**

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This publication comes with different versions
for offset, monitor, and printer output,
and in German, English and other languages (intended), see
<http://130.149.60.45/~farbmetrik/color>

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Two test charts at the front and the back cover

PE7011B:

16 and 8 step elementary hue circle with elementary colours
according to CIE R1-47, and DIN 33872-1 to 6

PE9011B:

5 and 16 step colour scales for the elementary hue red R_e
according to CIE R1-47, and DIN 33872-4

Six test charts in the annex

PE1011L:

Test chart 1 for colour rendering with 54 colours of the RECS-colour system.

PE2011L:

Test chart 2 for colour rendering with metameric colours for D65 and D50.

PE3011L:

Test chart 3 for colour rendering with metameric colours for A and P4000.

PE4000L:

1080 colours for colour measurement and output steering, start output.

PE4010L and PE4011L:

1080 colours for colour measurement with a 3D-linearization for the device and the elementary colour output.

RG9810L:

Achromatic test chart according to ISO/IEC 15775, ISO/IEC 24705, and ISO 9241-306, Annex D.

RG9610L:

Chromatic test chart with an ISO/IEC image according to ISO/IEC 15775, ISO/IEC 24705, and ISO 9241-306, proposed Annex E.

Remarks to the test charts

The **test charts no. 1 to 3 for colour rendering** (PE10, PE20, PE30) are used in lighting and image technology. The following visual evaluations and colorimetric specifications are possible between the real and reference light source (D65, D50, P4000, A) or the real and intended reproduction (metameric test colours are only possible in print with *CMYK* and *CMY0* colours):

Colour fidelity: Colour difference (CIELAB ΔE^*_{ab}) of the reproduction

Elementary hue location: Location of the four elementary hues (CIELAB Δh_{ab}) of the real and the intended reproduction

Hue scaling: Shift of the hues (CIELAB Δh_{ab}) in each hue sector

Metameric colours: Colour difference (CIELAB ΔE^*_{ab}) for real and reference light source (D65, D50, P4000, A), or for real and ideal (colorimetric) scanners

Colour preference: Colour difference (CIELAB ΔE^*_{ab}) with intended increase in lightness L^* and/or chroma C^*_{ab} .

The **international standards** ISO/IEC 15775 and ISO 9241-306 as well as the standard series DIN 33866-1 to -5, and DIN 33872-1 to -6 use 5 and 16 step *visual equidistant* colour series for input and output. The equal differences are usually evaluated visually. The colorimetric specification calculates the colour differences between the real and the intended output colours according to CIE-LAB (ISO 11644-4).

Information to reach the intended output colours is given by a technical description with a table at the two inner cover pages.

1 Colour Metric

The colour metric describes the definition and measurement of colours and the colour differences. The colour metric is based on the application of the publication CIE 15 *Colorimetry* of the *International Commission on Illumination* (CIE).

2 Colour and Colour Vision

The description of the quality of colour rendering is only possible on the basis of good knowledge of the visual properties of the human colour vision. Therefore it is essential to enlarge the basics by visual research. With support of the German Research Foundation (*DFG=Deutsche Forschungsgemeinschaft*) K. Richter (1979, 1985) has edited two BAM research reports. There are many other publications in the field of colour scaling, colour thresholds and elementary hues.

Important separate sections of *Colour and Colour Vision* are the psychological order of colours by the human perception. The psycho-physical description of the visual system is based on both the physical measurement and the perception

In the following sections the basic properties of the field *Colour and Colour Vision* are given.

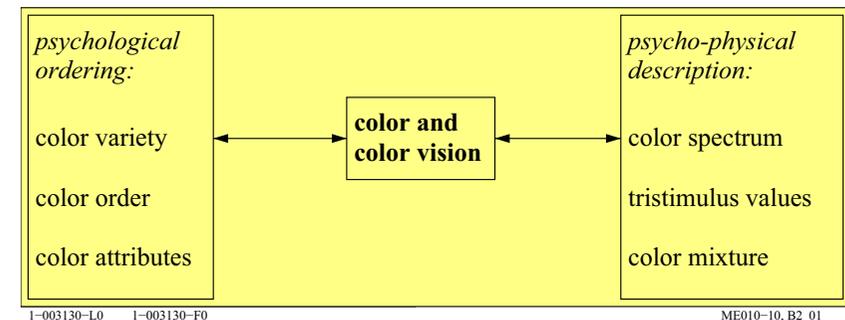


Fig. 1: Separate section of colour and colour vision

Figure 1 shows the important separate areas of colour and colour vision, which are described in the following by many colour figures.

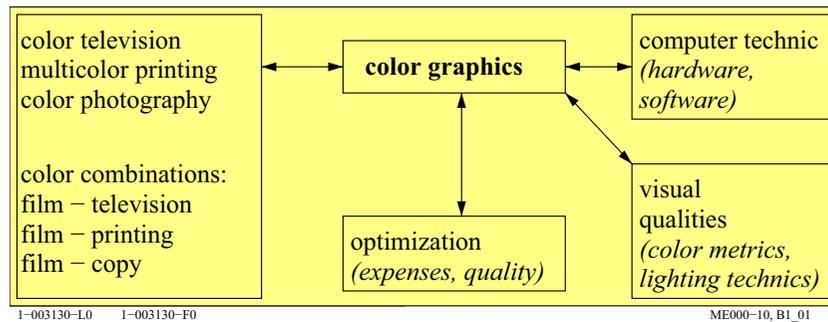


Fig. 2: Colour graphics as application of the visual properties of colour vision

Fig. 2 shows the field of colour graphics, which is essentially based on the visual properties of colour vision. In addition the properties of the colour reproduction properties and the computer technic has to be considered to optimise the many applications.

3 Colour Multiplicity

All that we see has colour. Colours form the elements of our visual sensations. Different from these sensations are the materials and processes which produce colours. In the following we will order the colour multiplicity. This leads us to colours with equal colour attributes.

According to *Judd* and *Wyszecki* (1975) people with normal colour vision can distinguish about 10 million different colours. A classification by common attributes is thus necessary to order this multiplicity.

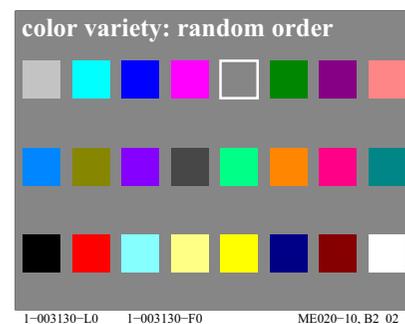


Fig. 3: Colour multiplicity

Fig. 3 shows a random arrangement of colour samples which first of all can be separated into groups of achromatic and chromatic colours.

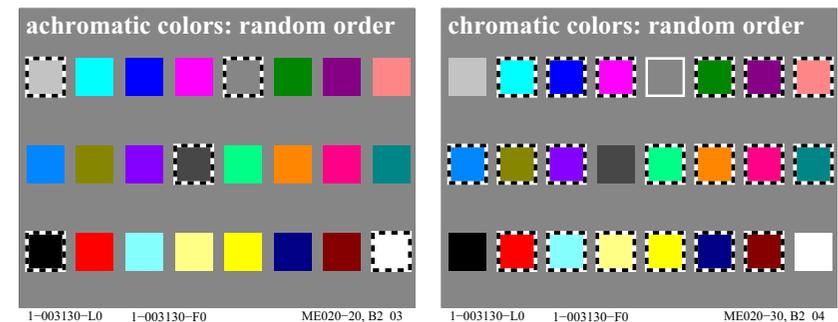


Fig. 4: Achromatic and chromatic colours

In Fig. 4 out of the random arrangement of colours, the achromatic group of colours is marked (*left*) and the chromatic group is marked (*right*).

Achromatic colours, intermediate colours	Chromatic colours, Elementary colours	Chromatic colours, Device colours
<i>five achromatic colours:</i>	"Neither-nor"-colours	TV, Print (PR), Photo (PH)
<i>N</i> black (French noir)	<i>four elementary (e) colours:</i>	<i>six device (d) colours:</i>
<i>D</i> dark grey	$R = R_e$ red	$C = C_d$ cyan blue (cyan)
<i>Z</i> central grey	<i>neither yellowish nor bluish</i>	$M = M_d$ magenta red (magenta)
<i>H</i> light grey	$G = G_e$ green	$Y = Y_d$ yellow
<i>W</i> white	<i>neither yellowish nor bluish</i>	$O = R_d$ orange red (red)
	$B = B_e$ blue	$L = G_d$ leaf green (green)
	<i>neither greenish nor reddish</i>	$V = B_d$ violet blue (blue)
	<i>two intermediate colours:</i>	
$C_e = G50B_e$ green-blue	$J = Y_e$ yellow (French jaune)	
$M_e = B50R_e$ blue-red	<i>neither greenish nor reddish</i>	

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Table 1 Elementary and device colours in information technology

Table 1 shows the definition of the elementary colours (index e) and the device colours (index d) of the information technology. There are four elementary colours $RGBY_e$ and six device colours $RGBCMY_d$. For some applications the visual intermediate colours C_e (blue-green) and M_e (blue-red) are added to the four elementary colours and produce then six colours (bottom left). Table 1 includes 5 achromatic colours $NDZHW$ between black N (= French noir) towards mean grey Z to White W . All others are chromatic colours.

The names O , L , and V are used in many standard documents, (for example *ISO/IEC 15775*, *ISO/IEC 24705*, *ISO 9241-306*, *DIN 33866-1 to -5*, and *DIN 33872-1 to -6*). The names O , L , and V have the advantages to be short and they represent the appearance. They have the disadvantage to be not used by many

applications. In addition the letter L is also used for the luminance in all standards of the lighting technology, for example ISO/IEC/CIE 8589.

In the following therefore the letters R_d , G_d , and B_d will be used instead of the names O , L , and V . In Table 1 the device colours (*index d*) red R_d , green G_d and Blue B_d differ compared to the elementary colours (*index e*) red R_e , green G_e , and Blue B_e . For any undefined colours red, green, and blue the letters $R_$, $G_$ and $B_$ (*letter underscore*) will be used. The colours $RGB_$ are usually neither identical to the device colours R_d , G_d , and B_d nor to the elementary colours R_e , G_e , and B_e . The large advantage of the elementary colours red R_e , green G_e , and Blue B_e is the *visual definition* and the *device independent property* according to CIE R1-47:2009 “Hue Angles of Elementary colours”.

Today in the colour image technology for the specification of colours the digital technic is used. The minimum amount of colours is 4096 colours. The three device colours (*index d*) R_d (red colour), G_d (green colour) and B_d (blue colour) are used to produce 16 colour steps for each colour. For monitors and data projectors the additive mixture of these colours leads to 4096 (=16x16x16) mixture colours.

The three device colours are usually coded by the hexadecimal system. Therefore the 16 steps with the decimal values 0 to 15 are coded by 0 to 9 and the letters A to F for the values 10 to 15.

For the different colours Fig. 5 shows the appropriate specifications in the hexadecimal system. The three specifications are one by one for the RGB_d -colour data. According to their appearance the three colours are named $R_d=O$ (for orange-red), $R_d=L$ (for leaf-green), and $B_d=V$ (for violet-blue).

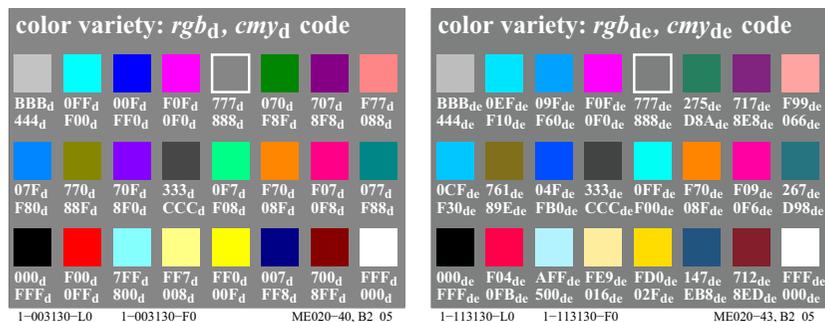


Fig. 5: rgb_d , cmy_d -colour code and rgb_{de} , cmy_{de} -colour code

In Fig. 5 (*left*) the achromatic device colours (*index d=device*) have three equal hex digits. For chromatic colours at least two of the three are different. The device colours rgb_d and cmy_d are transformed between rgb_d - and cmy_d -data for the device colour output according to the 1-minus-relation. For example the rgb_d -data 00F_d for blue are transferred to the cmy_d -data FF0_d. If in the colour file these two colorimetric definitions are used then equal or different output colours may be produced. With a file according to DIN 33872-4 the equality of the output is tested for both definitions, see <http://www.ps.bam.de/De14/10L/L14e00NP.PDF>

Fig. 5 (*right*) shows the rgb_{de} - and cmy_{de} -code for the elementary colour output (*index de = device to elementary data*) by three hex digits. Again the rgb_{de} -data are transferred to the cmy_{de} -data according to the 1-minus-relation. However, for example Blue (first row, third colour) is now defined by the hex number 09F_{de} instead of 00F_d in Fig. 5 on page 7. The Blue of Fig. 5 (*right*) is the device Blue B_d and the Blue in Fig. 5 (*left*) is the elementary Blue B_e . Both look different, B_d appears reddish and B_e neither greenish nor reddish. For the standard *sRGB*-Monitor and the standard offset print the hex numbers for the production of Blue B_e are different.

For colour scales, which intent equally spaced visual colour scales, for example an equally spaced 16 step grey series, the symbol * (*star*) for the colour coordinate is used. For example the *visual attribute* lightness L^* uses the symbol * (*star*) and the measurement term luminance L not. Similar one can add to the rgb_e data the symbol * (*star*), and call them rgb^*_e data. The interpretation of this symbol code indicates, that for example the hex-data series $rgb^*_e = 000$, 111, 222, ..., EEE, FFF produce a *visually equally spaced* grey series.

Instead of the hex data one can use numbers between 0.0 and 1.0. For example for the hex number 5 the decimal number is equal to 0.3333 (=5/F = 5/15). The information technology uses instead of 16 steps between 0 and 9, and from A to F the 256 steps between 00 and 9F, and from A0 to FF. One can transfer these hex numbers to decimal numbers. For example the hex number 55 is equal to the decimal number 0.3333 (=55/FF = 85/255).

For the output of the colorimetric equivalent *rapt*- and *cmy0*-colour data many problems occur in applications. The display output of the equivalent rgb and $cmy0$ colour data produces for example with the software products *Adobe Acrobat* (all versions above 3 under Mac and Windows) different output and with *Adobe FrameMaker* (Version 8, Windows, 2011) equal output. For example *PostScript* colour printers produce often different outputs and *PostScript*-black-white printers equal outputs. With test files according to DIN 33872-4 and -2 the equality of the output is tested, see <http://www.ps.bam.de/33872E>.

4 Colour Solid

Leonardo da Vinci (died 1519) ordered the multitude of colours by selecting six elementary” colours: one neutral or achromatic pair (white-black) and two chromatic pairs red-green and yellow-blue. The double cone of Fig. serves as a simplified model to illustrate his ideas. The vertical axis corresponding to the array of neutral colours (white to black) and the circumference corresponding to the pure chromatic colours.

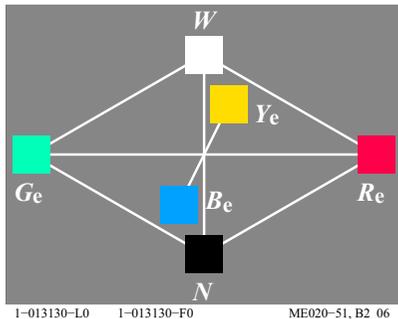


Fig. 6: Colour double cone

Fig. 6 shows the double cone with the six “simple” colours. In Fig. 6 the letters stand for:

- W white
- N black (= noir)
- Y_e yellow
- B_e blue
- R_e red
- G_e green

The six “simple” colours are here the six “elementary” colours (index e).

The Technical Committee ISO TC 159/WG2/SC4 *Ergonomics, Visual Display Requirements* has recommended, to produce the four elementary colours $RYGB_e$ with the following four rgb^*_e -input data 100, 110, 010, and 001, see CIE R1-47. There are at least three methods to calculate the rgb_{de} -data (Index de = device to elementary colours) for the output device: by the device manufacturer, die image technology software or a frame file. The frame file method has been used to change all the rgb -data of the figures in this publication according to the output device ($sRGB$ or offset print). The frame file includes 729 (=9x9x9) rgb - and CIELAB-data (colour measurement data) of the output device.

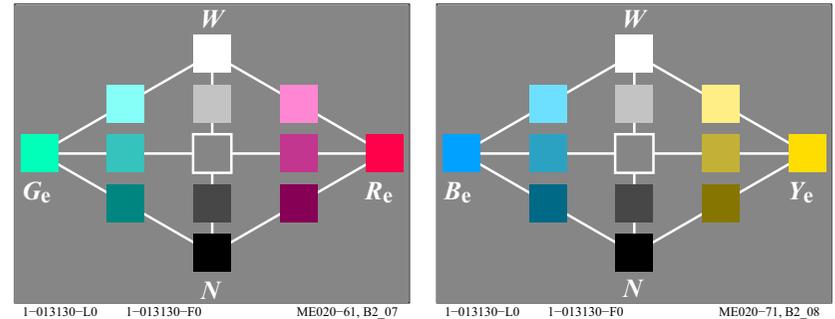


Fig. 7: RG_e - and YB_e -hue plane cut

Fig. 6 shows the colour double cone with many intermediate steps in the vertical plane cuts red-green (*left*) and yellow-blue (*right*). The achromatic (white-black) axis is located in the middle.

5 Elementary Colours

In any hue circle there are four chromatic colours which are perceptually simple, compare Table 1 on page 6. We call them elementary colours, and we distinguish elementary red, yellow, green, and blue.

Experimentally, bracketing within a hue circle permits easy determination of elementary yellow. Elementary yellow is called a "neither-nor" colour (neither reddish nor greenish) as opposed to yellow-greens which are called "as-well-as" colours (yellow as well as green) in a hue circle. The "neither-nor" colours" are often called unique in the literature.

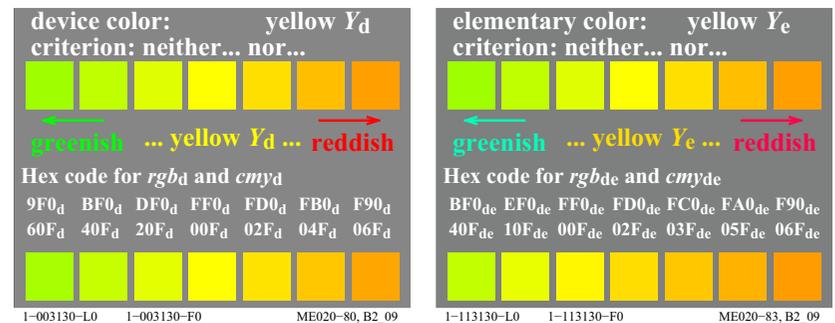


Fig. 8: Device and elementary colour with criterion for elementary yellow Y_e

Fig. 8 describes the criterion for the determination of the elementary colour yellow Y_e out of a hue circle in the yellow region. For the rgb_d -input data (1, 1, 0) or FF0 usually the device colour yellow Y_d is produced. The criterion for elementary yellow as *neither greenish nor reddish* is approximately fulfilled here (*left*). The rgb_{de} -input data (1, 0,86 0) = FD0 produce the intended elementary yellow Y_e with the visual property *neither greenish nor reddish*. The example is given for the standard offset print (*right*). The difference between device yellow Y_d and elementary yellow Y_e is small in offset print. However for blue it is large.

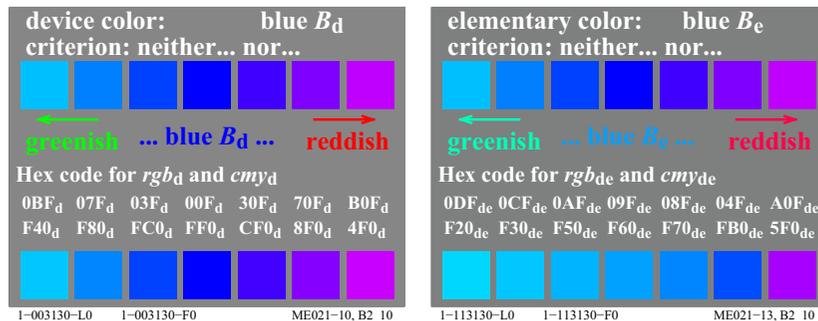


Fig. 9: Device and elementary colour with criterion for elementary blue B_e

Fig. 9 describes the criterion for the determination of the elementary colour blue B_e out of a hue circle in the blue region. For the rgb_d -input data (0, 0, 1) or 00F usually the device colour blue B_d is produced. The criterion for elementary blue as *neither greenish nor reddish* is not fulfilled here (*left*). The rgb_{de} -input data (1, 0,60 0) = 09F produce the elementary blue B_e with the visual property *neither greenish nor reddish*, in the example for the standard offset print (*right*).

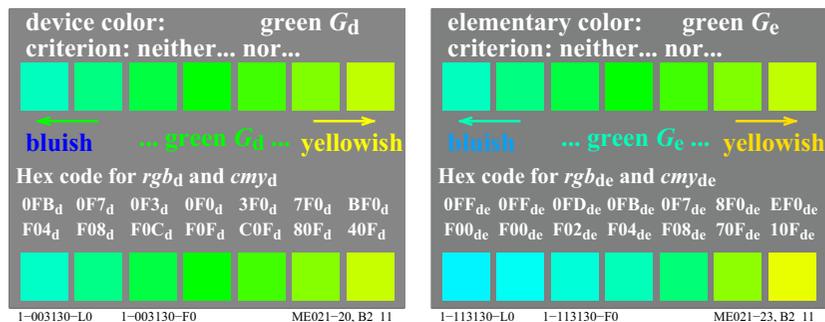


Fig. 10: Device and elementary colour with criterion for elementary green G_e

Fig. 10 describes the criterion for the determination of the elementary colour green G_e out of a hue circle in the green region. For the rgb_d -input data (0, 1, 0) or 0F0 usually the device colour green G_d is produced. The criterion for elementary green as *neither bluish nor yellowish* is not fulfilled here (*left*). The rgb_{de} -input data (0, 1 0,80) = 0FB produce the intended elementary green G_e with the visual property *neither bluish nor yellowish*. The example is given for the standard offset print (*right*). The difference between device green G_d and elementary green G_e is smaller compared to blue in offset print.

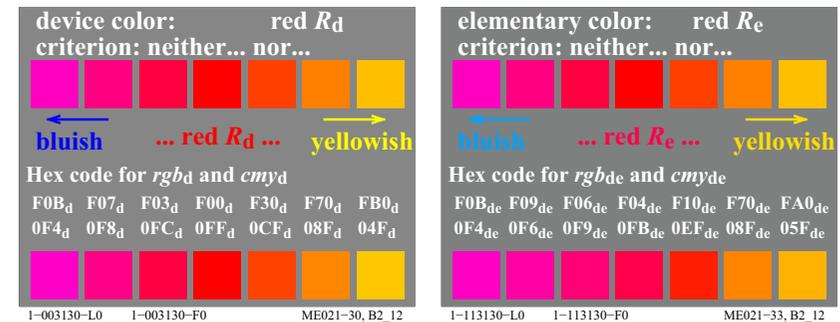


Fig. 11: Device and elementary colour with criterion for elementary red R_e

Fig. 11 describes the criterion for the determination of the elementary colour red R_e out of a hue circle in the red region. For the rgb_d -input data (1, 0, 0) or F00 usually the device colour red R_d is produced. The criterion for elementary red as *neither bluish nor yellowish* is not fulfilled here (*left*). The rgb_{de} -input data (1, 0, 0,27) = F04 produce the intended elementary red R_e with the visual property *neither bluish nor yellowish*, in the example for the standard offset print (*right*).

Under daylight and with 28 observers *K. Miescher* (1948) has experimentally determined the elementary colours out of a 400 step hue circle. The standard deviation was 4 steps for R_e , Y_e and G_e (1% = 4 of 400 steps) and 8 steps for B_e (2%), see CIE R1-47. The hue circle had a high chroma compared to the CIE-test colours no. 9 to 12, see Fig. 52 on page 51.

6 Symmetric Hue Circle

The colours on either side of the two perpendicular elementary hue axis R_e - G_e and Y_e - B_e become increasingly yellower or bluer, redder or greener respectively, as they depart from the achromatic centre.

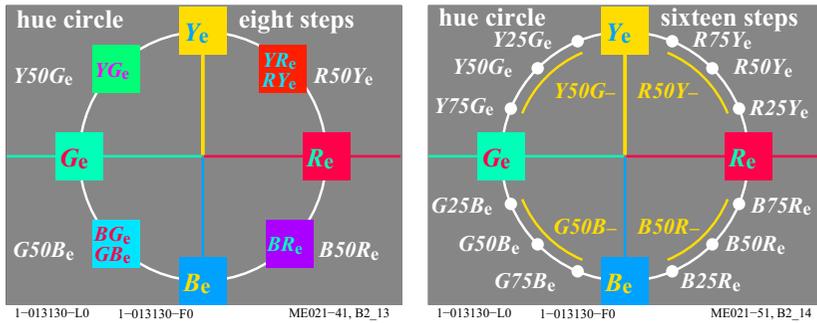


Fig. 12: Symmetric hue circle and names of intermediate colours

Fig. 12 shows the symmetric hue circle with the opposing elementary colours red - green and yellow - blue and the intermediate colours.

In most languages (for example German, English, French) yellow and blue is used first in combined colour names for example yellow-red YR and yellow-green YG , and blue-green BG and blue-red BR . This preferred naming in these languages is used in Fig. 12 (left). In Fig. 12 (right) the mathematical angle and the continuous naming RY_e , YG_e , GB_e , and BR_e is used.

In addition the CIELAB-colour system (ISO 11664-4/CIE S 014-4) uses for the hue h_{ab} the mathematical angle. The angle count starts at the angle of 0 degree for elementary red R_e and increases with the angle 90 degree for Y_e , 180 degree for G_e and 270 degree for B_e .

The CIELAB-colour system uses 100 steps between black and white. One uses 100 hue steps between two elementary colours. This produces in Fig. 12 the names for the intermediate colours. The information technology recommends hue outputs, which shift 25%, 50% and 75% from R_e towards yellow Y_e . The output on many devices produce undefined output hues which are located in a large range between R_e and Y_e .

A colorimetric information technology recommends to reach the visual intermediate colour with the hue $R50Y_e$. For many of the output devices the output hues for $R50Y_e$ are in a device dependent large range $R50Y_-$ (yellow range) and similar for the other intermediate hues $Y50G_e$, $G50B_e$ and $B50R_e$.

