**Weber-Fechner law in CIE 230:2019 for threshold colour differences of surface colours**

The *Weber-Fechner* law describes the lightness $L^*_r$ as ***logarithmic*** function of $L_r$.
The *Stevens* law describes the lightness $L^*_{CIELAB}$ as ***potential*** function of $L_r=Y/5$.
IEC 61966-2-1 uses a similar potential function $L^*_{IEC} = m\, L_r^{1/2,4}$.
The *Weber-Fechner* law is equivalent to the equation: $\Delta L_r = c\, L_r$ [1]
*Integration* leads to the logarithmic equation: $L^*_r = k \log(L_r)$. [2]
*Derivation* for $\Delta L^*_r=1$ leads to the linear equation: $L_r/\Delta L_r = k = 57$. [3]
For colours in offices the **standard contrast range** is **25:1**=90:3,6.

**Table 1: CIE tristimulus value Y, luminance L, and lightnesses L***

| Colour (matte) | Tritimulus value | office luminance | relative luminance | CIE lightness | relative lightness |
|---|---|---|---|---|---|
| *(contrast)* (25:1=90:3,6) | $Y$ | $L$ [cd/m²] | $L_r$ $=L/L_u$ | $L^*_{CIELAB}$ $\sim m\, L_r^{1/2,4}$ | $L^*_r$ $=k \log(L_r)$ |
| White W (paper) | 90 =18*5 | 142 =28,2*5 | 5 | 94 =50+44 | 40 $=k\log(5)$ |
| Grey Z (paper) | 18 | 28,2 | 1 | 50 | 0 $=k\log(1)$ |
| Black N (paper) | 3,6 =18/5 | 5,6 28,2/5 | 0,2 | 18 50−32 | −40 $=k\log(0,2)$ |

For the lightness range between $L^*_r=-40$ and 40 the constant is: $k=40/\log(5)=57$

een30−1n

**Weber-Fechner law in CIE 230:2019 for threshold colour differences of surface colours and two ranges 0,2 <= $L_r$ <=1 and 1 <= $L_r$ <=5**

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The *Stevens* law describes the lightness $L^*_{CIELAB}$ as ***potential*** function of $L_r=Y/5$.
IEC 61966-2-1 uses a similar potential function $L^*_{IEC} = m\, L_r^{1/2,4}$.
The *Weber-Fechner* law is equivalent to the linear equation: $\Delta L_r = c_i L_r$ $(i=0,1)$ [1]
*Integration* leads to the logarithmic equation: $L^*_r = k_i \log(L_r)$ $(i=0,1)$ [2]
*Derivation* leads for $\Delta L^*_r=1$ to the linear equation: $L_r/\Delta L_r = k_i$ ($k_0=46$, $k_1=63$) [3]
For colours in offices the **standard contrast range** is **25:1**=90:3,6.

**Table 1: CIE tristimulus value Y, luminance L, and lightnesses L***

| Colour (matte) | Tritimulus value | office luminance | relative luminance | CIE lightness | relative lightness |
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| *(contrast)* (25:1=90:3,6) | $Y$ | $L$ [cd/m²] | $L_r$ $=L/L_u$ | $L^*_{CIELAB}$ $\sim m\, L_r^{1/2,4}$ | $L^*_r$ $=k \log(L_r)$ |
| White W (paper) | 90 =18*5 | 142 =28,2*5 | 5 | 94 =50+44 | 44 $=k_1\log(5)$ |
| Grey Z (paper) | 18 | 28,2 | 1 | 50 | 0 $=k_0\log(1)$ |
| Black N (paper) | 3,6 =18/5 | 5,6 28,2/5 | 0,2 | 18 50−32 | −32 $=k_0\log(0,2)$ |

For the two lightness ranges it is $k_0=-32/\log(0,2)=46$ and $k_1=44/\log(5)=63$.

een30−2n

**Colour-line element of *Stiles* (1946) with „colour values" $L_P$, $M_D$, $S_T$**

**three separate colour-response functions**

$$F(L_P) = i \ln(1+9L_P)$$
$$F(M_D) = j \ln(1+9M_D)$$
$$F(S_T) = k \ln(1+9S_T)$$

***Taylor*-derivations:**

$$\Delta F(L_P,M_D,S_T) = \frac{dF}{dL_P}\Delta L_P + \frac{dF}{dM_D}\Delta M_D + \frac{dF}{dS_T}\Delta S_T$$
$$= \frac{9i}{1+9L_P}\Delta L_P + \frac{9j}{1+9M_D}\Delta M_D + \frac{9k}{1+9S_T}\Delta S_T$$

een31−1n

**Colour-line element of *Vos&Walraven* (1972) with „colour values" $L_P$, $M_D$, $S_T$**

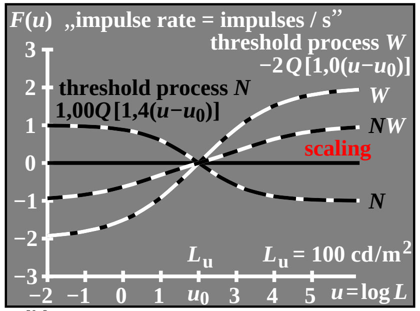
**three separate colour-response functions**