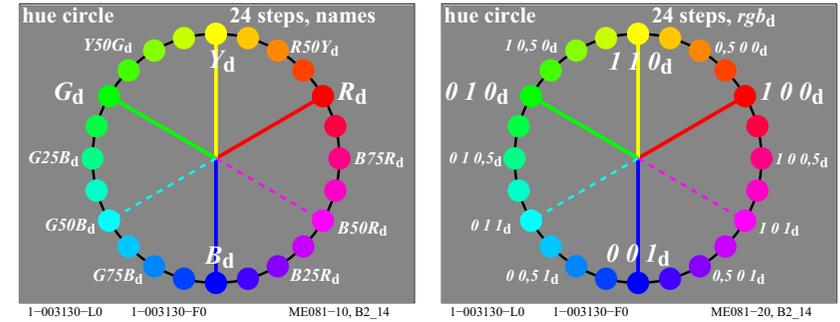


Fig. 13: 24 step device-hue circle in information technology

Fig. 13 shows a 24 step device-hue circle of the information technology. The example device colours RGB_d (left) and the corresponding rgb -input data $(1\ 0\ 0)_d$, $(0\ 1\ 0)_d$, and $(0\ 0\ 1)_d$ (right) are given. The intermediate hues Y_d , $G50B_d$ and $B50R_d$ of the device have the rgb -input data $(1\ 1\ 0)_d$, $(0\ 1\ 1)_d$, and $(1\ 0\ 1)_d$.

For applications in technology, design and art the range of the lighter colours between red towards yellow to green is more important compared to the range of darker colours between green towards blue to red. In addition in the yellow range the CIELAB chroma C^*_{ab} of surface colours is twice as large compared to the blue range, see the table with C^*_{ab} for 48 hues on the inner back cover. Therefore for equal angle difference the visual hue differences is twice as large in the yellow range compared to the blue range. Both reasons are used to increase the range between red towards yellow to green from 120 degree to 180 degree, and to decrease the range between green towards blue to red from 240 degree to 180 degree.

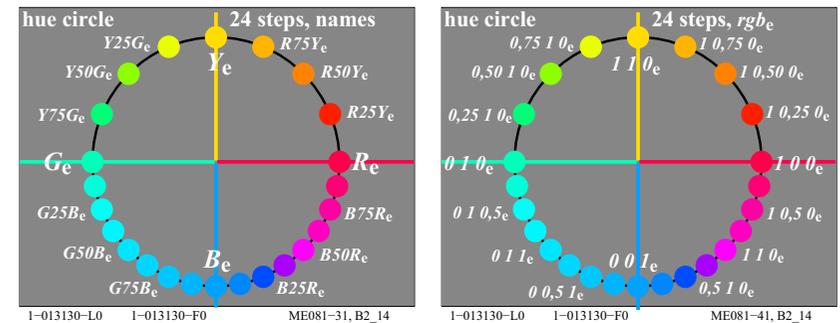


Fig. 14: 24 step elementary hue circle of the information technology

Fig. 14 shows the relation of the elementary hues $RYGB_e$ (left) and the rgb^*_e -input data (right) of the information technology for a 24 step hue circle. Fig. 14 produces the elementary hues $RYGB_e$ for the rgb^*_e -input data $(1\ 0\ 0)_e$, $(1\ 1\ 0)_e$, $(0\ 1\ 0)_e$ and $(0\ 0\ 1)_e$. The workflow *file - output* must produce the rgb_{de} -data for the intended output of the elementary hues. In the simplest case the device manufacturer may produce the transformation within his device. DIN 33872-5 includes a test chart in the formats *PDF* and *PS (PostScript)*. The output property *elementary hue output* is usually tested visually. In addition it may be specified by colorimetry.

The lower hue discrimination of surface colours in the *darker colour range* between green towards blue to red recommends to use only every second step in this darker area.

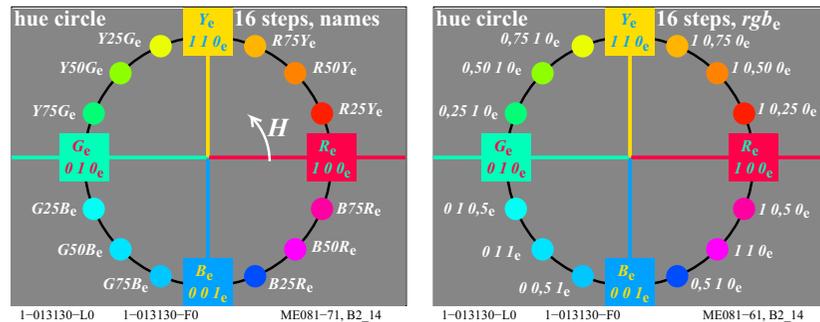


Fig. 15: 16 step Elementary hue circle of the information technology

Fig. 15 shows the relation of the elementary hues $RYGB_e$ (left) and the rgb^*_e -input data (right) in the information technology in a 16 step hue circle. The elementary hues $RYGB_e$ are produced for rgb_e -input data $(1\ 0\ 0)_e$, $(1\ 1\ 0)_e$, $(0\ 1\ 0)_e$ and $(0\ 0\ 1)_e$. The hue increases with the hue angle similar as the hue angle h_{ab} of the CIELAB-colour system (ISO 11564-4). According to CIE R1-47 the elementary hue angles have in the CIELAB-colour order system the hue angles $h_{ab} = 26, 92, 162,$ and 272 degree. Especially red R_e and green G_e are not located on the horizontal axis in the CIELAB-colour system.

7 Colours with Maximum Chroma

In a colour series with different amount of colorant, leading from whitish colours through chromatic colours to blackish colours, there is one colour that is perceived to exhibit maximum chroma.

Bracketing allows the determination of the “reddest” red in this colour series. This determination is made according to the criterion of whether the colour becomes more achromatic or chromatic, as well as whether it becomes whiter or blacker than the other colours of the series.

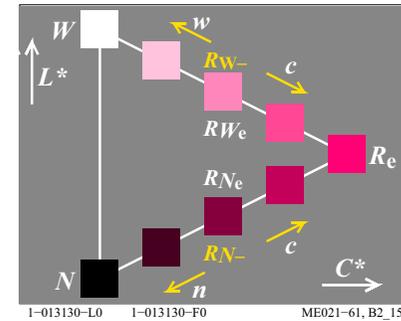


Fig. 16: Maximum chroma

In Fig. 16 one can easily determine the most chromatic red colour. The criteria for determining the colour of maximum chroma out of a colour series with different amounts of colorant matter are given. The designations stand for:

R_e red	W white	N black (= <i>noir</i>)
c more chromatic	w whiter	n blacker
C^* chroma	L^* lightness	

In the information technology usually the most chromatic colour of any hue (colour of maximum chroma C^*_{ab} in the CIELAB-colour system) is mixed with white W and black N . For the mixture between the most chromatic colour R_e and white W the CIE chromaticity difference is continuously decreasing, for the mixture of R_e with black N the chromaticity is approximately constant. The additive colour mixture on the colour monitor (display), and the often subtractive colour mixture in offset print will be discussed in section 20 on page 64.

In Fig. 16 the visual intermediate colours R_{we} and R_{ne} shall be produced in the middle between R_e and W or N . Similar compared to Fig. 10 on page 11 usually the output colours are produced in a large device dependent range (*yellow range*).

8 Colour Attributes Chroma and Lightness

Perceptually, three colour attributes specify a colour. Most colour systems choose hue as the first attribute, for example the *Munsell*-colour system, the colour system DIN 6164, and the *NCS*-colour system. These colour systems differ in the choice of the two other colour attributes. A comparison of the col-

our systems needs a similar coordinate system. In colorimetry a cut through the colour solid in a plane of constant hue is used. We use on the abscissa the chroma C^* and on the ordinate the lightness L^* .

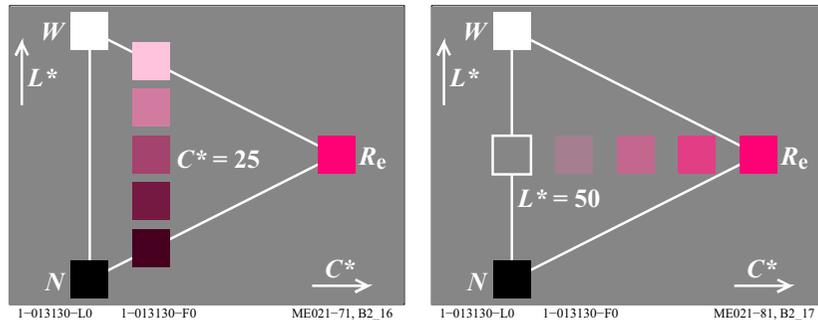


Fig. 17: Equal chroma C^* and equal lightness L^*

Fig. 17 (left) shows colours of constant hue and of the chroma $C^* = 25$. Colours of equal chroma are located on vertical series parallel to the grey axis. In colorimetry for the most chromatic red R_e the chroma $C^* = 100$ may be used. Then in Fig. 17 (left) the chroma is $C^* = 25$.

Fig. 17 (right) shows colours of constant hue and of the lightness $L^* = 50$. Colours of equal lightness are located on horizontal rows which are perpendicular to the grey axis. Colorimetry defines the lightness $L^* = 100$ for white W . Then in Fig. 17 (right) the colour series has the lightness $L^* = 50$.

At first colour scales of constant hue and equal chroma and lightness were used in the *Munsell*-colour system. This system includes colour samples of 40 different hues. Today in colour metrics the colour spaces (ISO 11564-4 and -5) define the coordinates chroma C^* (designations C^*_{ab} in CIELAB and C^*_{uv} in CIE-LUV) and lightness L^* .

In the colour order system *RAL-Design* the colour samples of 36 CIELAB-hues $h_{ab} = 0, 10, \dots, 350$ produce a grid with the chroma differences $\Delta C^*_{ab} = 10$ and $\Delta L^* = 10$.

9 Colour Attributes Brilliance and Whiteness

There are more than three colour attributes hue, chroma, and lightness. In a constant hue plane the further colour attributes blackness (opponent to brilliance) and whiteness (opponent to colour deepness) have a linear relation to chroma and lightness.

The colour attributes blackness and brilliance describe the same property. However, they change their attribute values in opposite directions similar as for lightness and darkness. Further whiteness and colour deepness count in opposite directions. Blackness is chosen as an important colour attribute in *Swedish Natural Color System (NCS)*. The NCS-Colour system chooses the colour attributes hue, blackness, and chroma (called chromaticness). The colour attribute lightness of the *Munsell*-colour system is not used.

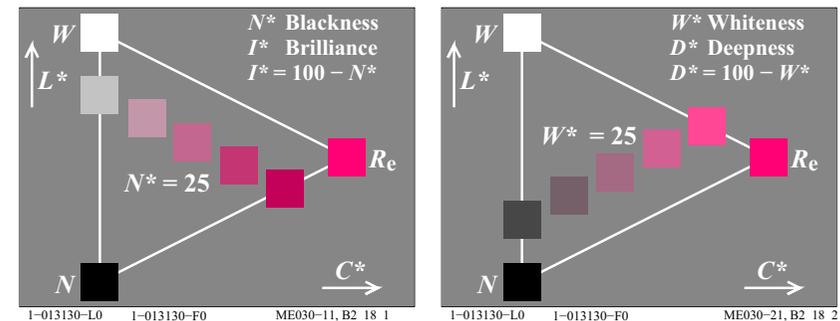


Fig. 18: Equal blackness N^* and whiteness W^*

Fig. 18 shows colours of equal blackness N^* (left) with the blackness $N^* = 25$ and of equal whiteness W^* (right) with the whiteness $W^* = 25$. Instead of the blackness N^* the colour attribute brilliance $I^* = 100 - N^*$ may be chosen. Instead of whiteness W^* the colour attribute colour deepness $D^* = 100 - W^*$ may be chosen.

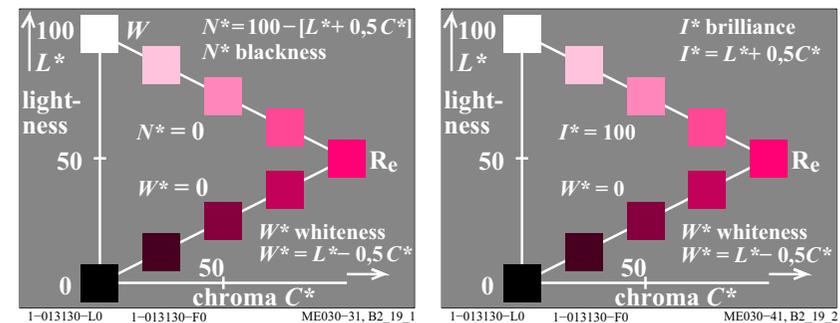


Fig. 19: Blackness N^* , whiteness W^* and brilliance I^*

Fig. 19 shows the relation of the three colour attributes blackness N^* , whiteness W^* , and brilliance I^* with the two colour attributes lightness L^* and chroma C^* . Fig. 19 shows the relations with linear equations.

It is expected that the linear relations are connected with the physiological achromatic and chromatic signals of Fig. 54 on page 54 and the chromatic values of Fig. 58 on page 61 (*bottom left*).

10 Colour Spectrum and Elementary Colours

10.1 Luminous Valence and Lightness

The daylight colour spectrum which can be produced by a prism, and which was examined by *Newton* (died 1727), includes the light radiation between the short wave violet-blue (approximately $R60B_c$) and the long wave yellowish red (approximately $J90R_c$). Colored lights differ in their *spectral power distribution of light radiation*. The spectral distribution of the light radiation which finally falls on the eye is changed by reflection from surfaces which appear colored, for example by use of the pigments of the chemical industry.

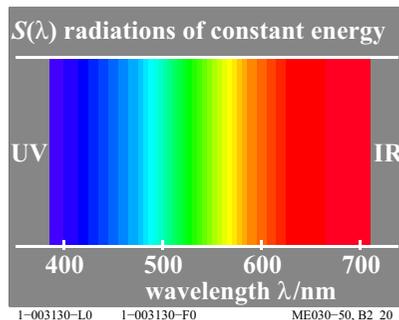


Fig. 20: Visible spectral region

Fig. 20 shows schematically the region of light radiation with all wavelength λ in the visible spectrum between approximately $\lambda = 380\text{nm}$ and $\lambda = 720\text{nm}$ ($1\text{nm} = 10^{-9}\text{m}$). The rays beyond these limits are named “ultra-violet” (UV) and “infrared” (IR). Fig. 20 shows a spectrum, which can be produced by a continuous interference filter in the slide plane of a projector. The interference filter has the property of letting through visible light radiation between approximately 380nm and 720nm in a continuous spectrum.

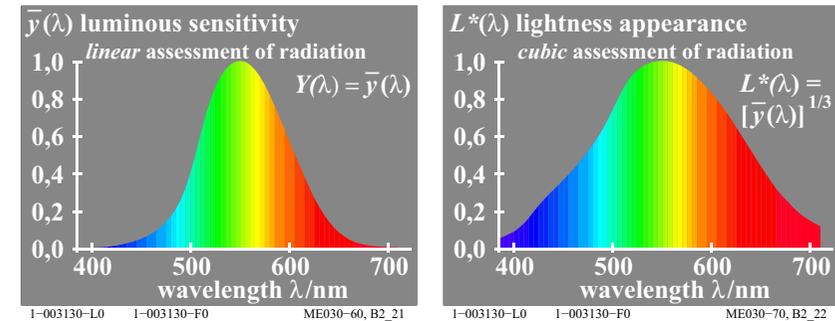


Fig. 21: Spectral luminous sensitivity and lightness appearance

In Fig. 21 (*left*) the luminous sensitivity $Y(\lambda)$ of the spectrum decreases from the middle yellow-green region to both ends. This property is a consequence of the spectral luminous efficiency $Y(\lambda) = V(\lambda) = y_q(\lambda)$ of the eye. This efficiency has a maximum near 555nm and decreases to less than 1% near 400nm and 700nm. The relative spectral luminous efficiency function $y_q(\lambda)$ specifies the *valence* (value) in the colour mixture of the spectral colours, for example of a band width 10nm and equal energy of light radiation. Therefore the value which is described by the luminous efficiency function $y_q(\lambda)$ may be also called luminous value or luminous valence.

In CIE 15 *Colorimetry* the tristimulus value Y with the normalization $Y_w = 100$ for white W is defined, compare Section 17 on page 47

Different from the *linear* function $Y(\lambda)$ is a *nonlinear* function $L^*(\lambda)$ which describes the lightness appearance of the spectral colours of equal light radiation. This nonlinear function decreases from the middle of the spectrum to both ends by a nonlinear relation which is approximately cubic for grey and quadratic for white surrounds, compare section 16 on page 42.

Fig. 21 (*right*) shows the lightness appearance $L^*(\lambda)$. The function $L^*(\lambda)$ decreases much less compared to the function $Y(\lambda)$ (*left*).

Note: CIE 15 defines the following relation between lightness L^* and the tristimulus value Y :

$$L^* = 116 [Y/100]^{1/3} - 16 \quad (Y > 0,8)$$

Approximations are the relations:

$$L^* = 100 [Y/100]^{1/3} \quad \text{and} \quad L^* = Y^{1/3} \quad \text{which is used for spectral colours in Fig. 21.}$$

10.2 Chromatic Value (Valence) and Chroma

In the colour mixture the spectrum is assessed by "luminous values" and in addition by "chromatic values".

The visible spectrum includes a continuous series of hues, and one can recognize in it three spectral elementary colours. The spectral elementary colours are located close to 475nm for elementary blue B_e , 503nm for elementary green G_e , and 575nm for elementary yellow Y_e .

Elementary red is located outside the visible spectrum and can be produced for example by a suitable mixture of the colours 400nm and 700nm. The purple colours produced in this way are specified by compensatory wavelengths compared to illuminant E (equal energy of light radiation). For elementary red R_e this result is the dominant wavelength $\lambda_{d,E} = 494\text{c nm}$, see Fig. 50 on page 47

At both spectral ends of the spectrum the red-green and yellow-blue chromatic values change the sign from negative to positive or in opposite direction.

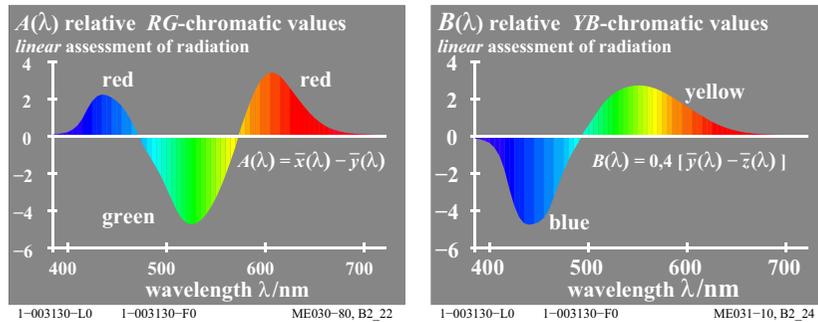


Fig. 22: RG-chromatic values and YB-chromatic values

Fig. 22 (left) shows the red-green-chromatic values $A(\lambda)$, which are the red-green-valences (values) in the colour mixture, as function of wavelength. The zero points are near 475nm and 574nm and specify the spectral elementary colours blue B_e and yellow Y_e .

Fig. 22 (right) shows the yellow-blue-chromatic values $B(\lambda)$, which are the yellow-blue-valences (values) in the colour mixture, as function of wavelength. The zero point is near 503nm and specifies the spectral elementary colour green G_e .

For the spectral colours of equal energy the luminous values and the red-green- and yellow-blue-chromatic values produce three numbers (a vector) for any wavelength λ , for example of the band width 10nm between 380nm and

720nm. In the 3-dimensional space a point is created for the coordinates red-green-chromatic value A , yellow-blue-chromatic value B and luminous value or tristimulus value Y of any wavelength. In Fig. 23 the points of all spectral colours are located on a 3-dimensional curve.

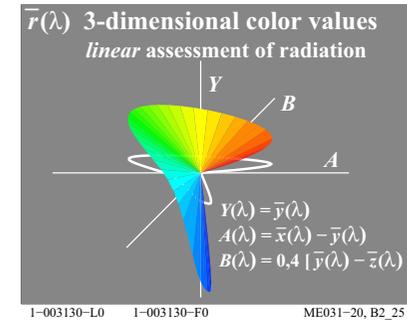


Fig. 23: 3-dimensional colour values

Fig. 23 shows the 3-dimensional colour values in the colour space (A, B, Y), and the projection in the plane (A, B) (white curve). The 3-dimensional curve cuts the plane (B, Y) near 475nm (elementary blue B_e) and 574nm (elementary yellow Y_e). The plane (A, Y) is cut near 503nm (elementary green G_e). Fig. 23 includes the linear relation between the luminous and chromatic values $Y(\lambda)$, $A(\lambda)$ and $B(\lambda)$ and the CIE tristimulus values $x_q(\lambda)$, $y_q(\lambda)$, and $z_q(\lambda)$.

There is a difference between chromatic value (valence in colour mixture) and chroma similar compared to the difference between luminous value and lightness.

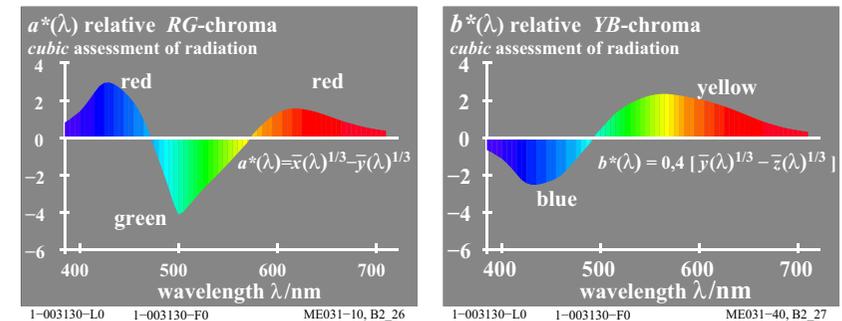


Fig. 24: RG-chroma and YB-chroma

Fig. 24 (left) shows the red-green-chroma $a^*(\lambda)$ which describes chroma appearance of reddish and greenish spectral colours. The zero points are near

475nm and 574nm and specify the spectral elementary colours blue B_e and yellow Y_e .

Fig. 24 (right) shows the yellow-blue-chroma $b^*(\lambda)$ which describes chroma appearance of yellowish and bluish spectral colours. The zero point is near 503nm and specifies the spectral elementary colour green G_e .

For the spectral colours of equal energy the lightness and the red-green-, and yellow-blue-chroma produce three numbers (a vector) for any wavelength λ , for example of the band width 10nm between 380nm and 720nm. In the 3-dimensional space a point is created for the coordinates red-green-chroma a^* , yellow-blue-chroma b^* and lightness L^* for any wavelength. In Fig. 25 the points of all spectral colours are located on a 3-dimensional curve.

It is useful to define the group term colorness (*German Farbheit*) which covers the terms lightness, red-green-chroma, yellow-blue-chroma, whiteness, blackness, deepness, and other visual colour attributes. Instead of the term chroma also the term chromaticness is used, for example in the *NCS-colour system*.

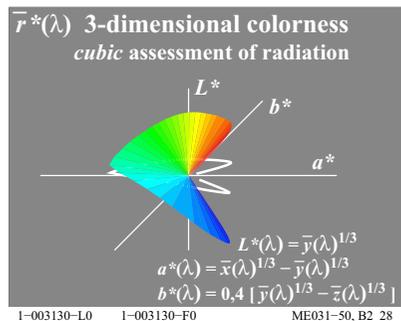


Fig. 25: 3-dimensional colorness

Fig. 25 shows the three colorness data L^* , a^* , b^* in the 3-dimensional colour space (a^* , b^* , L^*), and the projection in the plane (a^* , b^*) (*white curve*). The 3-dimensional curve cuts the plane (b^* , L^*) near 475nm (elementary blue B_e) and 574nm (elementary yellow Y_e). The plane (a^* , L^*) is cut near 503nm (elementary green G_e).

In Fig. 25 the projection of the 3-dimensional curve in the plane (a^* , b^*) is shown by a white curve. Fig. 25 includes the nonlinear relation of the spectral lightness and chroma $L^*(\lambda)$, $a^*(\lambda)$, and $b^*(\lambda)$ with the spectral tristimulus values $x_q(\lambda)$, $y_q(\lambda)$, and $z_q(\lambda)$.

11 Apparatus for Mixing Spectral Colours and Reflection

With a spectrophotometer one can measure at each wavelength the reflection of the radiation which falls upon a surface. By comparison of the reflection of a surface colour with the reflection of the ideal white surface one normally gets a spectral reflection curve with numerical values between 0,0 and 1.0 for each wavelength.

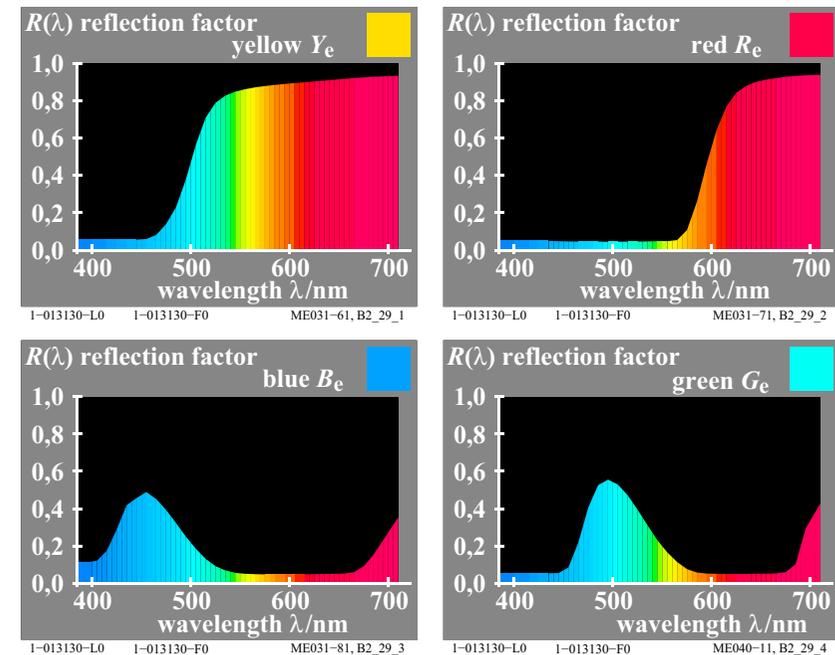


Fig. 26: Spectral reflection factor of the four elementary colours $RYGB_e$

Fig. 26 shows spectral reflection factors which are transferred to "masks" with corresponding transmission factors. With a spectral apparatus for mixing spectral colours one can produce optically the elementary colours $RYGB_e$.

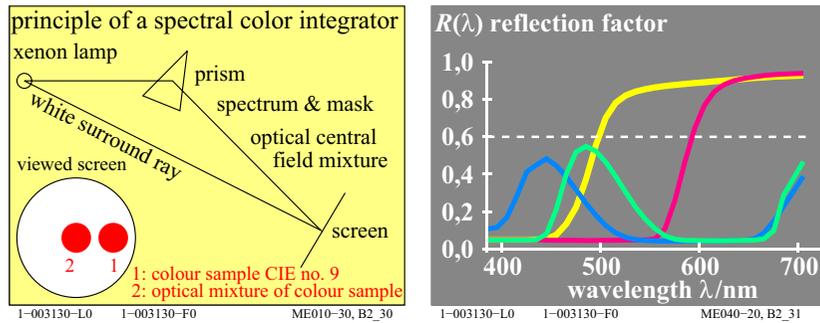


Fig. 27: Apparatus for mixing spectral colours and reflection factor as mask

Fig. 27 (left) shows the principle of an apparatus for mixing spectral colours. From a white xenon arc lamp two light paths are started.

The surround light path produces a white surround with the shape of a circular ring on the projection screen.

In the central-field light path the light is split by a prism into a spectrum. This spectrum is mixed optically and produces a circular white central field. The white light of the central and surround field is equal at the projection screen.

By the help of masks at the location of the spectrum some spectral colours may be partially or totally masked out. The remaining parts of the spectrum are mixed optically. Different masks will lead to different central field colours, for example to the CIE-test colour no. 9 (elementary red R_e according to CIE R1-47).

Fig. 27 (right) shows the masks for the elementary colours $RYGB_e$. The masks are produced according to the reflection factors $R(\lambda)$ of the CIE-test colours no. 9 (red R_e), no. 10 (yellow Y_e), no. 11 (green G_e) and no. 12 (blue B_e). According to CIE 13.3 these CIE-test colours and others are used for the specification of colour rendering properties of light sources. In addition a constant reflection factor $R(\lambda) = 0,6$ is shown which corresponds to a light grey colour.

12 Fluorescence

Fluorescence changes short wave absorption to longer wave radiation. Optical brighteners use this effect. With optical brighteners laundry and paper appears whiter and paints appear more luminous. Luminous red is used as warning colour. The luminous paints or fluorescent colours produce an extension of the normal colour gamut of normal (non-fluorescent) surface colours.

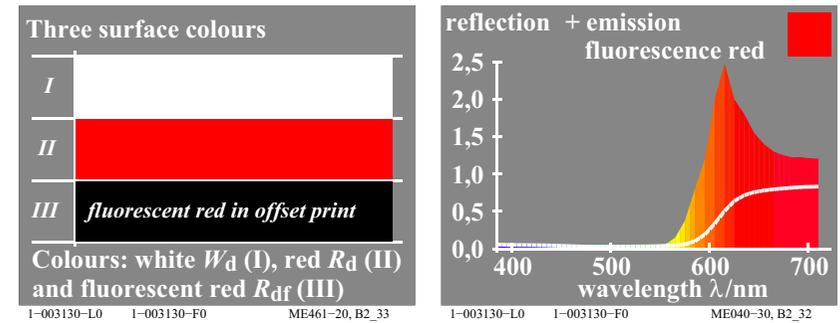


Fig. 28: Surface colours and reflection and emission of a fluorescent colour

Fig. 28 shows three surface colours white W_d (I), red R_d (II) and a fluorescent red R_{df} (III) (left), and the reflection and emission of a fluorescent colour red R_{df} (right). Fluorescent colours reflect more long wave (red appearing) light than a white diffuse reflecting sample. In Fig. 28 (right) the sum of the spectral emission and reflection is for the fluorescent colour red larger than 1,0 in the long wave spectral region. This surface colour appears especially luminous red. Therefore we call this colour a luminous colour.

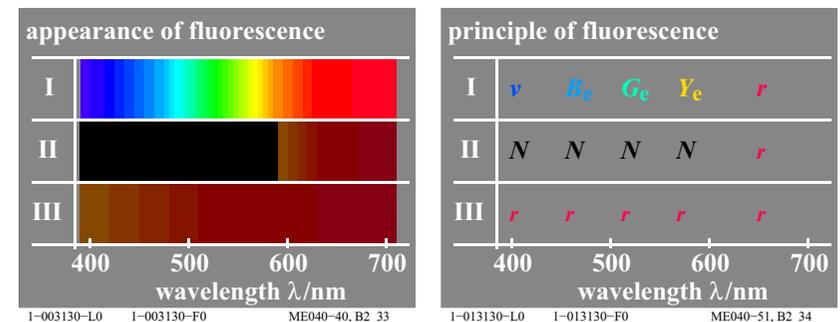


Fig. 29: Spectral appearance and principle of fluorescence

Fig. 29 shows the appearance (left) and the principle (right) of the fluorescence. The spectrum appears on an white surface (I), a normal red surface (II) and a fluorescent red surface (III) very different (left).

The change of the colour appearance of the spectrum can be demonstrated on different colour areas. One produces a spectrum with a continuous interference filter. The spectrum is projected on the three different colour areas I to III:

- The spectrum on a white surface (I) appears as the usual colour series violet-blue v , blue B_e , green G_e , yellow Y_e , and red r (*left*). This is indicated by letters according to the colours (*right*).
- The spectrum on a red surface (II) appears dark in the range violet-blue v to yellow Y_e , and reflects in the red region similar compared to white (*left*). The letter N (=black) indicates absorption in the range violet-blue v to yellow Y_e and the letter r reflection (*right*).
- The spectrum on a fluorescent red surface (III) appears red r in the whole spectral range between violet-blue v and red r (*left*). The letter r for the whole spectrum indicates this reflection property (*right*).

13 Retroreflection

Retroreflective materials appear as particular chromatic and luminous colours under special lighting and observing conditions. The colour is here produced by the illuminant, an achromatic (white appearing) material surface with special geometric reflection properties, and a transparent colour layer as a colour filter. This colour filter has different transmission factors depending on its colouring.

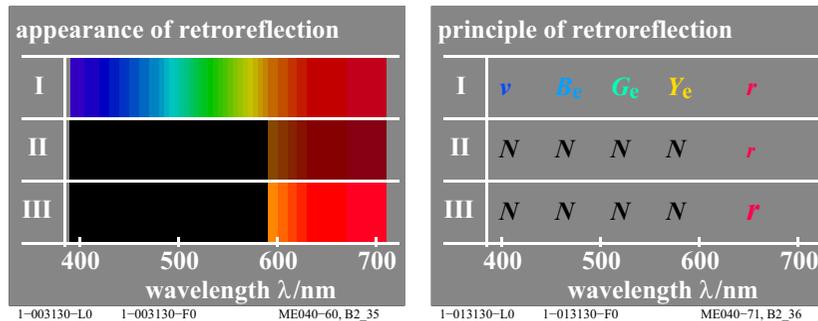


Fig. 30: Spectral appearance and principle of retroreflection

Fig. 30 shows the appearance (*left*) and the principle (*right*) of the retroreflection. The spectrum appears on an white surface (I) and the normal red surfaces (II) and a retroreflective red surface (III) very different (*left*).

The change of the colour appearance of the spectrum can be demonstrated on different colour areas. One produces a spectrum with a continuous interference filter. The spectrum is projected on the three different colour areas I to III:

- The spectrum on a white surface (I) and a normal red surface (II) is already described in Fig. 29.
- The spectrum on a retroreflective red surface (III) appears dark in the range violet-blue v to yellow Y_e , and may reflect in the red region more compared to white (*left*). The letter N (=black) indicates absorption in the range violet-blue v to yellow Y_e and the large letter r an increase of reflection (*right*). This reflection reaches a maximum when the direction of illumination and observation agree.

14 Colour Mixture

14.1 Dichromatic additive Colour Mixture

Miescher has called the additive mixture of two colours a *dichromatic* colour mixture. Similarly one calls a mixture of three colours a *trichromatic* colour mixture. The mixture of two compensatory colours, which may result in an achromatic colour, is according to Miescher (1961, 1965) called an *antichromatic* colour mixture.

Colours of any given spectral distribution can be produced by additive colour mixture with an apparatus for mixing spectral colours, compare Fig. 27 on page 25. It is even possible to obtain the *optimal colours*, representing the possible limits of surface colours. Among the optimal colours the *most chromatic* colours, for example the reddest red, are of large importance for image technology, see section 19 on page 61.

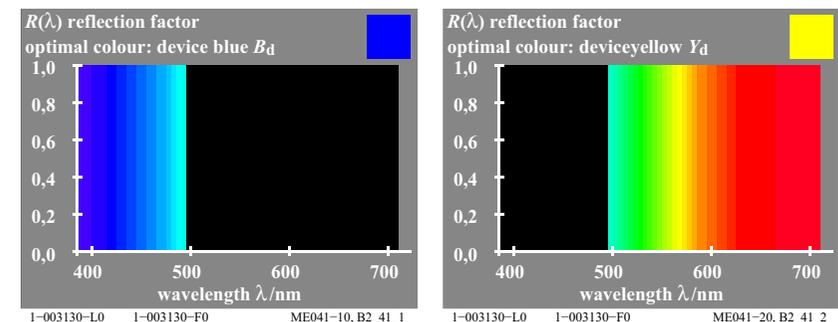


Fig. 31: Dichromatic additive optimal colours B_d , Y_d

Fig. 31 shows the dichromatic mixture of white. White W is produced by additive mixture of any pair of *compensatory* (or complementary) optimal colours

(for example blue B_d and yellow Y_d). In the following and in reproduction processes a greenish yellow which we call the device yellow Y_d ($Y = \text{yellow}$, $d = \text{device}$), and a reddish blue B_d is usually used.

In Fig. 31 (left) the reflection curve of the optimal colour blue B_d has a sharp transition between the value 1,0 and 0,0 at 490nm. The reflection curve has the value 1,0 between 380nm and 490nm and the value 0,0 between 490nm and 720nm.

In Fig. 31 (right) the reflection curve of the optimal colour yellow Y_d has the value 0,0 between 380nm and 490nm with a sharp cut off at 490nm. Between 490nm and 720nm the value is 1,0.

The additive mixture of both optimal colours B_d and Y_d results in an achromatic colour with a spectral reflection curve $R(\lambda)$ of the value 1,0 throughout, which appears white.

Fig. 31 shows two device colours yellow Y_d and blue B_d which are different compared to the elementary colours yellow Y_e and blue B_e .

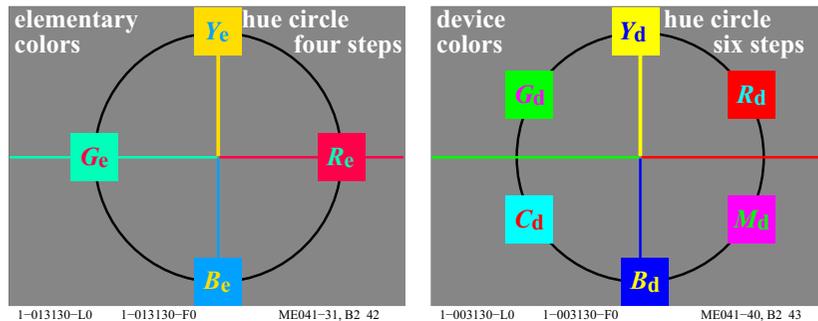


Fig. 32: Four elementary colours $YRGB_e$ and six device colours $RYGCBM_d$

Fig. 32 (left) shows the four elementary colours red R_e , yellow Y_e , green G_e , and blue B_e in a symmetric elementary colour circle.

Fig. 32 (right) shows the six chromatic colours $RYGCBM_d$ of a 6 step hue circle, which serves as basis for the colour reproduction. According to the location in the symmetric hue circle yellow Y_d appears slightly greenish compared to yellow Y_e and blue B_d appears reddish compared to blue B_e .

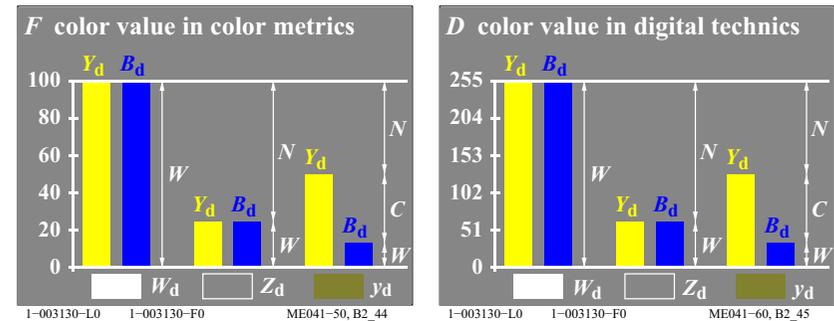


Fig. 33: Dichromatic colour value in colorimetry and digital technic

Fig. 33 shows the colour value F in colorimetry (left), and the colour values D in the digital technology (right). In colorimetry 100 steps and in digital technic 255 steps are used.

Fig. 33 shows mixture colours between a dominant colour yellow Y_d and the compensatory colour blue B_d : white W_d , central grey Z_d , and a yellow colour y_d (bottom right).

If one uses 100% of both the dominant colour Y_d and the compensatory colour blue B_d , then the achromatic mixture is the colour white W_d with the spectral reflection factor of the value 1,0 throughout. It is valid in the left part: white value $W = 100$, black value $N = 0$ and chromatic value $C = 0$. The mixture colour W_d is shown (bottom left).

If one uses only 25% of both the dominant colour Y_d and the compensatory colour blue B_d , then the achromatic mixture is the colour central grey Z . With the apparatus for spectral mixtures the masks may have only two jumps between 0,0 and 0,25. It is valid in the middle part for central grey Z_d : white value $W = 25$, black value $N = 75$, and chromatic value $C = 0$.

If the dominant colour Y_d is larger compared to the compensatory colour blue B_d , then a chromatic colour is produced which has the hue of the dominant colour. It is valid in the right part: white value $W = B_d = 15$, black value $N = 100 - Y_d = 50$, and chromatic value $C = Y_d - B_d = 35$.

The image technology leads to the reproduction of equidistant colour series of the colour attributes. For example equidistant lightness series $\Delta L^* = \text{constant}$ are described on a white surround by the square root of the colour values. For example the CIE tristimulus value Y with the values $Y = 1, 4, 9, 16, \dots, 81, 100$ produce the equal distant lightness series $L^* = 10, 20, 30, \dots, 90, 100$.

The coordinates of the colour attributes are the colorness F^* in colorimetry or the colorness D^* in the digital technic. The group term *colorness* covers the colour attributes lightness, blackness, whiteness, deepness and others. The group term colour values covers the colour values white value, black value, chromatic value and others. There is often a nonlinear (square root) relation between both group terms, for example between lightness L^* and the tristimulus Value Y on a white surround (white paper or white monitor).

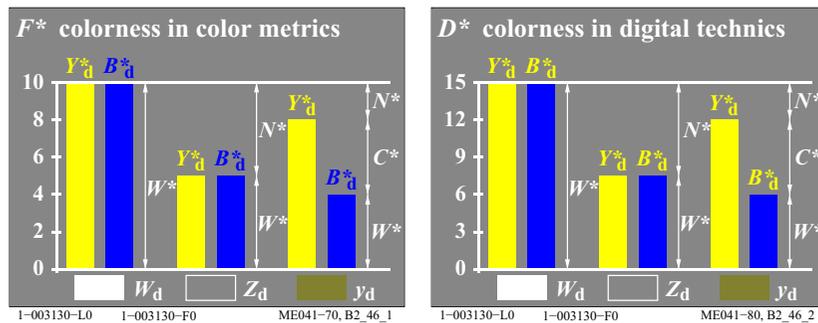


Fig. 34: Colorness in colorimetry and digital technic

Fig. 34 (*left*) shows the colorness $F^* = Y^*_d$ or B^*_d between 0 and 10 used in colorimetry (*left*). 10 steps are used in the *Munsell*-colour system. Fig. 34 (*right*) shows the colorness D^* between 0 and 15 in the digital technic. 15 steps are used in the European standard CEPT for Videotext (*Btx*).

14.2 Trichromatic additive Colour Mixture

White W_d may be produced by additive mixture of *three* optimal colours red R_d (or orange-red O), green G_d (or leaf-green L) and blue B_d (or violet-blue V). *Miescher* called this mixture with three basic colours a trichromatic mixture.

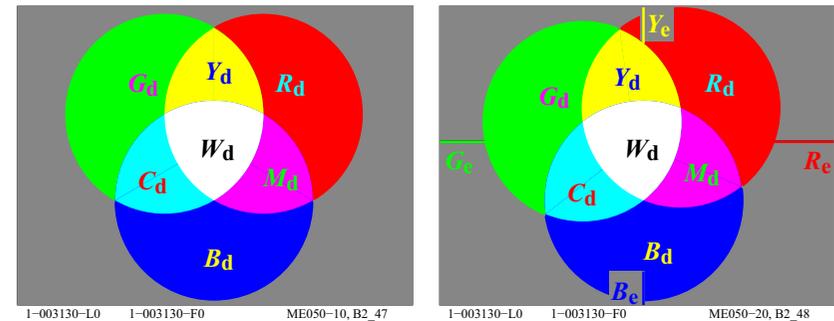


Fig. 35: Trichromatic additive colour mixture and location of elementary colours

Fig. 35 (*left*) shows die additive colour mixture with three basic colours red R_d (or orange-red O), green G_d (or leaf-green L), and blue B_d (or violet-blue V). They mix to three dichromatic mixture colours yellow Y_d , Cyan-blue C_d , and Magenta-red M_d . White W_d is the trichromatic mixture colour with the three basic colours.

Fig. 35 (*right*) shows die location of the additive basic colours, and the dichromatic mixture colours CMY_d , and the trichromatic mixture colour W_d in relation to the four elementary colours $RYGB_e$. It is necessary to consider the difference between R_d and R_e , and between G_d and G_e .

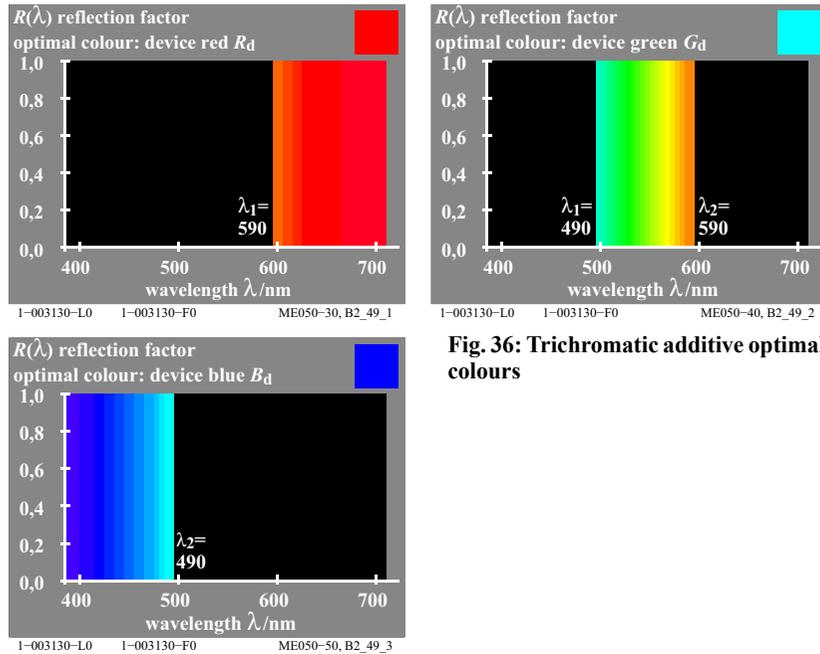


Fig. 36: Trichromatic additive optimal colours

Fig. 36 shows the three optimal colours red R_d , green G_d , and blue B_d , which mix additively to white. The additive mixture with different values of the three basic colours red R_d , green G_d , and blue B_d is of general importance.

In Fig. 36 the three colour values of the device colours R_d , green G_d , and blue B_d are ordered according to their values, in the example it is valid $R_d > G_d > B_d$.

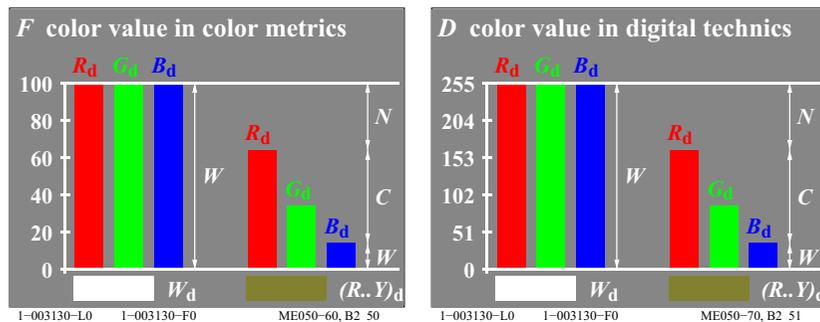


Fig. 37: Trichromatic colour values RGB_d in colorimetry and digital technic

Fig. 37 shows the colour values $F = R_d, G_d,$ and B_d between 0 and 100 in colorimetry (left) and the colour values $F = R_d, G_d,$ and B_d between 0 and 255 in the digital technic (right).

The relation with the black value N , the white value W and the chromatic value C of colours is shown.

Colour attributes of low and high colour metric	Mode of colour mixture	
	dichromatic	trichromatic
<i>low colour- or valence metric</i>	(for $Y_d \geq B_d$)	(for $R_d \geq G_d \geq B_d$)
white value W	B_d	B_d
black value N	$100 - Y_d$	$100 - R_d$
chromatic value C	$Y_d - B_d$	$R_d - B_d$
<i>high colour- or sensation metric</i>	(for $Y^*_d \geq B^*_d$)	(for $R^*_d \geq G^*_d \geq B^*_d$)
whiteness W^*	B^*_d	B^*_d
blackness N^*	$100 - Y^*_d$	$100 - R^*_d$
chromaticness C^*	$Y^*_d - B^*_d$	$R^*_d - B^*_d$

Table 2: Mode of colour mixture, colour value and colorness in colorimetry

Table 2 shows the two modes of colour mixture. The relation between the colour attributes and the colour values Y_d and B_d of the dichromatic colour mixture, and the colour values $R_d, G_d,$ and B_d of the trichromatic colour mixture are given.

The colour attributes of the high colour metric use the group term *colorness* (whiteness, blackness, chroma). In the table the *colorness* is specified by the * (star), for example the whiteness $W^* = B^*_d$.

The abbreviations in Fig. 33 on page 30 and in Fig. 37 on page 33 and Table 2 mean:

Fig. 33 on page 30 for $Y_d \geq B_d$:
 Y_d dominant colour B_d compensatory colour
 W white Z central grey y_d light yellow

Fig. 37 on page 33 for $R_d \geq G_d \geq B_d$
 R_d red G_d green
 B_d blue $(Y..R)_d$ yellow-red

In Table 2 the colour values of the dominant colour yellow Y_d and the compensatory colour blue B_d or the three basic colours red R_d , green G_d , and blue B_d are in a simple relation with the colour attributes: relative white value w , relative black value n , and relative chromatic value c of *Ostwald*.

It is valid, compare Fig. 33 on page 30, and Fig. 37 on page 33:

$$\begin{aligned} w \text{ relative white value} &= \text{white value} / 100 &= W / 100 \\ n \text{ relative black value} &= \text{black value} / 100 &= N / 100 \\ c \text{ relative chromatic value} &= \text{chromatic value} / 100 &= C / 100 \end{aligned}$$

The three colour values RGB_d of colorimetry or of the digital technic may be used to calculate the white value W , the black value N , and the chromatic value C . Based on the nonlinear relation between colour value and colorness the two ratios between white value / black value and whiteness / blackness are different.

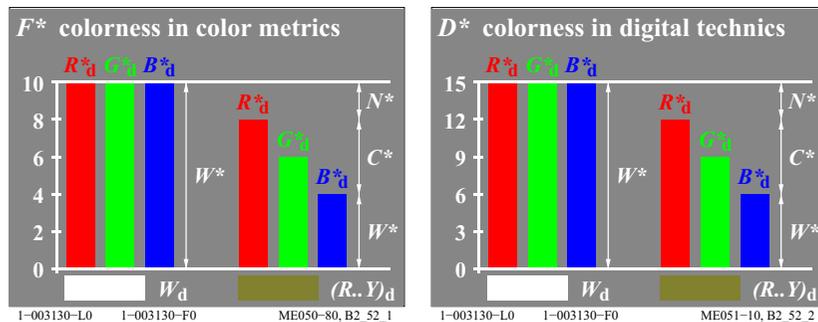


Fig. 38: Colorness RGB^*_d in colorimetry and digital technic

Fig. 38 shows the colorness R^*_d , G^*_d or B^*_d between 0 and 10 in colorimetry (*left*) and between 0 and 15 in the digital technic (*right*). The relation with the blackness N^* , the whiteness W^* and the chroma C^* of colours is given.

Note: In the CIELAB system the lightness L^* and the chroma C^* varies in the range between 0 and 100 instead of 0 and 10, for example in the *Munsell*-colour system.

The most known application of the additive colour mixture is the television and the computer colour monitor. Here the display output is mixed by many raster points red R_d , green G_d , and blue B_d . The luminance of these points is changed by the television signals or the computer image software. On the standard television monitor there are at least 1.2 million luminous points. The points are small and can not be seen separately in a viewing distance of about 3m under normal viewing conditions. An additive raster colour mixture is created on the screen.

14.3 Trichromatic subtractive Colour Mixture

The insertion of three appropriate colour filters in the path of the *same* light source leads (in a white surround) to black if nearly all light is absorbed. Contrary to the additive colour mixture described above, filters are put one upon the other in the path of only *one* light source.

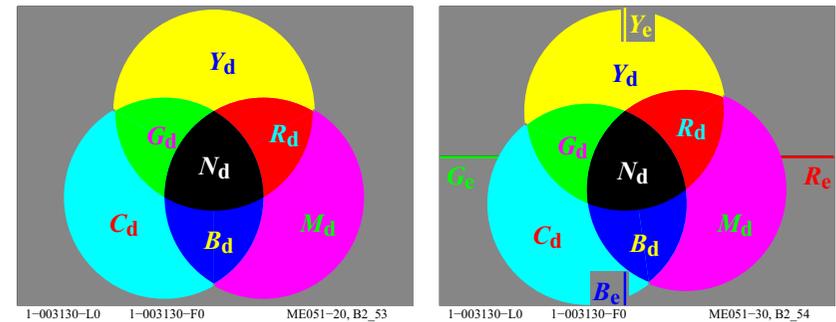


Fig. 39: Trichromatic subtractive colour mixture, and location of elementary colours

Fig. 39 (*left*) shows the subtractive colour mixture with the three basic colours cyan-blue C_d , magenta-red M_d and yellow Y_d . The three dichromatic mixture colours red R_d , green G_d , and blue B_d are produced. Black N_d (= noir) is the trichromatic mixture colour of the three basic colours. For subtractive colour mixture techniques *three* special filters are appropriate with spectral transmission curves similar to the optimal colours yellow Y_d , cyan-blue C_d and magenta-red M_d , see Fig. 40 on page 37.

Fig. 39 (*right*) shows the location of the subtractive basic colours CMY_d , and of the dichromatic mixture colours RGB_d , and of the trichromatic mixture colour N_d . The relative location compared to the four elementary colours $RYGB_e$ is shown. There is an important difference between R_e and R_d or M_d . In the printing area M_d is often named *red* instead of *magenta-red*. In addition there is a difference between B_e and B_d or C_d , which is often named *blue* instead of *cyan-blue* in the printing area.

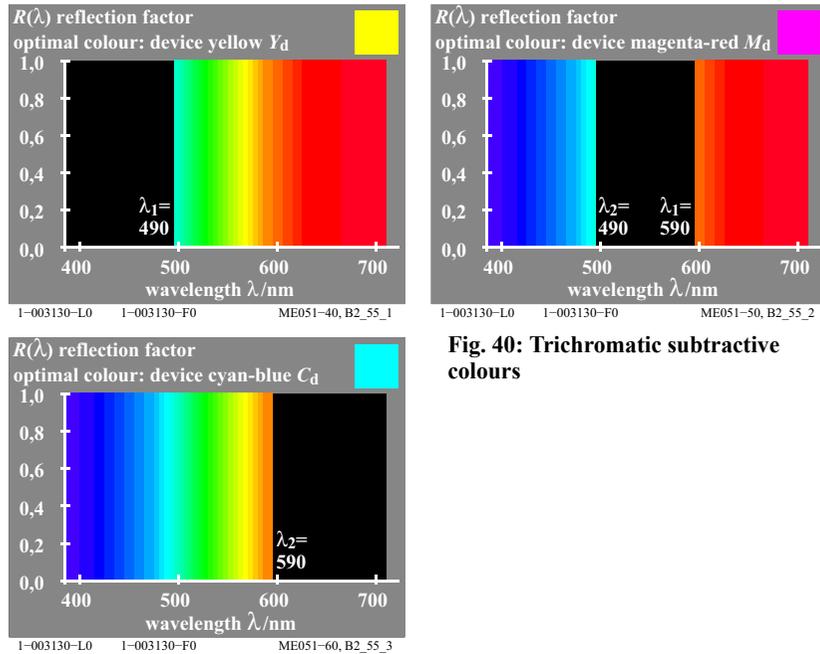


Fig. 40: Trichromatic subtractive colours

Fig. 40 shows die spectral reflection factors $R(\lambda)$ (or transmissions factors $T(\lambda)$ of filters) which are appropriate for the subtractive colour mixture: The optimal colour yellow Y_d with $R(\lambda) = 1$ starting at the wavelength 490nm, the optimal colour magenta-red M_d with $R(\lambda) = 1$ up to the wavelength 490nm and starting again at 590nm, and the optimal colour cyan-blue C_d with $R(\lambda) = 1$ up to the wavelength 590nm.

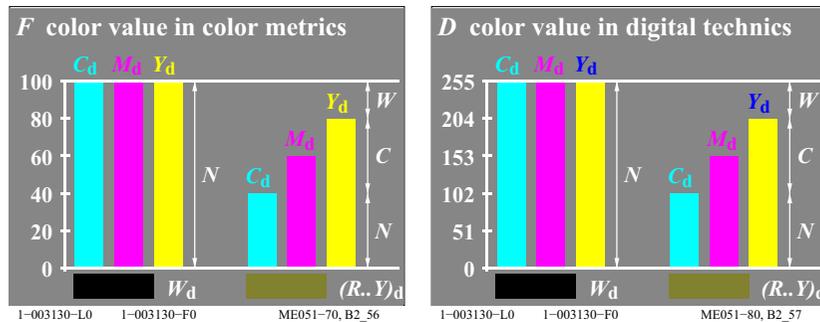


Fig. 41: Colour values CMY_d in colorimetry and digital technic

Fig. 41 shows the colour values F in the colorimetry (left), and the colour values D in the digital technic (right) for a trichromatic subtractive colour mixture.

The specification of the mixture colours based on three standard printing colours cyan-blue C_d , magenta-red M_d , and yellow Y_d is shown. If the colour value of yellow Y_d is dominant compared to the values of magenta-red M_d and cyan-blue C_d , then the mixture of Y_d and M_d leads at first to red R_d . Because of the large value of yellow Y_d , the mixed hue is a yellowish red colour $(R..Y)_d$.

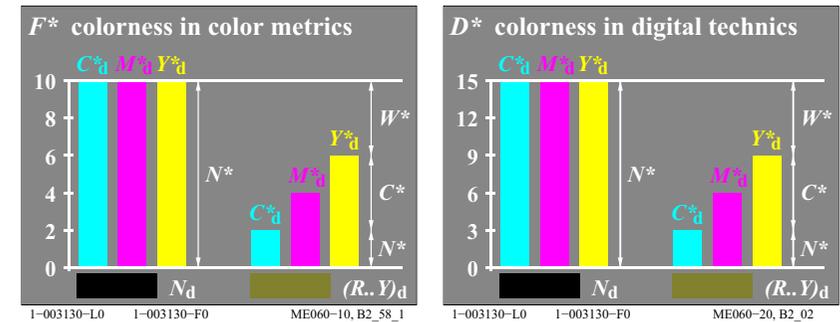


Fig. 42: Colorness CMY_d^* in colorimetry and in digital technic

Fig. 42 shows the colorness F^* in the colorimetry, and the colorness D^* in the digital technic for a trichromatic subtractive colour mixture.

The most known *technical application* of the *subtractive* colour mixture is in *colour photography*. In a colour reversal film (slide film) there are three filter layers, one after another, with colours cyan-blue C_d , magenta-red M_d , and yellow Y_d . The transmission factors of the layers are controlled by the exposure and the developing process.

In standard multicolour printing both additive and subtractive colour mixtures are involved and this kind of mixture is called an auto-typic mixture. The mixture is additive if two printing colours are printed side by side, and subtractive if the two transparent inks are printed on top of each other. One calls this type of mixture in printing a auto-typic mixture.

15 Spectral Radiation

Colours with the same appearance can be created by different spectral distribution of light radiation. In modern colorimetry metameric colours can be calculated with special numerical procedures, taking the illuminance into account.

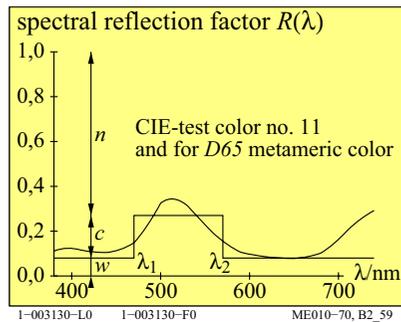


Fig. 43: Reflection factor of metameric colours

Fig. 43 shows the CIE-test colour no. 11 (green) according to CIE 13.3, and a metamer colour of rectangular reflection for the CIE standard daylight D65. Usually one tries to avoid using metamer colours on different parts of an industrial product since such colours will only match under one illuminant. By a change of the illuminant, for example from daylight to incandescent light, *colour differences* appear, the metamer colours no longer match.

Fig. 43 includes the *relative black value* n , the *relative chromatic value* c , and the *relative white value* w . The *Ostwald-equation* $n + c + w = 1$ is valid.

The two compensatory wavelength limits $\lambda_1=480\text{nm}$ and $\lambda_2=580\text{nm}$ for D65 belong to an optimal colour with the elementary hue green G_c . This optimal colour has the maximum chromatic value C_{AB} and creates a *colour half* according to *Ostwald*.

The corresponding *linear* rgb_e -colour values are for $n=0.73$ and $w=0.08$:

$$rgb_e = (w, (1-n), w) = (0.08, 0.27, 0.08)$$

The corresponding *nonlinear (visual)* rgb^*_e -colour values lead to (*for square root relation of white surround*):

$$rgb^*_e = (w^{1/2}, (1-n)^{1/2}, w^{1/2}) = (0.28, 0.52, 0.28)$$

For the reproduction of the CIE-test colour no. 11 in colour printing or on the colour display the rgb_{dc} -colour values (*de = device to elementary hue*) must be calculated and may get approximately the following values:

$$rgb_{dc} = (w, (1-n), w+0.20w) = (0.08, 0.27, 0.10)$$

According to Fig. 10 on page 11, and for production of the elementary colour green G_c the b -value increases by 20% ($=3/F\%=3/15\%$) from 0.08 to 0.10.

For this print output all the calculations shown here are done automatically by the software. The software uses the measurement data of 729 ($=9 \times 9 \times 9$) colours of the output device for the steering of the output.

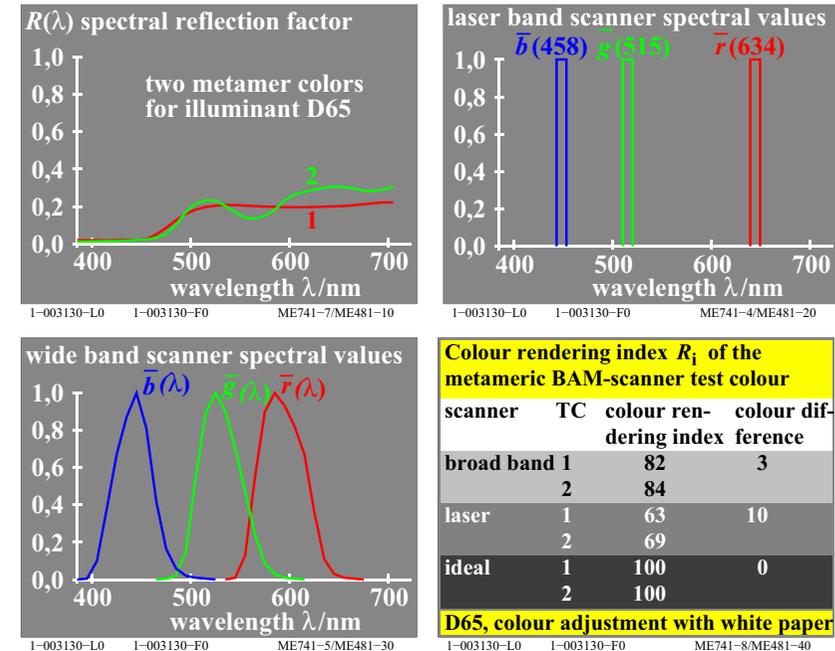


Fig. 44: Two metamer test colours, scanner values and colour rendering

Fig. 44 (*top left*) shows two metamer test colours which are scanned by a laser scanner and a broad band scanner. The scanner values are usually normalized for the white paper to $r=g=b=1$.

Depending on the type of scanner the two colours no. 1 and 2 in Fig. 44 (*top left*) produce usually two different rgb data sets. However, the two colours appear equal for the CIE- illuminant D65 and they have equal CIE-XYZ data.

The rgb -scan values are usually interpreted in the $sRGB$ -colour space according to IEC 61966-2-1, and are then transformed to CIE-XYZ data and to colour differences ΔE^* . For the ideal scanner, which has the broad band sensitivities of the CIE tristimulus values, the rgb values are equal. Maxima and minima of the spectral reflection curves, and the *real* spectral sensitivities of the scanner type, for example a laser or wide band scanner, determine the differences of the rgb scanner values. In Fig. 44 (*bottom right*) the colour differences ΔE^* are in the range 0 to 10.

In the ideal case the colour rendering index R_i according to CIE 13.3 has the value 100. It decreases according to the formula $R_i = 100 - 4,6 \Delta E^*$. For exam-

ple $R_i = 86 (=100 - 3 \cdot 4,6)$ for the colour difference $\Delta E^* = 3$ in Fig. 44 (bottom right).

In offset print and with colour printers the achromatic colours may be printed only with the achromatic colour black N_d or only with the three chromatic colours cyan-blue C_d , magenta-red M_d , and yellow Y_d , especially in images. The achromatic colours, which are printed by the black ink N_d , have approximately a constant reflection curve. The achromatic colours, which are printed by CMY_d , have usually up to three maxima and minima, compare Fig. 44 (top left).

Test and metameric colours for the CIE standard illuminants D65 and A, and the CIE-illuminants D50 and P40 are printed as test charts no. 1 to 3 in the annex with the format A4 landscape. The spectral reflection factors of the samples of three test charts are available. The rgb data are given and the many CIE data for six CIE illuminants D65, D50, P40, A, C, and E have been calculated. The colour samples are based on the 16-step colour circle of the *Relative Elementary colour system RECS*, compare DIN 33872-1 to -6.

CIE R1-47 defines the elementary hue angles for the CIE standard illuminant D65. Elementary yellow Y_e and blue B_e have the hue angles 92 and 272 degree. However, for CIE standard illuminant A the elementary hue angles may shift from 92 and 272 degree to 82 and 262 degree in CIELAB (for D65 and A). The exact values are unknown. Therefore the elementary hues under D65 appear not any more as elementary hues under the CIE standard illuminant A. For example the elementary blue B_e under the CIE standard illuminant D65 appears reddish under the CIE standard illuminant A. Similar the elementary yellow Y_e under the CIE standard illuminant D65 appears greenish under the CIE standard illuminant A. In future the CIE may define the change of the elementary hue angles for the different CIE illuminants.

If in offset print or with colour printers the achromatic colours are only printed with the three colorants CMY instead of *only* N , then this needs 3 times higher material resources. If achromatic colours are *only* printed with CMY then the price may increase by a factor 6. The price for the three chromatic inks CMY is usually twice compared to the achromatic ink N . In addition for the CMY print there are up to three maxima and minima of the reflection curves which may produce colour differences $\Delta E^* = 10$ for equal (metameric) colours, see Fig. 44 (bottom right).

In the annex, the test charts no. 2 and 3 (*PE2311L* and *PE3311L*) use the above two print technologies for the print of achromatic colours. The offset print of the test charts no. 2 and 3 has produced metameric achromatic colours for the four different CIE illuminants D65, D50, P40 and A. In addition there are metameric colours for a 8 step colour circle with half the maximum CIELAB chroma C^*_{ab} compared to the maximum chroma of offset print.

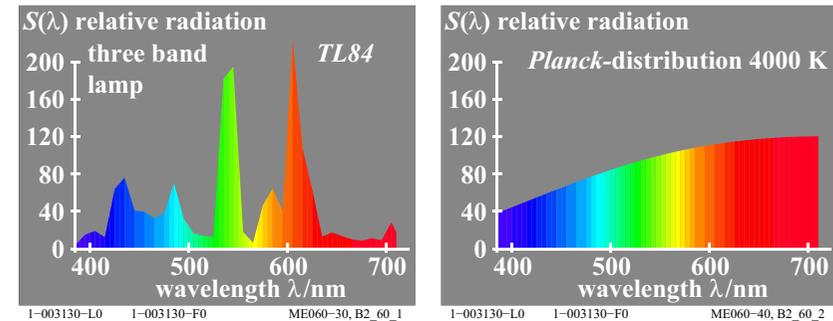


Fig. 45: Relative spectral distribution of the radiation

Fig. 45 shows the relative spectral distribution of the radiation $S(\lambda)$ of a three band fluorescent lamp of high luminous efficiency (energy saving lamp), and a (hypothetical) light source of the colour temperature 4000K according to the radiation law of *Planck*. Both illuminants appear equal white, although they have different spectral distributions.

Alternative illumination of chromatic test colours under these metameric lamps leads to differences in the colour appearance of the test colours. One speaks of this as *differences in the colour rendering properties*, compare CIE 13.3. The two *metameric* colours in Fig. 43 on page 39, which appear equal for daylight D65, appear *different* under the two illuminants TL84 and 4000K in Fig. 45.

The test chart no. 1 to 3 in the annex allow both a *visual evaluation* and a *colorimetric specification* of the *colour rendering properties* of LED lamps and of the *colour reproduction properties* in the field of information system technology. The test charts are based on the *Relative Elementary colour system RECS*, compare DIN 33872-1 to -6.

16 Contrast

Contrast, already known to *Leonardo da Vinci* and described in detail by *Goethe* (1749-1832), is one of the *most important principles of expression* in fine art, arts and crafts, and industrial design. Contrast is conditioned by the mutual influence of different parts of the visual field.

16.1 Achromatic contrast

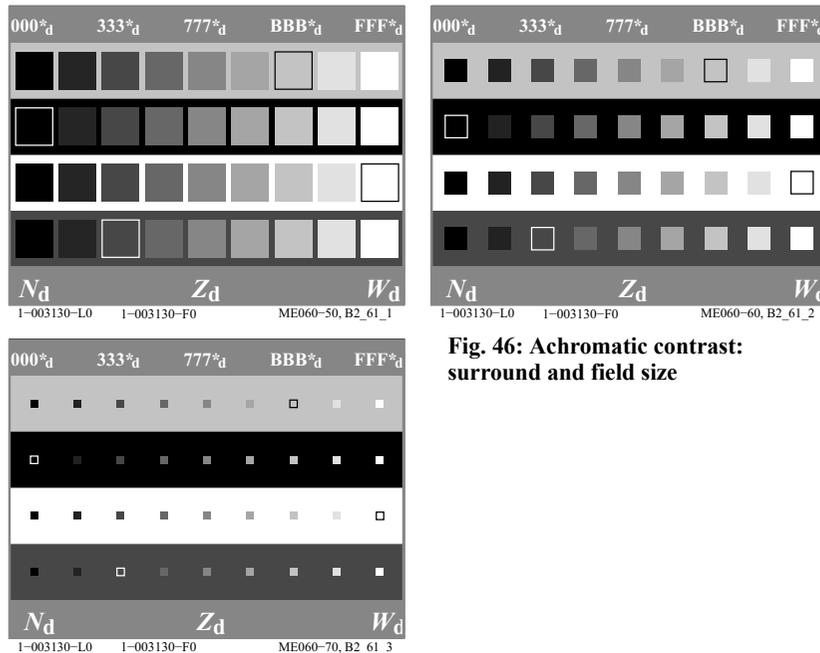


Fig. 46: Achromatic contrast: surround and field size

The perceived lightness of a central field in a light surround shifts in the opposite sense. For example the lightness of different steps of four *physically identical* grey series changes, depending on the surround luminance. Without a lighter reference field there is no grey or black.

Fig. 46 shows four achromatic grey series between black N_d and white W_d viewed against four surrounds, different in lightness. The $rgb*_d$ code (with a star) shall indicate, that on a medium grey surround the nine-step grey series is equally spaced, and has the CIELAB lightness $L^* = 15, 25, 35, \dots, 95$. In a white surround the samples appear *darker* and in a black surround *lighter* compared to a medium grey surround ($Z_d =$ central grey, here shown as rectangle at top and bottom).

According to *Miescher* (1961) an equally spaced scale with 100 steps on a white, medium grey and a black surround obey the following equations for the CIE tristimulus value Y :

- white surround: $L^*_W = 100 (Y_W / 100)^{1/2}$

According to this formula for a medium grey step with $L^*_W = 50$ the CIE tristimulus value is $Y_W = 25$.

- medium grey surround: $L^*_Z = 100 (Y_Z / 100)^{1/2.4}$

According to this formula for a medium grey step with $L^*_Z = 50$ the CIE tristimulus value is $Y_Z = 19$.

- black surround: $L^*_N = 100 (Y_N / 100)^{1/3.0}$

According to this formula for a medium grey step with $L^*_N = 50$ the CIE tristimulus value is $Y_N = 12,5$.

On a medium grey surround the medium grey step with $Y_Z = 19$ in the original has the lightness $L^*_Z = 50$. According to the above formulas $Y_W = 19$ has on a white surround the lightness $L^*_W = 44$, and $Y_N = 19$ has on a black surround the lightness $L^*_N = 58$.

The formulae given for the ranking of the greys on different surrounds provides only a first step in describing contrast by taking the surround into account. The absolute luminance also influences this ranking.

With increasing luminance, the discrimination of individual grey steps increases. White appears more and more white and black more and more black with increasing luminance. This means that the sensory colour difference between white and black also increases. If the illuminance of the grey series is increased from 500 lux to 5000 lux, then the discrimination increases by approximately 20%. This effect seems small compared to the change of the illuminance by the factor 10 (1000%).

Fig. 46 includes three separate figures with different field size of the central fields compared to the surround. The largest contrast influence by the surround occur by a central field with a viewing size of about one degree (1°), and if the field size of the surround is at least 10 times larger ($>10^\circ$).

16.2 Chromatic contrast

The colour of a chromatic surround shifts all the colour attributes of the central field in the opposite direction.

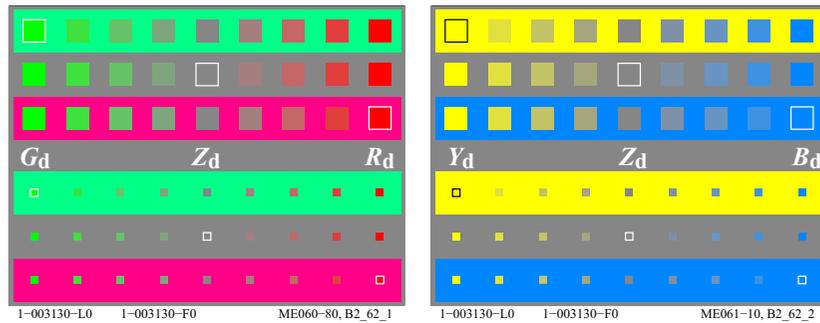


Fig. 47: Chromatic contrast: surround and field size

Fig. 47 shows three physically identical chroma series with equally spaced steps viewed against a medium grey surround Z_d and two surrounds red R_d and green G_d . In Fig. 47 the red samples appear on a green surround redder compared to a red surround. The green samples appear on a red surround greener compared to a green surround. In addition the grey samples Z_d appear in red and green surround not achromatic, but are influenced in *opposite directions*.

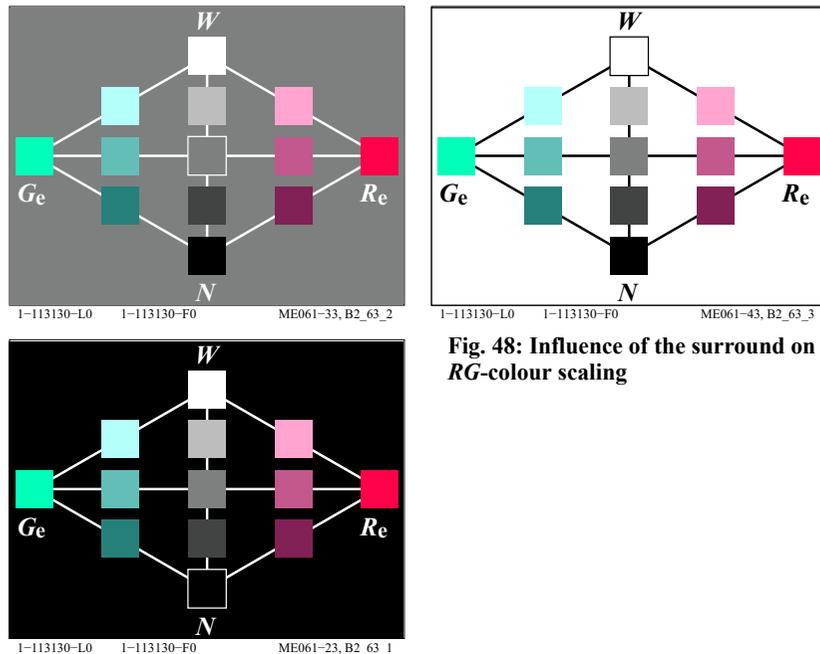


Fig. 48: Influence of the surround on RG-colour scaling

Fig. 48 shows a further important property of the achromatic and chromatic contrast in a hue plane red-green.

The colour multiplicity and the colour gamut appear on a medium grey surround larger compared to a white and black surround. On a black surround many colours appear luminous, and the important component “blackish” is missing. On a white background many colours appear “blackish” and the important attribute “luminous” is missing. On a medium grey background both colour attributes “blackish” and “luminous” are present with an appropriate part compared to the natural viewing.

Fig. 48 shows that all red colours have a better agreement in hue with the elementary red hue R_e compared to Fig. 7 on page 10. In Fig. 48 a 3-dimensional linearization has been used, to calculate (from the undefined rgb -input data in the file) the intended rgb_{de} -coordinates (*Index de = device to elementary hue*). The rgb_{de} -coordinates produce for all red colours the CIELAB-hue $h_{ab}=26$ in the output, which is defined in CIE R1-47 for the elementary hue red R_e .

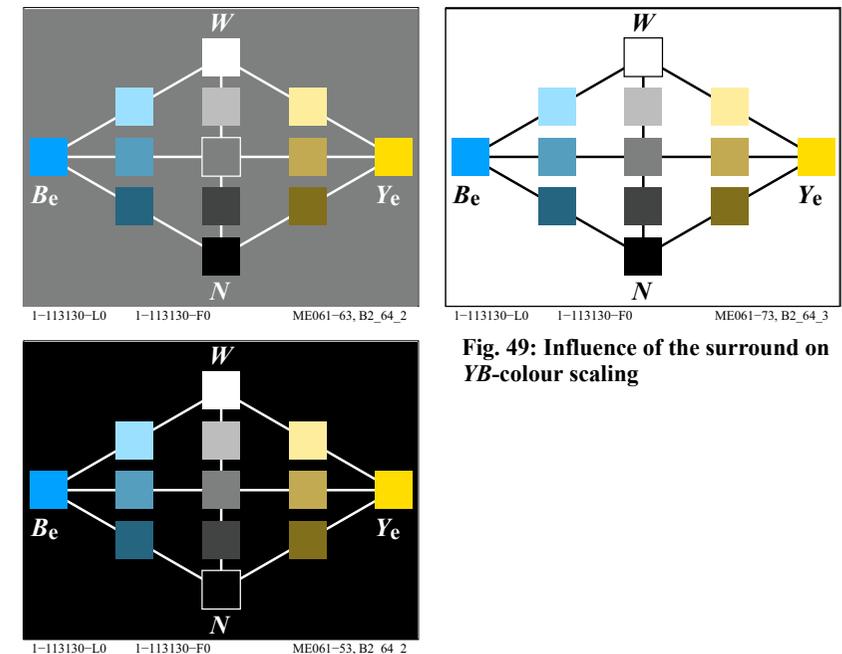


Fig. 49: Influence of the surround on YB-colour scaling

Fig. 49 shows the colour multiplicity and the colour gamut in a hue plane yellow-blue. Similar as for the hue plane red-green, again on a medium grey back-

ground the two colour attributes “blackish” and “luminous” are present with an appropriate part compared to the natural viewing.

The appearance change of colours by the surround colours depends on physiological processes in the eye. Up to now there are only first steps to describe these processes, compare also the section 18 on page 53.

17 Standard Colour Value and Colour Measurement

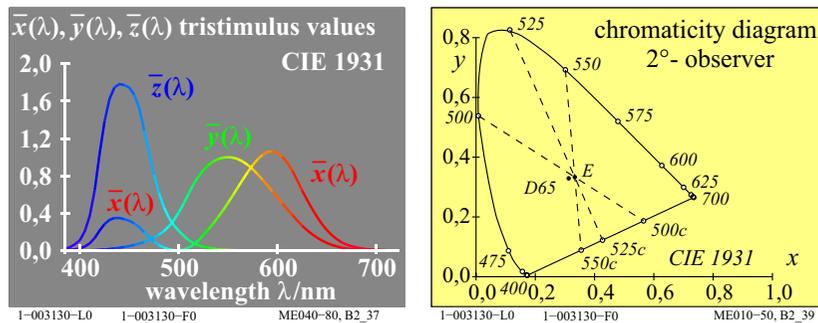


Fig. 50: CIE tristimulus values and CIE chromaticity diagram for 2° observer

Fig. 50 shows the three CIE spectral tristimulus values $x_q(\lambda)$, $y_q(\lambda)$, and $z_q(\lambda)$ for CIE illuminant E (equal energy radiation) between 380nm and 720nm. In Fig. 50 the curves show the colour values of the spectral colours. There are three tristimulus functions which may be specified roughly by three colours blue $z_q(\lambda)$, green $y_q(\lambda)$, and red $x_q(\lambda)$.

For the spectral colours the CIE has defined the spectral chromaticity

$$\begin{aligned} x(\lambda) &= x_q(\lambda) / [x_q(\lambda) + y_q(\lambda) + z_q(\lambda)] \\ y(\lambda) &= y_q(\lambda) / [x_q(\lambda) + y_q(\lambda) + z_q(\lambda)] \\ z(\lambda) &= z_q(\lambda) / [x_q(\lambda) + y_q(\lambda) + z_q(\lambda)] = 1 - x(\lambda) - y(\lambda) \end{aligned}$$

The spectral tristimulus values, the physical spectral radiation, and the spectral reflection of samples allow to calculate the CIE tristimulus values X , Y and Z and the CIE standard chromaticities x , y and z .

$$\begin{aligned} x &= X / (X + Y + Z) \\ y &= Y / (X + Y + Z) \\ z &= Z / (X + Y + Z) = 1 - x - y \end{aligned}$$

Examples of these calculations for the three additive optimal colours are given by *K. Richter* (1996), pages 276-277, for download see (288 pages, 2,8 MB) <http://130.149.60.45/~farbmetrik/BUA4BF.PDF>

In Fig. 50 the CIE spectral chromaticities define the border of the CIE chromaticity diagram. Together with the so called purple line a closed area is defined. The purple line is created by the connection of the chromaticities of the short and long wave spectral colours, approximately $\lambda = 400\text{nm}$ and $\lambda = 700\text{nm}$.

All colours, for example surface colour, spectral, and optimal colours, have chromaticities within or at the edge of the chromaticity diagram.

The chromaticities x and y and the CIE tristimulus value Y , which is normalized to the value 100 for a reference white, specify a colour as well as the CIE tristimulus values X , Y , and Z . The numerical values of the CIE tristimulus values X , Y , and Z are between 0 and 100 according to CIE 15 (“Colorimetry”) for the CIE illuminant E. The chromaticity coordinates are always smaller than 1.0. The x and y coordinates specify chromaticity points on a rectangular (x, y) diagram.

In the (x, y) chromaticity diagram there is a whole colour series with different CIE tristimulus values Y ($0 \leq Y \leq 100$) that belong to each chromaticity point. Therefore colours with a constant chromaticity point in the region “yellow” of the chromaticity diagram can appear either approximately black (for example with $Y=4$) or as a chromatic light yellow (with $Y=90$). Both the chromaticity and the tristimulus value Y are required to define a colour stimulus.

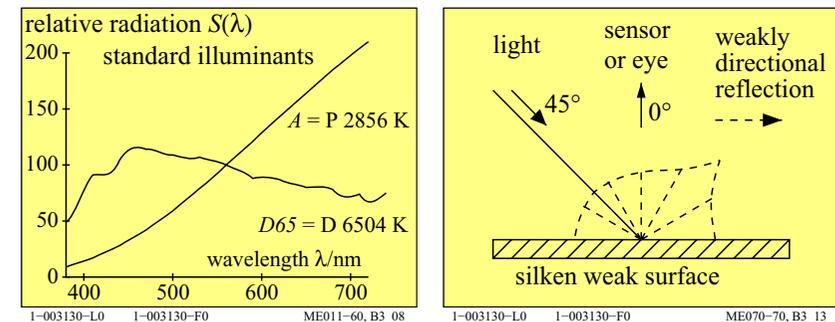


Fig. 51: Radiation of illuminants D65 and A, and CIE-measurement geometry

The standard tristimulus values are dependent on the illuminant, for example the CIE standard illuminants D65 or A, see Fig. 51 (left). The illumination angle of the sample surface is usually 45°. The viewing or measuring angle is usually 0°, see Fig. 51 (right). The silky weak surface of the standard offset print produces usually a diffuse reflection in all directions. However, there is some more reflection in the mirror or gloss direction -45°. Therefore black colours appear blacker under 0° compared to -45°. The CIE tristimulus value is $Y=2,5$ for the angle 0° and approximately $Y=5$ for the angle -45°.

color valence metric (color data: linear relation to CIE 1931 data)		
linear color terms	name and relationship to CIE tristimulus or chromaticity values	notes:
luminous value	$Y = y (X + Y + Z)$	
chromatic value	<i>linear chromatic value diagram (A, B)</i>	
red-green	$A = [X / Y - X_n / Y_n] Y = [a - a_n] Y$ $= [x / y - x_n / y_n] Y$	$n=D65$ (backgr.)
yellow-blue	$B = -0,4 [Z / Y - Z_n / Y_n] Y = [b - b_n] Y$ $= -0,4 [z / y - z_n / y_n] Y$	
radial	$C_{ab} = [A^2 + B^2]^{1/2}$	
chromaticity	<i>linear chromaticity diagram (a, b)</i>	<i>compare to linear cone excitation</i>
red-green	$a = X / Y = x / y$	$L / (L+M) = P / (P+D)$ $S / (L+M) = T / (P+D)$
yellow-blue	$b = -0,4 [Z / Y] = -0,4 [z / y]$	
radial	$c_{ab} = [(a - a_n)^2 + (b - b_n)^2]^{1/2}$	

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Table 3: Colour coordinates of low colour metric or colour valence metric

Table 3 shows the coordinates of the low colour metric or the colour valence metric. All these coordinates are *linear* transformations of the CIE tristimulus values X, Y, Z or the CIE chromaticities x, y and the tristimulus value Y . The main coordinates are the chromatic values $A, B,$ and C and the chromaticities $a, b,$ and c_{ab} . The chromatic values are dependent on the chromaticities of the surround (index n). Usually the chromaticity $x = 0,3127$ and $y = 0,3390$ of the CIE standard illuminant D65 is used.

The CIE-receptor sensitivities of the human colour vision $l_q(\lambda), m_q(\lambda),$ and $s_q(\lambda)$ according to CIE 170-1 are linear functions of the CIE spectral tristimulus values $x_q(\lambda), y_q(\lambda),$ and $z_q(\lambda)$. The luminous efficiency $y_q(\lambda)$ is approximately the sum of $l_q(\lambda)$ and $m_q(\lambda)$. Section 18 on page 53 shows the CIE-receptor sensitivities.

In Table 3 the chromaticity a (red or green content) is roughly equal to the ratio $L / (L+M)$ and the chromaticity b (blue or yellow content) is equal to the ratio $S / (L+M)$. In the literature instead of the letters L, M, S the letters P, D, T are

used, according to the three colour vision deficiencies $P=Protanop, D=Deutanop,$ and $T=Tritanop$.

Anomalies of colour vision, either of the colour receptors in the retina or in the neural signal transmission, result in partially or totally defective colour vision. Defective colour vision occurs in 8% of men, but only 0,5% of woman (ratio 16:1). Most of these people confuse red and green colours. They appear grey or greyish for these persons.

Persons with colour vision deficiencies shall not take some professions, which depend on normal colour vision, for example pilots, bus or taxi drivers, and print technicians. Only very less people confuse the colours yellow and blue. Even less people perceive all colours as achromatic (white, grey, and black). For tests of defective colour vision there are test charts, for example those of *Ishihara* (1953) in which digits or symbols are used. Observers with normal and defective colour vision see different digits or symbols on these test charts. The anomaloscope of *Nagel* according to DIN 6160 allows the determination the degree of the colour vision deficiency.

Higher colour metric (color data: nonlinear relation to CIE 1931 data)		
nonlinear color terms	name and relationship with tristimulus or chromaticity values	notes
lightness	$L^* = 116 (Y / 100)^{1/3} - 16 (Y > 0,8)$ Approximation: $L^* = 100 (Y / 100)^{1/2,4} (Y > 0)$	CIELAB 1976
chroma	<i>nonlinear transform chromatic values A, B</i>	
red-green	$a^* = 500 [(X / X_n)^{1/3} - (Y / Y_n)^{1/3}]$ $= 500 (a' - a'_n) Y^{1/3}$	$n=D65$ (backgr.) CIELAB 1976
yellow-blue	$b^* = 200 [(Y / Y_n)^{1/3} - (Z / Z_n)^{1/3}]$ $= 500 (b' - b'_n) Y^{1/3}$	
radial	$C^*_{ab} = [a^{*2} + b^{*2}]^{1/2}$	
chromaticity	<i>nonlinear transform chromaticities x/y, z/y</i>	<i>compare to log cone excitation</i>
red-green	$a' = (1 / X_n)^{1/3} (x / y)^{1/3}$ $= 0,2191 (x / y)^{1/3}$ for D65	$\log [L / (L+M)]$ $= \log [P / (P+D)]$ $\log [S / (L+M)]$ $= \log [T / (P+D)]$
yellow-blue	$b' = -0,4 (1 / Z_n)^{1/3} (z / y)^{1/3}$ $= -0,08376 (z / y)^{1/3}$ for D65	
radial	$c'_{ab} = [(a' - a'_n)^2 + (b' - b'_n)^2]^{1/2}$	

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Table 4: Colour coordinates of high colour metric or sensation colour metric

Table 4 shows the coordinates of the high color metric or sensation colour metric. The lightness L^* , the chroma coordinates a^* , b^* , and C^*_{ab} , and the chromaticities a' , b' , and c^*_{ab} are the most important coordinates of the high or sensation colour metric. In Table 4 the nonlinear chromaticities a' and b' serve as alternate to calculate the chroma coordinates a^* , b^* , and C^*_{ab} of the CIELAB-colour space. The chromaticity (a' , b') for CIELAB is not defined in CIE 15 and ISO 11664-4. The *nonlinear* chromaticity (a' , b') seems roughly similar compared to the *linear* chromaticity (u' , v') of CIELUV in CIE 15.

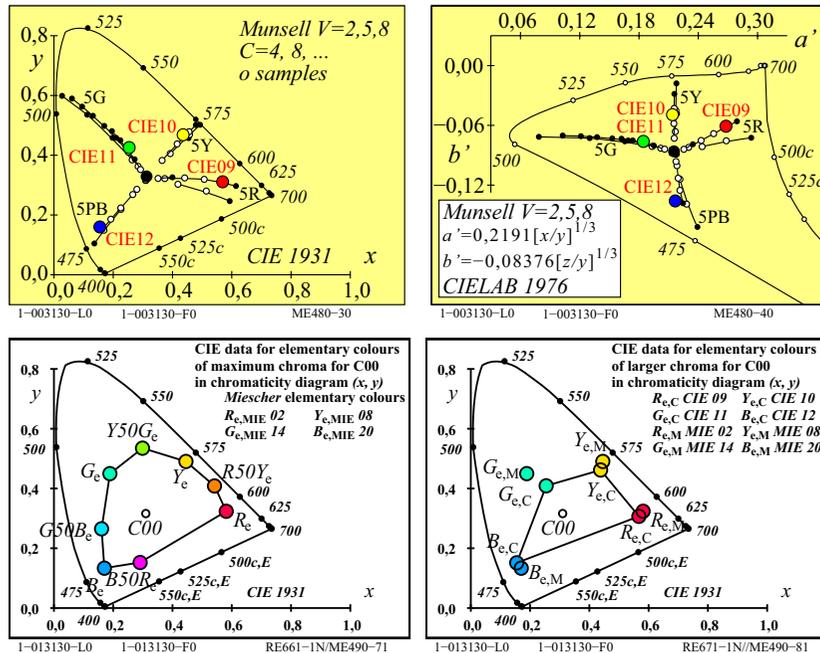


Fig. 52: Munsell, Miescher, and CIE colours in (x, y) and (a', b')

Fig. 52 shows the four elementary hues $RYGB_e$ and in addition the Miescher-intermediate hues $R50Y_e$, $Y50G_e$, $G50B_e$, and $B50R_e$ (bottom left). The colours of the real (o) and the extrapolated (•) four Munsell hues 5R, 5Y, 5G, and 5PB of Value 2, 5, and 8 are shown (top left and right). In the chromaticity diagram (a' , b') the location is more on straight lines compared to (x, y) . The chroma of the four elementary colours of the Miescher-hue circle is larger compared to the four CIE-test colours no. 9 to 12 according to CIE 13.3 (bottom right). The Miescher-hue circle has been produced by 11 appropriate colour inks. In addition the chroma is larger compared to the hue circle of the Relative Elementary

Colour System RECS. This hue circle is produced by the three colours CMY_d of standard offset print, compare Fig. 59 on page 63 (bottom left and right).

The chromaticity diagram (a' , b') is defined in Table 4. It has a large extension to the chromaticity of the wavelength $\lambda=400\text{nm}$. For the application in the information technology this is not a problem, because these colours do usually not exist in both offset printing and on colour monitors. All real colours are located within the region of Fig. 52 (top right).

Fig. 52 shows die elementary colours of Miescher in the chromaticity diagram (x, y) . the colour samples are not located on a circle around the chromaticity of the CIE illuminant C, which is also used in the Munsell-colour system. The chromaticity differences of the CIE illuminants C and D65 are small. Die elementary colours yellow Y_e and blue B_e , and the achromatic point D65 are approximately on a line. Therefore one can mix additively the colours Y_e and B_e in a appropriate mixture ratio to the achromatic colour D65. The elementary colours red R_e and green G_e are *not* located together with D65 on a line. Therefore R_e and G_e mix additively to yellowish-green, yellowish or yellowish-red colours and never to the achromatic colour D65.

Table 3 on page 49 shows colour attributes of the low and high colour metric. It is assumed, that the complementary device colours yellow Y_d and blue B_d mix additively to white. Then one can only mix achromatic, yellow or blue hues. If the yellow value is larger compared to the blue value ($Y_d > B_d$), then the colour values white, black and the chromatic values are calculated according to Table 3 on page 49.

The dichromatic and trichromatic mixture is of large importance for the colour information technology. In the information technology for example on a colour monitor, or with a colour projector, or in multicolour printing all colours are reproduced by three basic device colours RGB_d or CMY_d . The additive colour mixture on a monitor, and a data projector produces with two basic colours and the third colour a dichromatic mixture, for example

$$\begin{aligned} W_d &= R_d + (G_d + B_d) = R_d + C_d \\ W_d &= G_d + (B_d + R_d) = G_d + M_d \\ W_d &= B_d + (R_d + G_d) = B_d + Y_d \end{aligned}$$

In the colour information technology usually a large colour gamut in the reproduction is intended. For this the colours with the maximum chromatic value C_{AB} and approximately of the largest chroma C^*_{ab} are produced by complementary optimal colours with compensatory wavelength limits λ_d and λ_c . Then the dichromatic mixture will lead to white. Colorimetric solutions for a large colour gamut in the output are given in section 19.

18 Special Properties of Colour Vision

The properties of colour vision depend on the properties of the three receptors *LMS* or *PDT* for the daylight vision.

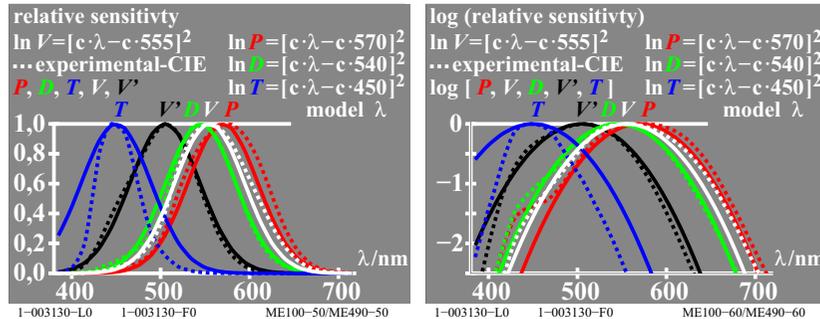


Fig. 53: Relative receptor sensitivities *PDT* (or *LMS*), $V(\lambda)$, and $V'(\lambda)$

Fig. 53 shows the receptor sensitivities *PDT* (according to the colour vision deficiencies P =Protanop, D = Deutanop, and T =Tritanop or *LMS* according to CIE 170-1). The maximum sensitivities are near the wavelength 570, 540, and 450nm.

The long wave receptor (*L* or *P*) has the maximum sensitivity *not* in the region red, but in the yellow-green region. The wavelength $\lambda_m = 570\text{nm}$ of the maximal sensitivity is smaller compared to the dominant wavelength $\lambda_d = 575\text{nm}$ of elementary yellow Y_c . If a logarithmic vertical axis is used, then a parable with approximately an equal shape for all receptors is appropriate. In this case the sum and the differences have special properties:

$$\begin{aligned} \log V(\lambda) &= \log P(\lambda) + \log D(\lambda) \quad (\text{new maximum } 555\text{nm with } 570 \text{ and } 540\text{nm}) \\ \log P(\lambda) &= \log V(\lambda) - \log D(\lambda) \quad (\text{known maximum } 570\text{nm with } 555 \text{ and } 540\text{nm}) \\ \log R(\lambda) &= \log P(\lambda) - \log D(\lambda) \quad (\text{new maximum } 600\text{nm with } 570 \text{ and } 540\text{nm}) \end{aligned}$$

Again in the last line a parable shape is created, and in addition the missing *red sensitivity* $R(\lambda)$ with a maximum at 600nm is defined.

The luminous sensitivity $V(\lambda)$ has a large importance for the colour vision. $V(\lambda)$ serves as basic for the definition of the luminance. According to CIE 15 *Colorimetry* $V(\lambda)$ is calculated linearly by the *Grassmann-law*, for example by:

$$V(\lambda) = P(\lambda) + D(\lambda)$$

The calculation according to the above logarithmic formulas leads to

$$V_{\log}(\lambda) = 10^{\lceil \log P(\lambda) + \log D(\lambda) \rceil}$$

The difference between $V(\lambda)$ and $V_{\log}(\lambda)$ is about 1% for the two wavelength 400 and 700nm compared to the maximum near 555nm, see *K. Richter* (1996). The colour threshold is also near 1%. Therefore for many applications both calculation methods are approximately equal. The spectral luminous sensitivity $V(\lambda)$ is of special importance for the lighting technology. The following ratios calculated with $V(\lambda)$ have special importance for the colour field, for example

$$\begin{aligned} A(\lambda) &= R(\lambda) / V(\lambda) && \text{spectral chromaticity red-green} \\ B(\lambda) &= -T(\lambda) / V(\lambda) && \text{spectral chromaticity yellow-blue} \end{aligned}$$

In applications these ratios correspond to the ratios X / Y and Z / Y . In Table 3 on page 49 this ratios (together with a weighting factor) are called the red-green and the yellow-blue chromaticities a and b . Both define the chromaticity diagram (a, b). For the use of the chromaticity diagram (a, b) instead of the chromaticity diagram (x, y) in applications, and for the description of colour thresholds, see *K. Richter* (1996).

Further properties of colour vision can be described with physiological colour signals in the retina of monkeys. *A. Valberg* (2005) has described many physiological signals as function of chromaticity and luminance of both central and surround fields.

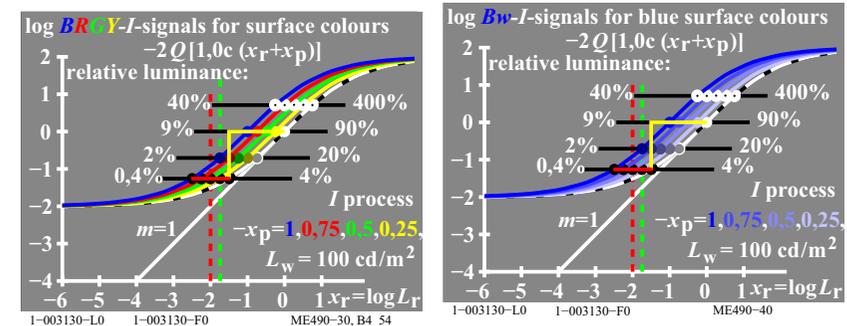


Fig. 54: Colour signals of chromatic and blue colours

Fig. 54 shows in principle the measured colour signals of central field colours with increasing luminance in a white surround. Both scales are logarithmic. The surround (white background w) luminance is 100 cd/m^2 . For the office the illuminance 500 lux is recommended. This corresponds to the luminance 142 cd/m^2 for the white standard offset paper. This luminance is in the range of daylight vision between about 1 cd/m^2 and 10000 cd/m^2 .

In Fig. 54 the *I*-Signals (I =Increment) for achromatic and chromatic colours follow an S-shape curve which saturates at 0,9% and 9000% compared to the

white surround with the value 90%. The curve for chromatic colours (*left*) are shifted compared to the achromatic colours to the left. The curves for chromatic blue colours (*right*) shift with the chromaticity difference to D65 in the chromaticity diagram (*a, b*) to the left. For all chromatic colours equal signals are therefore created for a lower luminance L compared to the surround luminance $L_w=100 \text{ cd/m}^2$ (w = white background).

According to *Ostwald* (1920) optimal colours of maximum chroma are defined by a “colour half”, which has compensatory wavelength limits. The tristimulus value Y and the chromatic value C_{ab} can be calculated according to Table 3 on page 49. The tristimulus value Y and the chromatic value C_{ab} are related linearly. The ratio C_{ab}/Y may serve to describe the shift to the left in Fig. 54 on page 54.

The slope of the S-shape signal curve is largest in the middle. Therefore here the largest luminance discrimination $L / \Delta L$ is expected. The threshold for achromatic and chromatic colours is expected at a luminance, which is at maximum by a factor 36 smaller compared to the white surround. The number 36 may be calculated by the ratio of the tristimulus values $Y_w=90$ and $Y_N=2,5$ of white W and black N .

Probably the luminances of the largest luminance discriminability $L / \Delta L$ are similar to the luminances of the G_0 colours of *Evans* (1967). The G_0 colours appear in a white surround neither blackish nor luminous. According to *Evans* the tristimulus value Y_s at the colour threshold (s =threshold) is for all colours by the factor 30 smaller compared to the tristimulus value of the G_0 colours.

A CIE report of the committee CIE 1-83 *Validity of Formulae for Predicting Small Colour Differences* (Chairman *K. Richter*, DE) with a description of colour thresholds may be produced during 2014. A CIE report CIE R1-57 *Border between blackish and luminous colours* (Reporter *T. Seim*, NO) is planned during 2013.

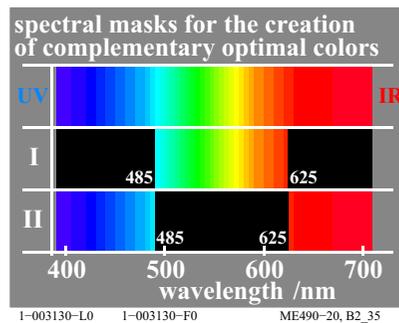
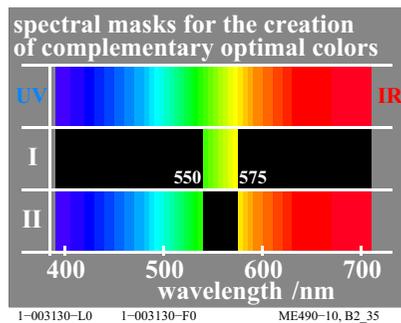


Fig. 55: Complementary optimal colours of different band width

Fig. 55 shows complementary optimal colours with different band width (*left and right*). Such complementary optimal colours are created as edge spectra of white-black and black-white, if one observes the edges of different size with a prism. Already *Goethe* (1830) has observed, that there is equal colour discrimination in the *positive* and *negative* spectrum for neighbouring locations within the continuous complementary colour series.

T. Holsmark and A. Valberg (1971) have mixed the spectral colours of a *positive* and *negative* slit with an apparatus for mixing spectral colours. The *negative* and *positive* slit produces very different optimal colours, for example yellow and blue (*left*) or cyan and red (*right*). For the appearance of a colour difference (threshold) the shift of the slit was approximately equal for the complementary optimal colours.

An improved colour metric for the description of colour thresholds needs therefore equal and anti symmetric coordinates. The chromatic values A and B of Table 3 on page 49 have these property. The chroma coordinates a^* and b^* of the CIELAB space have *not* this property. A colorimetry for colour thresholds, which shall consider the results, is planned in 2014 in a CIE report of the Committee CIE 1-83.

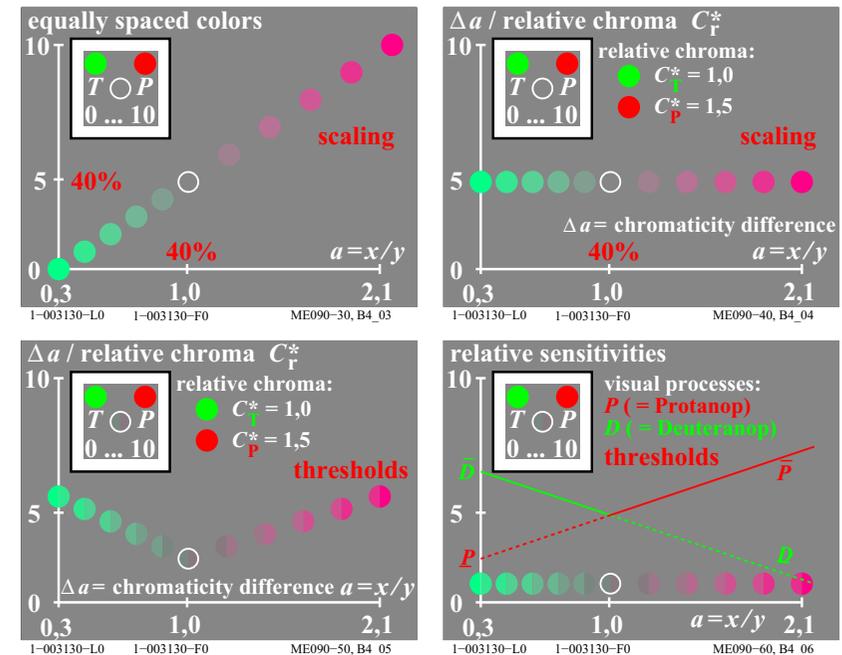


Fig. 56: colour scaling and colour thresholds of the colour series $T - D65 - P$

Fig. 56 shows in principle some experiments and results about colour scaling and colour thresholds as function of the chromaticity $a = x/y$. The colour scaling and the colour thresholds are shown for colours of equal luminance L , in this case with the constant tristimulus value $Y=18$.

Fig. 56 (top left) shows a colour series between a very chromatic turquoise colour T via grey ($D65$ daylight) up to a very chromatic purple-red P . This colour series is approximately equally spaced. The experimental situation is shown in Fig. 57 (top left). In a white surround there is a quadratic grey surround. In this grey surround two "end-colours" were presented, here turquoise T and purple-red. In the lower field it was possible to produce colours of equal luminance between the two end-colours T and P .

The observer gets a fixed scale between the steps 0, 5 and 10 for T , $D65$ and P . By a random process within the experiments digits between 0 and 10 were produced. For 1 the observer shall produce a chromatic turquoise, for 7 a medium chromatic purple, for 5 the achromatic grey of the chromaticity $D65$. The goal of the production of a visual equidistant colour scale both between T and $D65$ and between $D65$ and P was explained explicitly to the observer.

Fig. 56 (top right) shows the results of the experiments in $T - P$ direction. The difference Δa between two neighbouring colour steps (divided by the relative chroma, which was 1 for $T - D65$, and 1,5 for $D65 - P$) as function of the coordinate $a = x/y$ shows a straight line. Equal chromaticity differences Δa (divided by 1 and 1,5) correspond to equal chroma differences. There is a simple description of equal chroma differences by equal differences of a coordinate of the low colour metric (here $a = x/y$).

In addition the visible colour thresholds, this are just noticeable colour differences, has been determined along the same colour series $T - D65 - P$. At first we assumed, that the chromaticity difference Δa for the threshold may be smaller, for example by a constant factor 30. However, the results are different.

Fig. 56 (bottom left) shows the experimental situation. In a white surround there was a grey quadratic surround. In this grey surround two "end-colours" were shown, here turquoise T and purple-red P . In the lower circular field all colours between the two end-colours could be produced. In two half circle equal amounts of T or P could be added. In general for a colour threshold about 1% of the two end-colour was necessary to recognize a colour difference.

Fig. 56 (bottom left) shows the chromaticity difference Δa for colour thresholds as function of the coordinate $a = x/y$ of the low colour metric. The differences Δa for colour thresholds change in the range 1 to 3. The difference Δa is smallest for grey ($D65$), and increase linearly towards T and P . At grey about 30

thresholds correspond to a chroma step. At purple-red P and turquoise T 10 thresholds correspond to a chroma step.

The BAM-research results of *K. Richter* (1985) are in agreement with other results, for example of *Inamura and Yaguchi* (2011). In principle two different kinds of metric are necessary to describe for example the *MacAdam*-ellipses (at threshold) and the colour order systems which apply the colour scaling.

Fig. 56 (bottom right) shows the *relative sensitivities* of two colour vision processes in red-green direction. In each section an other colour vision process determines the recognition of colour thresholds. According to this model the chromaticity difference Δa is small for achromatic colours and large for chromatic colours. According to Fig. 56 (bottom left) the experimental results are opposite. The colour vision model with the colour signals as function of luminance and chromaticity can explain this property.

Fig. 54 on page 54 shows the colour signals of blue colours with an increasing chromaticity difference Δb compared to the achromatic series (right). The *largest luminance* discrimination $L / \Delta L$ is reached on a *horizontal* line and for *decreasing* luminance of blue colours compared to the achromatic white. This is described by the *largest* slope of all the signals on a *horizontal* line (slope change of the signals). The *luminance* discrimination $L / \Delta L$ on a *vertical* line decreases for this blue colours of *equal* luminance, because the slope of the signal curve decreases. If one in addition assumes a linear relation between ΔL and Δb on a *vertical* line, then Δb *decreases* for blue colours of *equal luminance* according to Fig. 54 on page 54.

Fig. 56 (bottom right) seem to show that the *relative sensitivities* increase with the chromaticity difference. This is not true and may be described as follows. The luminance of the black threshold is by a factor 1:36 lower compared to white luminance. According to *Evans* (1974) the luminance of the chromatic threshold is usually lower compared to the black luminance. This result is in agreement with Fig. 56. However with increasing chromaticity difference, for colours of equal luminance the signal slope decreases and therefore the chromatic threshold decreases which seems opposite to Fig. 56 (bottom right).

The research results require at least a colour metric for colour thresholds and a colour metric for scaling, and if possible with transitions. In applications the colour thresholds are important for the determination of small colour differences. The equal spacing of larger colour differences is important for the spacing of colour rendering properties. Colour samples in colour order systems have usually colour differences around 30 colour thresholds (or $\Delta E_{ab}^* = 10$). An example is the colour order system *RAL-Design* (1993), which is based on *CIELAB* and has sample differences of $\Delta E_{ab}^* = 10$ in any hue plane, and for 36 hues.

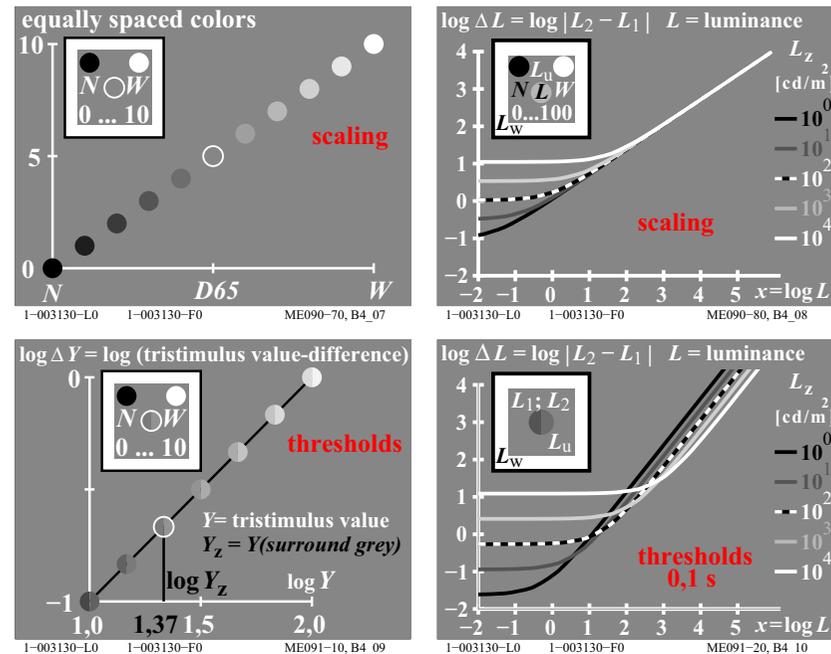


Fig. 57: Luminance scaling and thresholds of lightness series $N - Z - W$

Fig. 57 shows in principle experiments and results of colour scaling and colour thresholds as function of the luminance L . Instead of the luminance L one can also use the tristimulus value Y , which represents a relative luminance and is always normalized to 100 for white. The formula

$$Y = 100 L / L_w$$

uses the central field luminance L and the surround field luminance L_w (outer white frame in the experimental situation, see Fig. 57).

Fig. 57 (top left) shows the equally spaced colour (lightness) scaling for a central field luminance series in the two regions $N - D65$ and $D65 - W$.

Fig. 57 (top right) shows the measured central field luminance differences ΔL as function of the central field luminance L . A \log scale is used on both axis. As a parameter the surround luminance is given. The black-white curve is valid for the surround field luminance $L_z = 100 \text{ cd/m}^2$ of the grey surround u . The luminance $L_z = 100 \text{ cd/m}^2$ corresponds to a medium illuminance of 1500 lux ($=5 \cdot \pi$

$\cdot 100 \text{ lux}$). The factor five is used for a medium grey with the reflection factor 0,2.

Fig. 57 (bottom left) shows the results of colour thresholds along the grey series. For the central field and only a part of the grey scale the tristimulus value difference ΔY (proportional ΔL) is a function of the tristimulus value Y . The threshold ΔY is constant and 1% of the central field tristimulus value Y . The constant slope near the value 1 (or 0,9) is based on the law of *Weber-Fechner* $\Delta Y / Y = \text{constant}$ or $\Delta L / L = \text{constant}$.

Fig. 57 (bottom right) shows in addition the results for very dark and very light colours for a luminance range of six \log units (in Fig. 57 (bottom left) only one unit is shown). The parameter surround-field luminance describes especially the large change of the black threshold with the surround-field luminance. For small central-field luminances L a constant black threshold ΔL_s ($s = \text{threshold}$) is reached. Luminance differences smaller ΔL_s are not visible.

A comparison of Fig. 57 (top and bottom right) shows, that the luminance differences $\Delta L_{\text{scaling}}$ for equally spaced grey series and $\Delta L_{\text{thresholds}}$ for luminance thresholds are not proportional along the same gray scales. The different slopes (about 0,9 and 0,45) are the basis for this statement. One may explain the differences along the grey scale by two visual processes in white-black direction, see *K. Richter* (1996).

Also in Fig. 56 on page 57 for colours of equal luminance along the colour series $T - D65 - P$ two different slopes are necessary for the chromaticity differences $\Delta a_{\text{scaling}}$ for scaling and $\Delta a_{\text{threshold}}$ for thresholds.

A colour vision model for the description of both scaling and thresholds results and the transitions is missing up to now.

19 Elementary Colours and Colour Information Technology

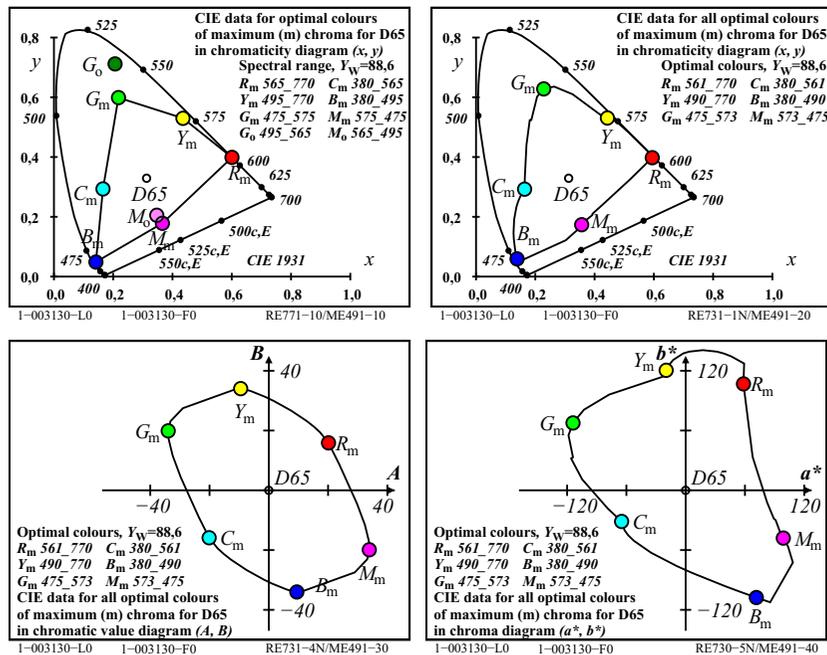


Fig. 58: Three complementary and all optimal colours of maximum chroma

Fig. 58 shows three optimal colour pairs R_m - C_m , Y_m - B_m , and G_m - M_m . The two colours of each pair are complementary, mix to white and are called dichromatic. The chromatic values A and B defined in Table 3 on page 49 are shown in Fig. 58 (bottom left). The chromatic value C_{ab} is equal for the dichromatic optimal colours.

For all optimal colours approximately instead of a triangle in the standard chromaticity diagram (x, y) now an ellipses is produced in the chromatic value diagram (A, B).

The anti symmetry in the chromatic value diagram (A, B) is a requirement for an efficient description of the equal threshold for complementary optimal colours. For this experimental result by *Holtsmark and Valberg* (1969), compare Fig. 55 on page 56.

Dichromatic optimal colours, for example R_m and C_m , include for red and cyan-blue the complementary spectral ranges 565nm to 770nm and 380 to 565nm, compare the description in Fig. 58 (top left). For example the wavelength limits $\lambda_1=380$ nm and $\lambda_2=565$ nm are located with D65 approximately on a line.

The colour cyan-blue C_m of the spectral range 380nm to 565nm (“colour half”) has the largest chromatic value C_{ab} . For example an additional spectral colour red with $\lambda_r=600$ nm mix to a more whitish cyan-blue C_m , and the chromatic value C_{ab} decreases.

Fig. 58 (top left) shows in addition two optimal colours G_o and M_o , which produce a triangle in the standard chromaticity diagram (x, y) together with $RYCB_m$. The spectral range 495 to 565nm of the colour G_o is smaller compared to the range 475 to 575nm (with compensatory wavelength limits) of the colour G_m . Green G_o is therefore darker compared to G_m . In the standard chromaticity diagram (x, y) the chromaticity difference between G_o and D65 is larger as the one between G_m and D65. However, for the chromatic value it is opposite and it is valid $C_{ab,G_o} < C_{ab,G_m}$.

The area of the basic and mixture colours in any chromaticity diagram is therefore not appropriate to specify the colour gamut. However, the colour area in a chromaticity diagram is often used to specify the colour gamut in many IEC and ISO standards. A more appropriate specification uses the chromatic value or chroma area.

Experimental results of *Miescher and Weisenhorn* (1961) with optimal colours in a white surround have shown, that the dichromatic optimal colours which have all the largest chromatic value, have at the same time the largest chroma. However, in many cases the band width was a little smaller compared to the colour half with compensatory wavelength limits.

Fig. 58 (top right) shows all dichromatic optimal colours as continuous curve in the standard chromaticity diagram (x, y) (top right), the chromatic value diagram (A, B) (bottom left) and the CIELAB-chroma diagram (a^* , b^*) (bottom right). These complementary optimal colours have all the maximum chromatic value C_{ab} (bottom left). The calculated wavelength limits (top right) for D65 differ slightly compared to the approximation for the three CIE illuminants D65, E, and C (top left).

The CIELAB-colour system is mainly based on colour scaling of the *Munsell*-colour order system and requires nonlinear coordinates.

For the description of colour thresholds for example the colour vision models of *Guth* (1972) require only linear coordinates.

All dichromatic optimal colours have the same chromatic value C_{ab} . In CIELAB for dichromatic optimal colours the chroma is in the red-yellow region about twice compared to the complementary region cyan-blue. Therefore the CIELAB-definition of chroma C^*_{ab} may be wrong by a factor 2.

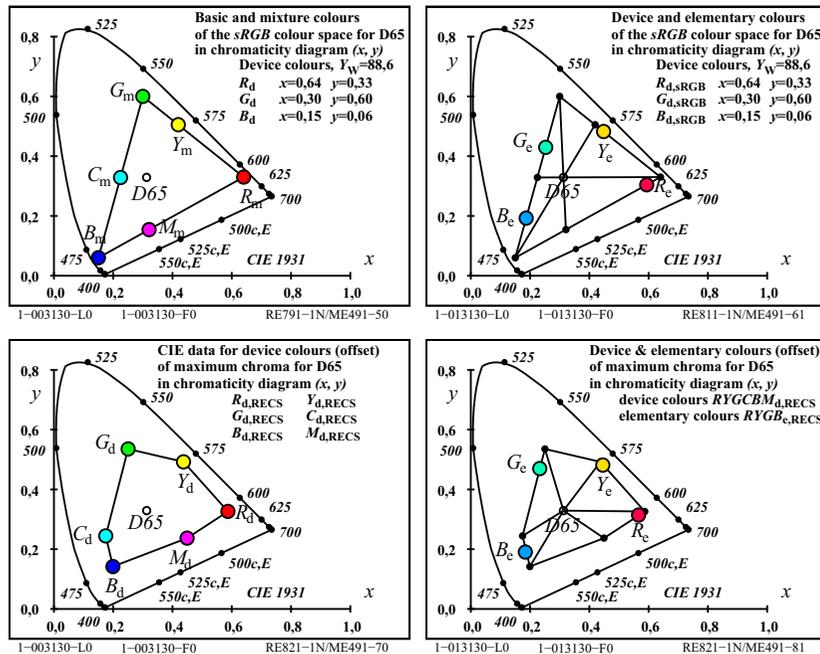


Fig. 59: Device- and elementary colours of the colour spaces *sRGB* and *RECS* (Offset)

Fig. 59 shows the device colours of a *sRGB* standard display, and of the *Relative Elementary Colour System RECS* (standard offset) (top and bottom left). The device independent elementary hues $RYGB_e$ according to CIE R1-47 with the CIELAB-hue angles $h_{ab} = 26, 92, 162,$ and 272 degree (larger balls) may be mixed from six device colours $RYGCBM_d$ (d =device) (top and bottom right).

In the *Relative Elementary Colour System RECS* with about 2000 colours a 16-step hue circle with the four elementary hues $RYGB_e$ as anchor hues is printed, see *RECS*. In each of the four sectors there are three intermediate hues. For the 16 hues there are 5- and 16-step colour series in standard offset printing on standard offset paper.

20 Device independent Elementary Colour Output

For the elementary colours the report CIE R1-47 defines the hue angles $h_{ab,e} = 26, 92, 162$ and 272 which allows a device-independent hue output on any colour device. Fig. 59 on page 63 shows the solution for the *sRGB*-colour space (standard display) and the *RECS*-colour space (standard offset). For the visual rgb^*_e data $(1,0,0)_e, (1,1,0)_e, (0,1,0)_e, (0,0,1)_e$ the device colours of maximum chroma $C^*_{ab,d}$ with the hue angles $h_{ab,e} = 26, 92, 162,$ and 272 are produced. This leads to device specific values for lightness L^*_d and chroma $C^*_{ab,d}$.

As a future next step, and in addition to the definition of the CIELAB-elementary hue angles, the definition of a special lightness L^*_e and a special chroma $C^*_{ab,e}$ for the elementary colours is appropriate. A first definition may be included in 2013 in the report CIE R1-57 “Border between blackish and luminous colours” (Reporter T. Seim, Norway).

For the illuminant D65 the dichromatic optimal colours (index o and e) with the largest chromatic value $C_{ab,oe}$, and the CIELAB elementary hue angles $h_{ab,oe} = 26, 92, 162,$ and 272 may be located at the border “neither blackish nor luminous”. These colours have the following *device-independent* lightness L^*_{oe} and chroma $C^*_{ab,oe}$.

colour	rgb^*_{oe}	L^*_{oe}	$C^*_{ab,oe}$	$h_{ab,oe}$	x_{oe}	y_{oe}	Y_{oe}
R_e	1 0 0	75	65	26	0,57	0,33	48
Y_e	1 1 0	89	136	92	0,47	0,51	73
G_e	0 1 0	79	120	162	0,19	0,52	55
B_e	0 0 1	60	69	272	0,17	0,19	25

It is unknown, if the here calculated optimal colours are located on the visual border between “neither blackish nor luminous”. The *fluorescent* red colour printed in Fig. 28 on page 26 and which appears luminous is above this border.

With displays and special software one perhaps may reach the *natural* colour data LCh^*_{oe} of CIELAB for the standard display luminance 142cd/m^2 . This luminance is required for the standard illuminance 500 lux in offices, and the white paper of offset printing with the standard reflection factor $R(\lambda) = 0.886$.

For surface colours (with no fluorescence and retroreflection) it seems impossible to reach the CIELAB values for green G_e and blue B_e . The shape of the reflection factors of these surface colours is too different compared to the reflection factor of the intended optimal colours, compare Fig. 26 on page 24.

21 Affine Colour Reproduction

In colour image technology the colour gamut is limited in any hue plane approximately by the triangle white - most chromatic colour - black. Both for displays and printing there is approximately an additive mixture at the border of this triangle. Solutions for the reproduction of both the border and within the triangle are most important.

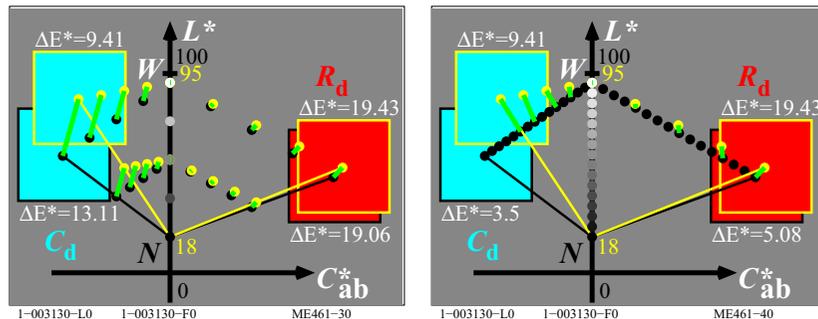


Fig. 60: Affine colour reproduction and minimum colour difference ΔE^*_{ab}

Fig. 60 shows the affine colour reproduction (*left*) and a reproduction with the smallest colour difference ΔE^*_{ab} (*right*) between monitor colours (*yellow balls*) and standard printing colours (*black balls*).

Fig. 60 (*left*) shows that for the hue cyan-blue 20% of the monitor colours and 30% of the printing colours are outside the common reproduction area. The affine colour reproduction uses the whole colour gamut of both devices.

Fig. 60 (*right*) shows the present solution of most colour management methods. The goal is the smallest colour difference ΔE^*_{ab} . In this case 30% of the cyan-blue printing area is not used. The ICC-colour management standard allows company specific solutions and therefore a wide variety of outputs may be produced for example by test charts according to DIN 33872-1 to -6, see <http://www.ps.bam.de/33872E>

For a filled out DIN-form of output questions see (*look for many others nearby*) <http://130.149.60.45/~farbmetrik/LE95>

Any user can ask the device manufacturer for solutions in agreement with DIN 33872-1 to -6 or other international standards, for example ISO/IEC 15775.

This paper may have shown that both the *device-independent hue* reproduction and the *affine* reproduction are possible. One possible next step the *device-independent colour* reproduction may be based on CIE data of Fig. 58 on page 61 for optimal colours of maximum chromatic value and section 21.

22 References

Standards and similar publication

- CIE 13.3:1995, Method of measuring and specifying colour rendering of light sources.
 CIE 15: 2004, Colorimetry, 3rd edition.
 CIE 170-1:2006, Fundamental chromaticity diagram with physiological axes
 CIE R1-47:2009, Hue angles of elementary colours, see <http://div1.cie.co.at/>
 ISO 11664-4:2008(E)/CIE S 014-4/E:2007: Joint ISO/CIE Standard: Colorimetry — Part 4: CIE 1976 $L^*a^*b^*$ Colour Space
 DIN 33872-1 to -6 (2010), Information technology - Office machines - Method of specifying relative colour reproduction with YES/NO criteria, see <http://www.ps.bam.de/33872E>
 ISO/IEC TR 24705 (2005) Information technology - Office machines - Method of specifying relative colour reproduction with YES/NO criteria

Author publications

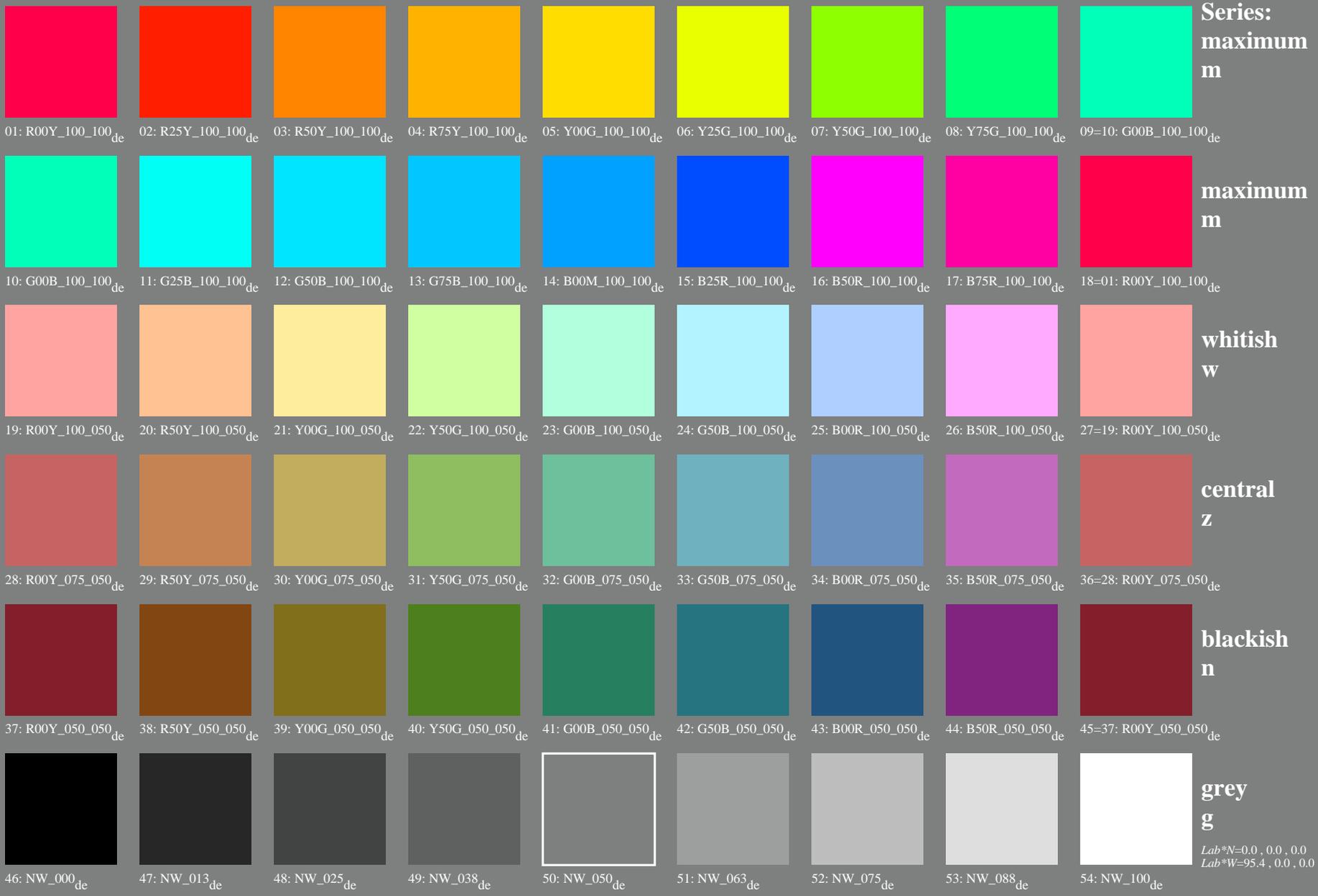
- Evans, R. M. and S. B. Swenholt (1967), Chromatic Strength of Colors; Dominant Wavelength and Purity, J. opt. Soc. Amer. 57, S. 1319-1324
 Holtsmark, T. and Valberg, A. (1969), Colour discrimination and hue, Nature, Volume 224, October 25, S. 366-367.
 Miescher, K. (1948), Neuermittlung der Urfarben und deren Bedeutung für die Farbordnung, Helv. Physiol. Acta 6, C12-C13
 Miescher, K., Richter, K. und Valberg, A. (1982), Farbe und Farbsehen, Beschreibung von Experimenten fuer die Farbenlehre, Farbe + Design, no. 23/24
 Newhall, S.M., D. Nickerson and D.B. Judd (1943), Final report of the O.S.A. subcommittee on the spacing of the Munsell colors. J. opt. Soc. Amer. 33, S. 385-418
 Richter, K. (1979), BAM-Forschungsbericht Nr. 61, Beschreibung von Problemen der höheren Farbmetrik mit Hilfe des Gegenfarbsystems, 99 Seiten, ISSN 0172-7613
 Richter, K. (1980), Cube root colour spaces and chromatic adaptation, Color Research and Application 5, no. 1, pages 25-43
 Richter, K. (1985), BAM-Forschungsbericht Nr. 115, Farbempfindungsmerkmal Elementaruntunnt und Buntheitsabstände als Funktion von Farbart und Leuchtdichte von In- und Umfeld, 118 Seiten, ISBN 3-88314-420-7
 Richter, K. (1996), Computergrafik und Farbmetrik - Farbsysteme, *PostScript* und geräteunabhängige CIE-Farben, VDE-Verlag, 288 pages with about 500 colour figures and many additional references, see <http://www.ps.bam.de/buche>
 Richter, K (2011) ISO-CIE trend for the description of colour threshold data by new coordinates based on the device independent elementary colour coordinates of the report CIE R1-47:2009, see http://130.149.60.45/~farbmetrik/CIE_ISO_10.PDF
 Valberg, A. (2005), Light, Vision, Color, Wiley, ISBN 0470 849037, 462 pages.

Test chart 1 for color rendering: 54 standard colours for D65; standard display (sRGB); rgb->rgb*de

See similar files: <http://130.149.60.45/~farbmetrik/PE10/PE10LOFP.PDF> /.PS
Technical information: <http://www.ps.bam.de> or <http://130.149.60.45/~farbmetrik>

TUB registration: 20120401-PE10/PE10LOFP.PDF /.PS
application for measurement of monitors and data projectors

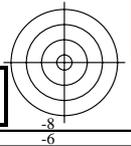
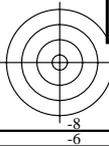
TUB material: code=rh4ta



J-113130-L0 PE100-73

TUB-test chart PE10; Colour rendering in image technology
54 standard colours, colorm=1, deintp=1, sRGB

input: rgb/cmyk -> rgb_de
output: 3D-Linearization to rgb*de

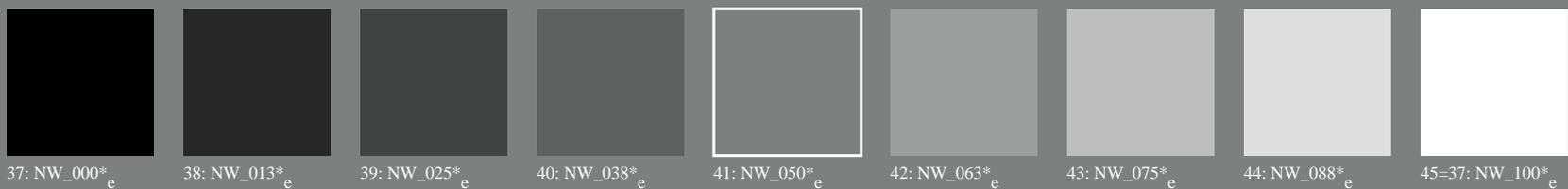
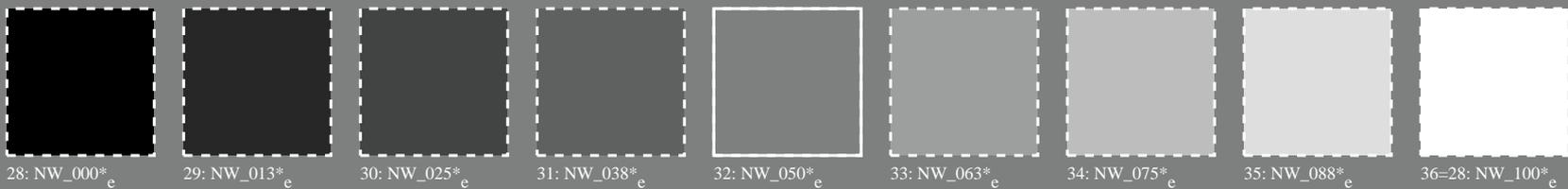


Test chart 2 for color rendering: metameric colours D65 and D50; standard display (sRGB); rgb->rgb*de



metameric colours only possible for offset print and colour printers

metameric m D50



metameric colours only possible for offset print and colour printers

metameric m D50

See similar files: http://130.149.60.45/~farbmetrik/PE20/PE20LOFP.PDF /.PS
Technical information: http://www.ps.bam.de or http://130.149.60.45/~farbmetrik

TUB registration: 20120401-PE20/PE20LOFP.PDF /.PS
application for measurement of monitors and data projectors

TUB material: code=rha4ta

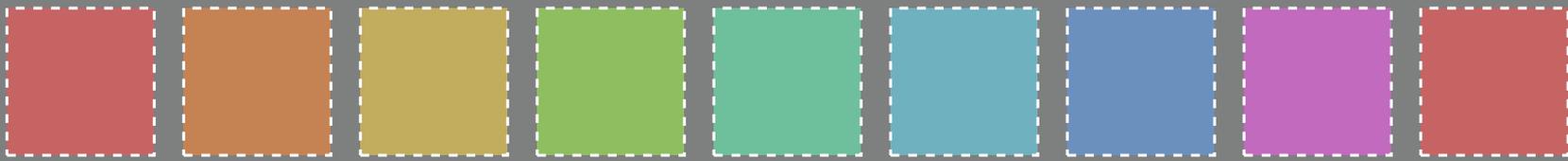


See similar files: <http://130.149.60.45/~farbmetrik/PE30/PE30LOFP.PDF> / .PS
Technical information: <http://www.ps.bam.de> or <http://130.149.60.45/~farbmetrik>

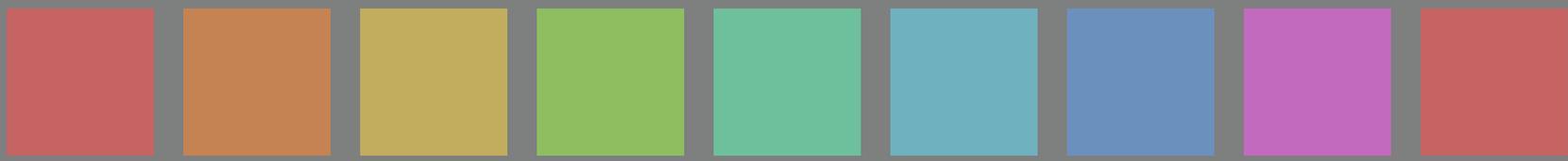
TUB registration: 20120401-PE30/PE30LOFP.PDF /.PS
application for measurement of monitors and data projectors

TUB material: code=rha4ta

Test chart 3 for color rendering: metameric colours A and P4000; standard display (sRGB); rgb->rgb*de



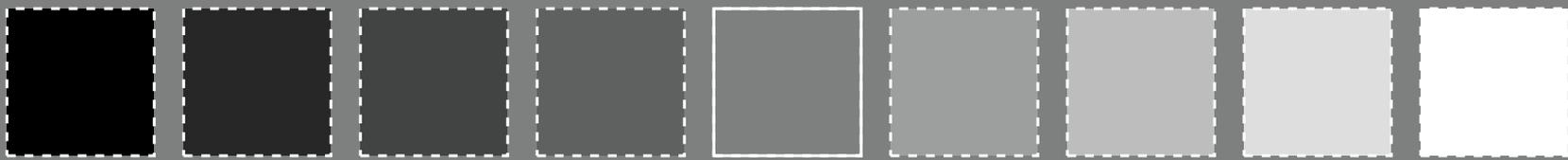
Series:
metameric
m
A



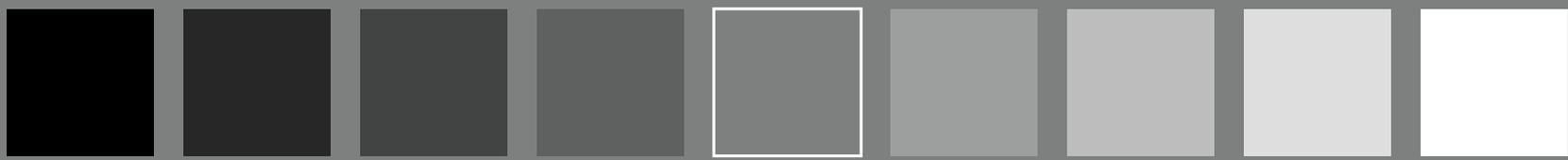
central
z
A/P4000

metameric colours only possible for offset print and colour printers

metameric
m
P4000



metameric
m
A
Lab*N=0.0, 0.0, 0.0
Lab*W=95.4, 0.0, 0.0



grey
g
A/P4000

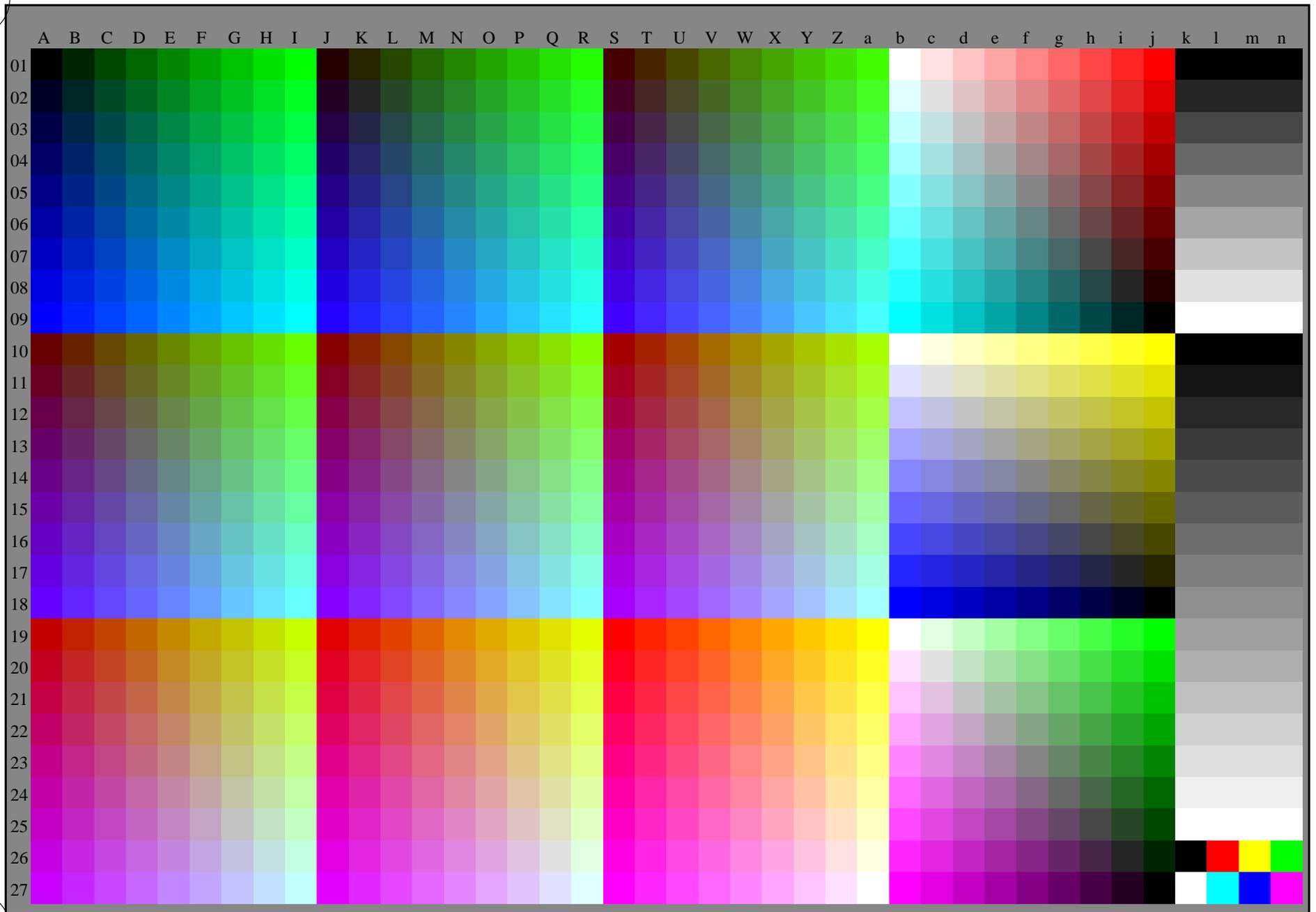
metameric colours only possible for offset print and colour printers

metameric
m
P4000



See similar files: <http://130.149.60.45/~farbmetrik/PE40/PE40L0NP.PDF> / .PS
Technical information: <http://www.ps.bam.de> or <http://130.149.60.45/~farbmetrik>

TUB registration: 20120401-PE40/PE40L0NP.PDF /.PS TUB material: code=rh4ta
application for measurement of sRGB display, no separation (RGB)



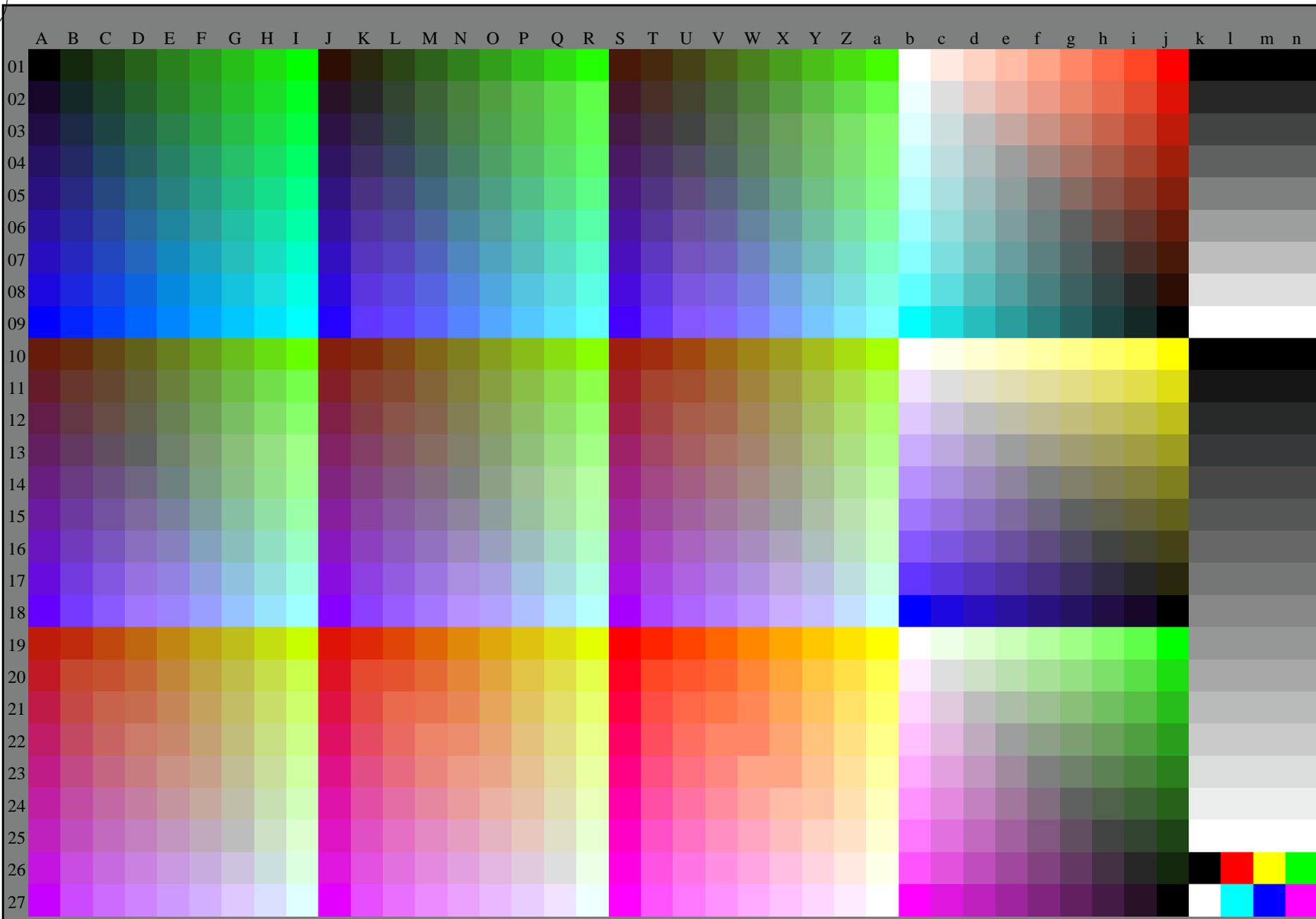
TUB-test chart PE40; standard test chart with 1080 colours
1080 standard colours, colorm=0, deintp=0, rgb

input: *rgb/cmyk* -> *rgb_d*
output: Transfer to *rgb_d*



See similar files: <http://130.149.60.45/~farbmetrik/PE40/PE40LOFP.PDF> / .PS
Technical information: <http://www.ps.bam.de> or <http://130.149.60.45/~farbmetrik>

TUB registration: 20120401-PE40/PE40LOFP.PDF /.PS TUB material: code=rh4ta
application for measurement of sRGB display, no separation* (RGB)



TUB-test chart PE40; standard test chart with 1080 colours
1080 standard colours, colorm=1, deintp=0, rgb*

input: rgb/cmyk -> rgb_{dd}
output: 3D-Linearization to rgb*_{dd}

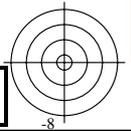
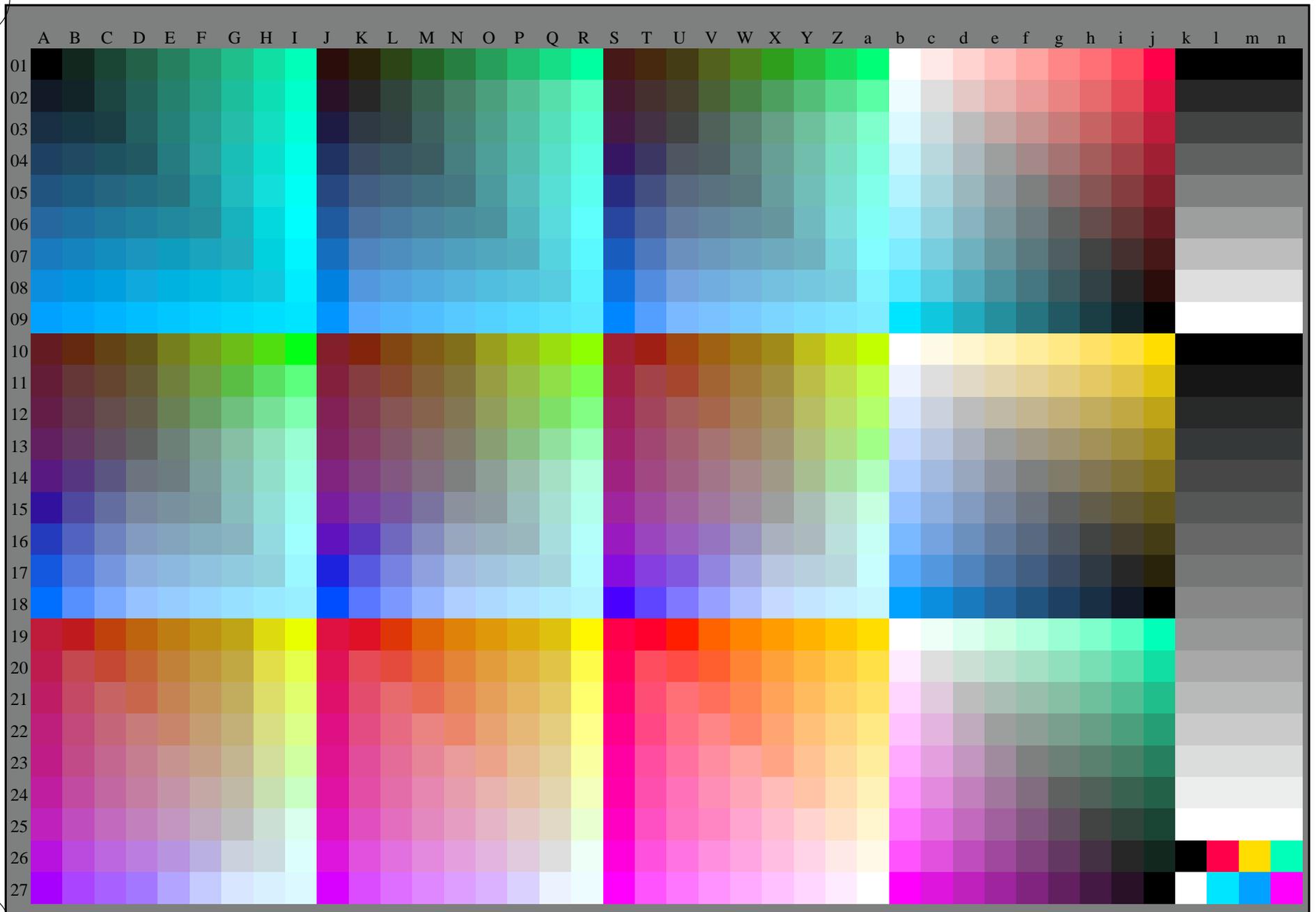




See similar files: <http://130.149.60.45/~farbmetrik/PE40/PE40LOFP.PDF> / .PS
Technical information: <http://www.ps.bam.de> or <http://130.149.60.45/~farbmetrik>

TUB registration: 20120401-PE40/PE40LOFP.PDF /.PS
application for measurement of sRGB display, no separation* (RGB)

TUB material: code=rh4ta



1=113130-L0

PE400-7N, Page 2/1-L

Test chart G with 40x27=1080 colours; digital equidistant 9 or 16 step colour scales; Colour data in column (A-n): **rgb (A_n), colorm = 1**

TUB-test chart PE40; standard test chart with 1080 colours
1080 standard colours, colorm=1, deintp=1, **rgb***

input: **rgb/cmyk** -> **rgb_{de}**
output: 3D-Linearization to **rgb*_{de}**

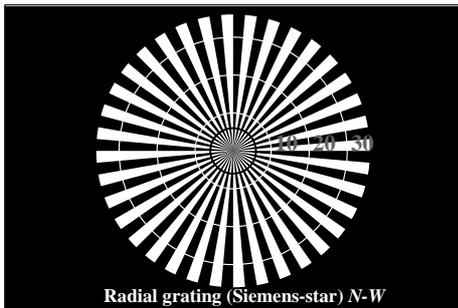
1=113130-F0

C M Y O L V

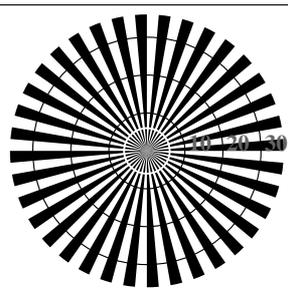
http://130.149.60.45/~farbmetrik/RE98/RE98L0FP.PDF /.PS; 3D-linearized output
F: 3D-Output Linearization (OL) data RE98/RE98LE30FP.DAT in File (F), Page 2/1

See: <http://www.ps.bam.de/ME16>, <http://130.149.60.45/~farbmetrik/RE98/>
Technical information: <http://www.ps.bam.de> or <http://130.149.60.45/~farbmetrik>

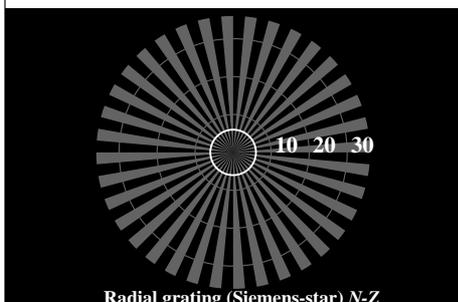
TUB registration: 20120401-RE98/RE98L0FP.PDF /.PS
application for measurement of offset print, separation cmyn6* (CMYK)
TUB material: code=rh4ta



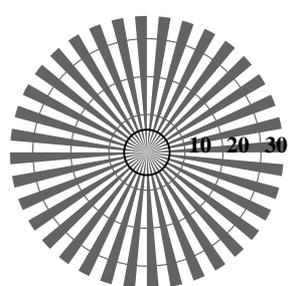
Radial grating (Siemens-star) N-W



Radial grating (Siemens-star) W-N



Radial grating (Siemens-star) N-Z



Radial grating (Siemens-star) W-Z

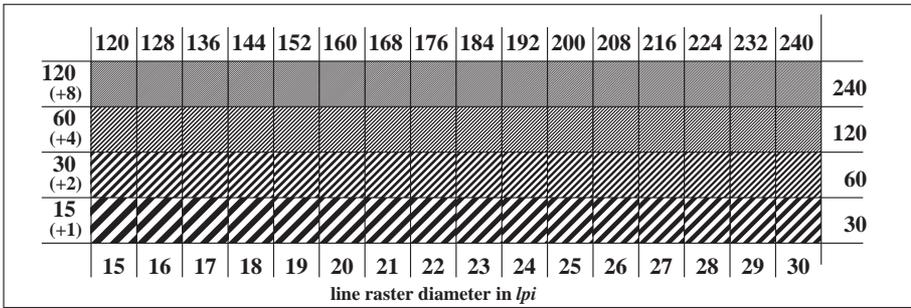
RE980-3, Element A: Radial gratings (Siemens-stars) N-W, W-N, N-Z and W-Z; PS operator: w* setgray

background step 0 Hex code 7 E 2 8 F

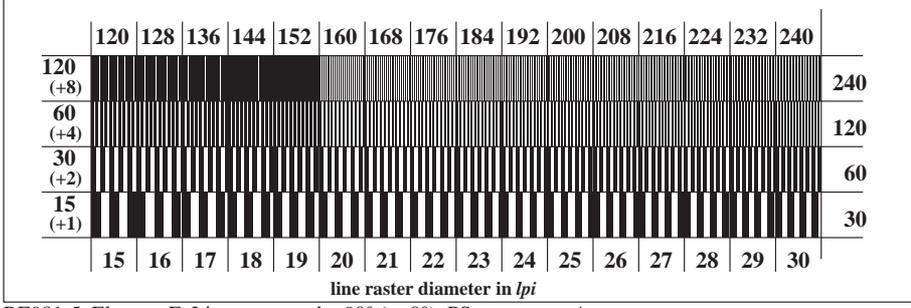
ring step 0-1 Hex code 8 F 0 6 D

Landolt-rings W-N code: background-ring

DM110-1, Element D: Landolt-rings W-N; PS operator: w* setgray



RE981-3, Element E: Line raster under 45° (or 135°); PS operator: w* setgray



RE981-5, Element F: Line raster under 90° (or 0°); PS operator: w* setgray

$L^*/Y_{intended}$ (absolute)	18.0/2.5	37.3/9.7	56.7/24.6	76.1/49.9	95.4/88.6	N_0 (min.)	W_1 (max.)
$w^* = I^*_{CIELAB, r}$ (relative) w^*_{input}	0,000	0,250	0,500	0,750	1,000	N_0 (min.)	W_1 (max.)

RE980-5, Element B: 5 visual equidistant L*-grey steps + N0 + W1; PS operator: w* setgray

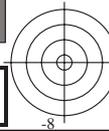
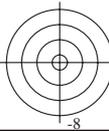
$L^*/Y_{intended}$ (absolute)	18.0/2.5	23.2/3.8	28.3/5.6	33.5/7.8	38.6/10.5	43.8/13.7	49.0/17.6	54.1/22.1	59.3/27.3	64.4/33.3	69.6/40.2	74.8/47.9	79.9/56.5	85.1/66.2	90.2/76.8	95.4/88.6
No. and Hex code	00;F	01;E	02;D	03;C	04;B	05;A	06;9	07;8	08;7	09;6	10;5	11;4	12;3	13;2	14;1	15;0
$w^* = I^*_{CIELAB, r}$ (relative) w^*_{input} / w^*_{output}	0,000	0,067	0,133	0,200	0,267	0,333	0,400	0,467	0,533	0,600	0,667	0,733	0,800	0,867	0,933	1,000

RE980-7, Element C: 16 visual equidistant L*-grey steps; PS operator: w* setgray

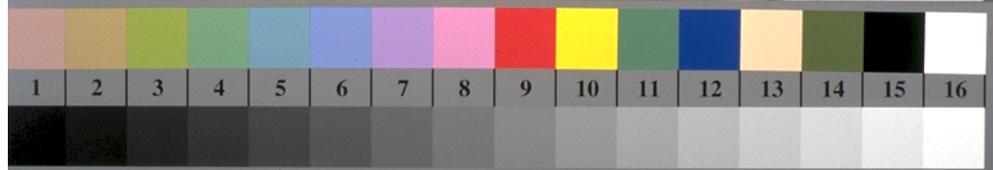
PE4300A_120830.TXT, 1080 colors, Separation cmyn6*

ISO 9241-306-test chart ME16; test chart no. 3 of ISO/IEC 15775
for recognition of 16 grey steps; standard contrast range 88.6 : 2.5

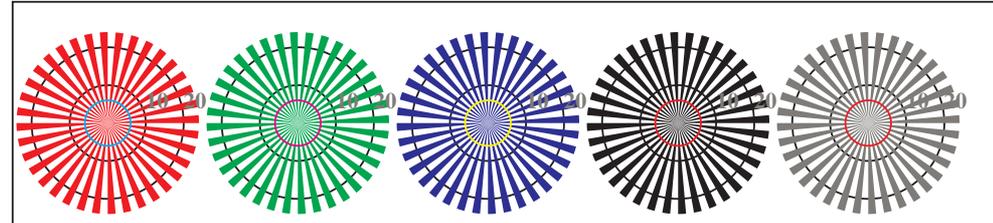
input: rgb/cmyk -> rgb_{dd}
output: 3D-Linearization to cm^yk*_{dd}



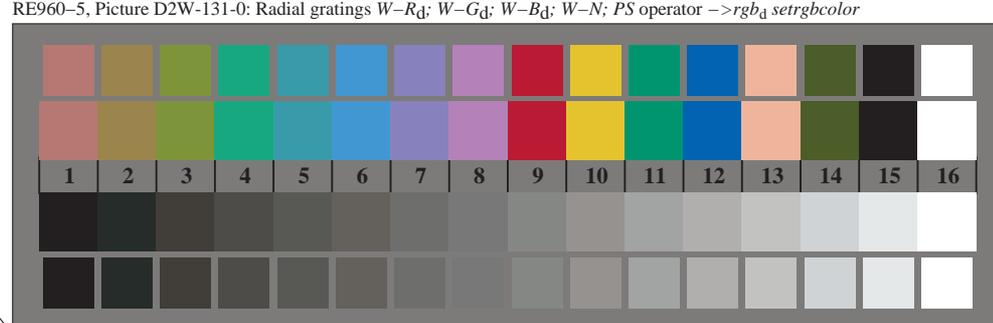
http://130.149.60.45/~farbmetrik/RE96/RE96L0FP.PDF /.PS; 3D-linearized output
F: 3D-Output Linearization (OL) data RE96/RE96LE30FP.DAT in File (F), Page 2/1



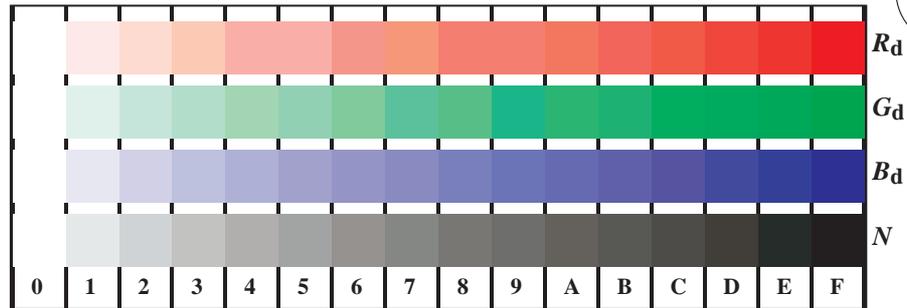
RE960-7, Picture B1-131-0: Flower motif, 14 CIE-test colours and 2 + 16 grey steps (sf); PS operators *settransfer*, 3 colorimage



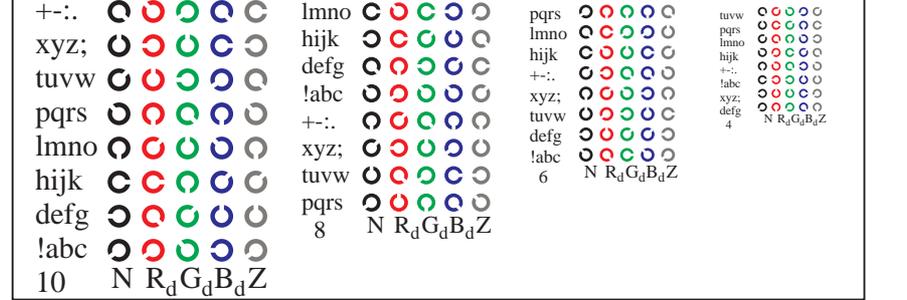
RE960-5, Picture D2W-131-0: Radial gratings W-R_d; W-G_d; W-B_d; W-N; PS operator ->rgb_d setrgbcolor



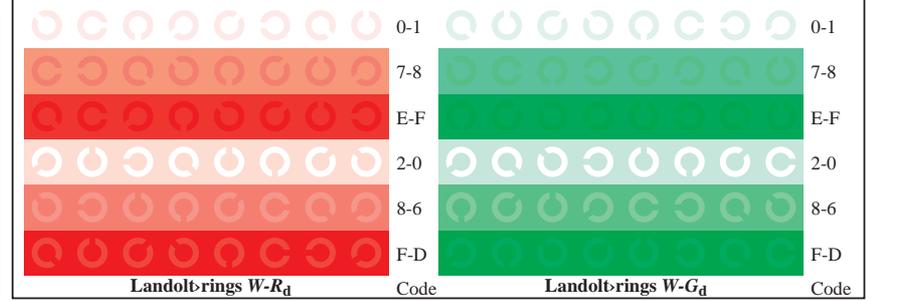
RE960-7, Picture D3W-131-0: 14 CIE-test colours and 2 + 16 grey steps; PS operator ->rgb_d setrgbcolor



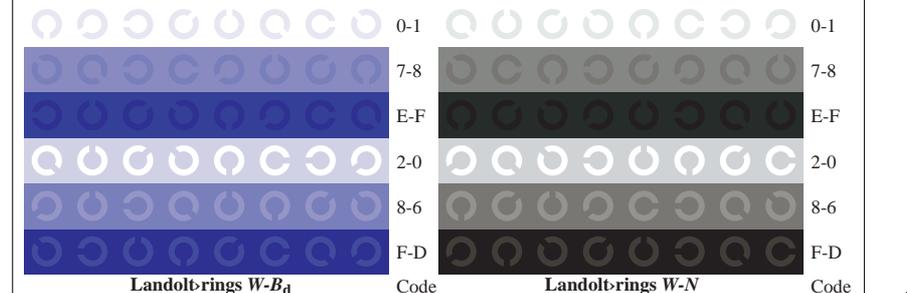
RE961-1, Picture D4W-L-131-0: 16 equidistant steps W-R_d; W-G_d; W-B_d; W-N; PS: ->rgb_d setrgbcolor



RE961-3, Picture D5W-131-0: Script and Landolt-rings N; R_d; G_d; B_d; Z; PS operator ->rgb_d setrgbcolor



RE961-5, Picture D6W-L-131-0: Landolt-rings W-R_d; W-G_d; PS operator ->rgb_d setrgbcolor



RE961-7, Picture D7W-L-131-0: Landolt-rings W-B_d; W-N; PS operator ->rgb_d setrgbcolor

RE96: Test chart 4 according to ISO 15775, TR 24705
Test chart according to ISO-9241-306, Annex E, C_{Y5}=36:1

input: rgb/cmyk -> rgb_{dd}
output: 3D-Linearization to rgb*_{dd}

TUB registration: 20120401-RE96/RE96L0FP.PDF /.PS
application for measurement of offset print, separation cmykn6* (CMYK)
TUB material: code=rh4ta

See: http://www.ps.bam.de/ME16, http://130.149.60.45/~farbmetrik/RE96/
Technical information: http://www.ps.bam.de or http://130.149.60.45/~farbmetrik

Colour, Colour Vision and Elementary Colours in Colour Information Technology

Acknowledgements

The *Karl Miescher-Foundation for Colour Science Support* has supported this offset print *Colour and Colour Vision* in German and English, and internet versions in additional languages

Since 2000 the professors *H. Kaase* and *S. Voelker* have continuously supported the exhibition *Colour and Colour Vision* at the department Lighting Technology of the Berlin University of Technology (TU).

Remarks about the history of the exhibition Colour and Colour Vision

During the years 1963/64 the exhibition *Colour and Colour Vision* has been developed by *K. Richter* under the leadership of *Dr. Karl Miescher* in the Laboratory of Colour Metrics at the Physical Institute of the University of Basle/Switzerland. The colour exhibition was shown for six months at the *Swiss National Exhibition Expo 64* in Lausanne/CH. After the *Expo 64* the colour exhibition was shown for 30 years in the high school *Mathematisch-Naturwissenschaftliches Gymnasium* in Basle.

In 2000 the exhibition was rebuilt with financial support of the *Karl Miescher-Foundation* at the Berlin University of Technology. In Berlin *K. Richter* has continuously enlarged the exhibition by some new colour developments in the field of colour information technology. Since 2000 many student and people with interests in colour visited the exhibition.

Notes to the former editions and the edition 2012

In 1964 the first edition comes without colour figures for the *Swiss National Exhibition Expo 64* in Lausanne and in languages German, French and Italian.

In 1982 after the second edition in 1978 the third edition with 50 colour figures comes in the colour journal "Farbe + Design" in German. In addition a special print comes in German and English. Coauthor of this edition was *Prof. Dr. Arne Valberg*, Trondheim/Norway.

The fourth edition 2012 comes with 135 colour figures, which in addition show new developments in the field of colour information technology. There are version for offset, monitor, and printer output, and internet versions in German and English. In addition internet versions are intended in the languages French, Spanish, and Italian.

For download of the last internet editions and the order of offset versions see <http://130.149.60.45/~farbmetrik/color>

Purpose and application of the special print 2012

The special prints serve for educational purposes and as introduction in the field of colour science. Different application fields of colour are connected without basic knowledge on colour but with some technical knowledge, for example

Visual basics and properties of colour vision

Colour measurement and colour metrics

Relative Elementary Colour System RECS in colour information technology.

For additional studies a colour book (only in German) is recommended with the title *Computer graphic and colorimetry - Colour systems, PostScript, and device independent CIE colours*. This book was edited by the VDE-Verlag in 1996 and has descriptions of about 500 colour figures in German and English. The colour figures may be used separately for educational purposes. The PDF files of this book and the colour figures are available as free download, see <http://130.149.60.45/~farbmetrik/buche.html>

The secretariat of the department Lighting Technology at the Berlin University of Technology may arrange guides on request through the exhibition *Colour and Colour Vision*, see <http://www.li.tu-berlin.de>

Copyright of the fourth edition of *Colour and Colour Vision*

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Fax: +49 30 84509039

email: klaus.richter@me.com

Technical remarks about the table at the inner back cover:

For a 48 step hue circle the table shows in column 2 the *rgb* input data and CIELAB-colour measurement data *LabCh**. In column 3 the data are interpolated for the next even number of a CIELAB-hue angle h_{ab} ($0 < i < 360$). The *rgb* system (s=standard) relates the *rgb*-data (1,0,0), (1,1,0), (0,1,0), (0,1,1), (0,0,1), and (1,0,1) to the angles 30, 90, 150, 210, 270, and 330. Similar according to CIE R1-47 the angles 26, 92, 162, 217, 272, and 329 are related to the angles of the elementary colour system. In addition for both systems the CIELAB data are interpolated as function of the hue angle *i*. For any *rgb*-data set (except $r=g=b$) it is valid according to DIN 33872-1

$$i = 360 \operatorname{atan} \left\{ \frac{[r \sin(30) + g \sin(150) - b \sin(270)]/[r \cos(30) + g \cos(150)]}{[r \cos(30) + g \cos(150)]} \right\}$$

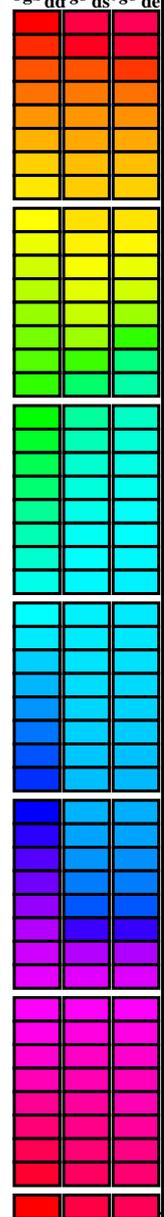
The index *i* produces from two tables with the angle *i* between 0 and 360 degree both the CIELAB-data *LabCh**, and the related *rgb* data for the device colour or the elementary colour output system.

In application the intended CIELAB-colour data are calculated for any hue angle *i* from the CIELAB data *LabCh** of colours with maximum chroma C^*_{ab} , and of white *W* and black *N*. To produce an intended colour the data rgb_{dd} (device to device output) and rgb_{de} (device to elementary output) use a 3D-linearization in the CIELAB-colour space.

Data of maximum color M in colorimetric system sRGB standard device; no separation, D65 for input or output; Six hue angles of the 60 degree standard colours RYGBM_s: h_{ab,ds} = 30.0, 90.0, 150.0, 210.0, 270.0, 330.0;

Six hue angles of the device colours RYGBCM_d: h_{ab,d} = 40.0, 102.9, 136.0, 196.4, 306.3, 328.2; Six hue angles of the elementary colours RYGBM_e: h_{ab,e} = 25.5, 92.3, 162.2, 217.0, 271.7, 328.6

Table with columns: h_{ab,d}, h_{ab,s}, h_{ab,e}, r_{gb}*_dd64M, LAB*_ddx64M (x=LabCh), r_{gb}*_dxx361M, LAB*_dxx361M (x=LabCh), r_{gb}*_dsx361M, LAB*_dsx361M (x=LabCh), r_{gb}*_dex361M, LAB*_dex361M. Rows contain numerical data for various colorimetric parameters.



See similar files: http://130.149.60.45/~farbmetrik/PE91/PE91.HTM
Technical information: http://www.ps.barn.de or http://130.149.60.45/~farbmetrik

TUB registration: 20120401-PE91/PE91LONP.PDF /.PS
application for measurement of sRGB display, no separation (RGB)
TUB material: code=rh4ta

Input and output: Colorimetric Television Luminous System TLS00a for relative CIELAB hue $h_{ab,a,rel} = h_{ab}/360 = 25/360 = 0.07$

$H^*_e = R00Y_e$

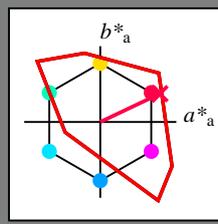
Data for any device (d) or elementary (e) colour:

HIC^*_e

hue text for the colours of this page:

$H^*_e = R00Y_e$

triangle lightness T^*



TLS00a; adapted (a) CIELAB data

Name	$L^*=L^*_a$	a^*_a	b^*_a	$C^*_{ab,a}$	$h^*_{ab,a}$
$R_{e, Ma}$	50.9	78.3	37.3	86.7	25
$Y_{e, Ma}$	83.7	-3.4	84.5	84.5	92
$G_{e, Ma}$	85.1	-64.6	20.7	67.9	162
$C_{e, Ma}$	79.0	-34.2	-25.7	42.8	216
$B_{e, Ma}$	59.2	1.7	-56.6	56.6	271
$M_{e, Ma}$	57.1	94.1	-57.4	110.3	328
$N_{e, Ma}$	0.0	0.0	0.0	0.0	0
$W_{e, Ma}$	95.4	0.0	0.0	0.0	0
$R_{e, CIE}$	39.9	58.7	27.9	65.0	25
$Y_{e, CIE}$	81.2	-2.8	71.5	71.6	92
$G_{e, CIE}$	52.2	-42.4	13.6	44.5	162
$B_{e, CIE}$	30.5	1.4	-46.4	46.4	271

Data for maximum colour (Ma):

$LabCh^*_{e, Ma}: 50\ 78\ 37\ 86\ 25$

$HIC^*_{e, Ma}: R00Y_100_100_e$

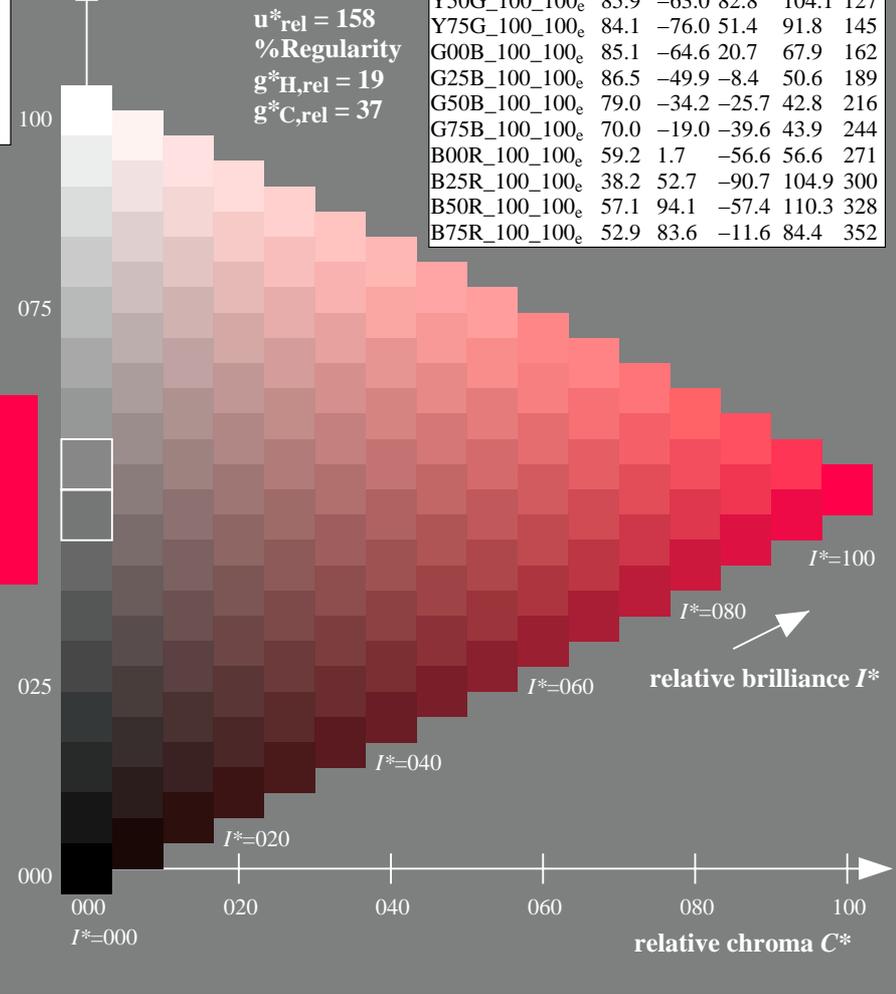
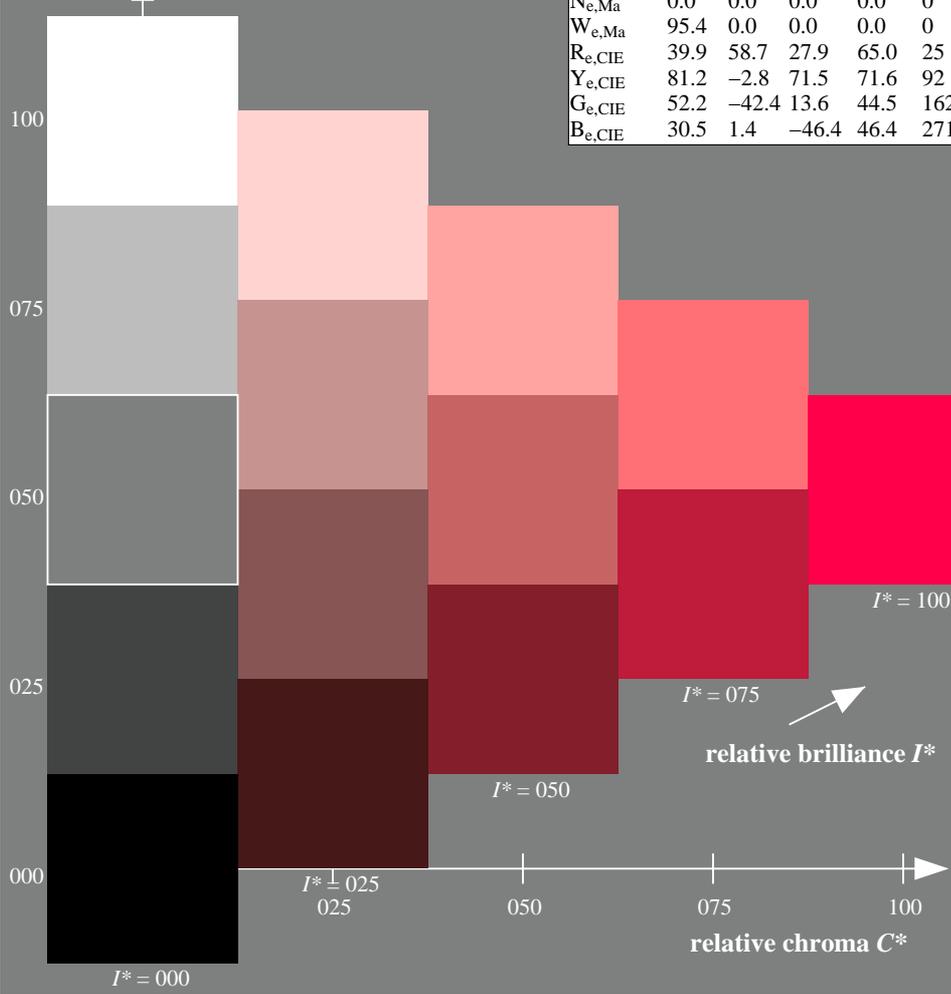
$rgbic^*_{e, Ma}$:

1.0 0.0 0.26 1.0 1.0

triangle lightness T^*

TLS00a; adapted (a) CIELAB data

H^*_e	$L^*=L^*_a$	a^*_a	b^*_a	$C^*_{ab,a}$	$h^*_{ab,a}$
$R00Y_100_100_e$	50.9	78.3	37.3	86.7	25
$R25Y_100_100_e$	51.3	74.4	64.8	98.7	41
$R50Y_100_100_e$	63.1	42.7	70.8	82.7	58
$R75Y_100_100_e$	73.5	18.3	77.7	79.8	76
$Y00G_100_100_e$	83.7	-3.4	84.5	84.5	92
$Y25G_100_100_e$	91.0	-29.9	88.9	93.8	108
$Y50G_100_100_e$	85.9	-63.0	82.8	104.1	127
$Y75G_100_100_e$	84.1	-76.0	51.4	91.8	145
$G00B_100_100_e$	85.1	-64.6	20.7	67.9	162
$G25B_100_100_e$	86.5	-49.9	-8.4	50.6	189
$G50B_100_100_e$	79.0	-34.2	-25.7	42.8	216
$G75B_100_100_e$	70.0	-19.0	-39.6	43.9	244
$B00R_100_100_e$	59.2	1.7	-56.6	56.6	271
$B25R_100_100_e$	38.2	52.7	-90.7	104.9	300
$B50R_100_100_e$	57.1	94.1	-57.4	110.3	328
$B75R_100_100_e$	52.9	83.6	-11.6	84.4	352



%Gamut
 $u^*_{rel} = 158$
%Regularity
 $g^*_{H,rel} = 19$
 $g^*_{C,rel} = 37$

See similar files: <http://130.149.60.45/~farbmetrik/PE90/PE90LOFP.PDF> /PS
Technical information: <http://www.ps.bam.de> or <http://130.149.60.45/~farbmetrik>

TUB registration: 20120401-PE90/PE90LOFP.PDF /PS
application for measurement of sRGB display, no separation* (RGB)
TUB material: code=rh4ta