$$F(L_P) = -2i\sqrt{L_P}$$
$$F(M_D) = -2j\sqrt{M_D}$$
$$F(S_T) = -2k\sqrt{S_T}$$

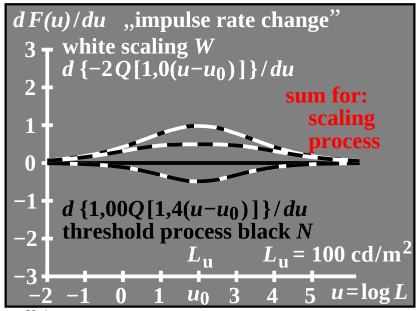
***Taylor*-derivations:**

$$\Delta F(L_P,M_D,S_T) = \frac{dF}{dL_P}\Delta L_P + \frac{dF}{dM_D}\Delta M_D + \frac{dF}{dS_T}\Delta S_T$$
$$\Delta F(L_P,M_D,S_T) = \frac{i}{\sqrt{L_P}}\Delta L_P + \frac{j}{\sqrt{M_D}}\Delta M_D + \frac{k}{\sqrt{S_T}}\Delta S_T$$

een31−2n




een30−3n




een30−4n

**„achromatic response"-description sub function $q[k(u-u_0)]$**

with $u = \log L$ ($L$ = luminance)
$u_0 = \log L_u$ ($L_u$ = surround luminance)

$$q[k(u-u_0)] = 1 + 1/[1+\sqrt{2}\,e^{k(u-u_0)}]$$

**sub function values:**

$q[k(u-u_0) \to +\infty] = 1$
$q[k(u-u_0) = 0] = \sqrt{2}$
$q[k(u-u_0) \to -\infty] = 2$

een31−3n

**„achromatic response"-description function $Q_{1m}[k(u-u_0)]$**

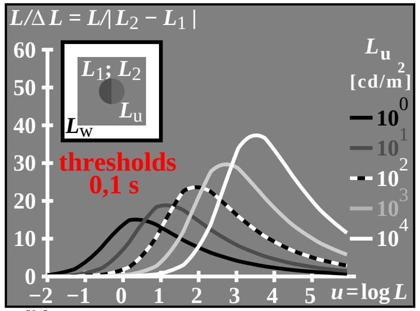
with $u = \log L$ ($L$ = luminance)
$u_0 = \log L_u$ ($L_u$ = surround luminance)

$$Q_{1m}[k(u-u_0)] = \frac{l}{\ln\sqrt{2}} \ln q[k(u-u_0)] - m$$

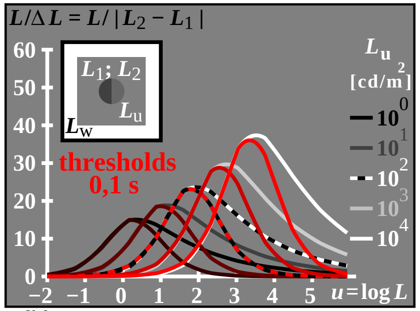
**function values with $l = m = 1$:**

$Q[k(u-u_0) \to +\infty] = -1$
$Q[k(u-u_0) = 0] = 0$
$Q[k(u-u_0) \to -\infty] = 1$

een31−4n




een30−5n




een30−6n

**„achromatic response" discrimination as function of relative light density $h = \ln H = k(u-u_0)$, ln = natural log.**

$$Q' = \frac{d}{dH}[\ln\{1+1/(1+\sqrt{2}H)\}]/\ln\sqrt{2}$$
$$= -\sqrt{2}/[\ln\sqrt{2}\,(1+\sqrt{2}H)(2+\sqrt{2}H)]$$

**function values:**

$Q'[k(u-u_0) \to +\infty] = 0$
$Q'[k(u-u_0) = 0] = -0{,}5$
$Q'[k(u-u_0) \to -\infty] = 0$

een31−5n

**luminance discrimination possibility $L/\Delta L$ as function of $H$**

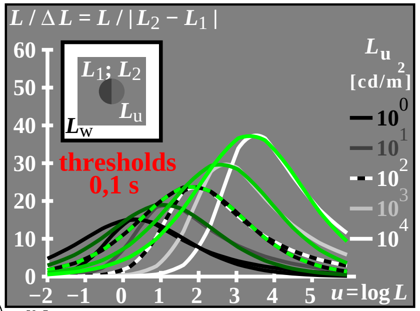
with: $L = 10^u$ $H = e^h = 10^{\log e\, k(u-u_0)}$
$dL/du = \ln 10\, L$ $dH/du = k\,H$

*it follows:* $L/\Delta L = [k\,H/(dH \ln 10)]$

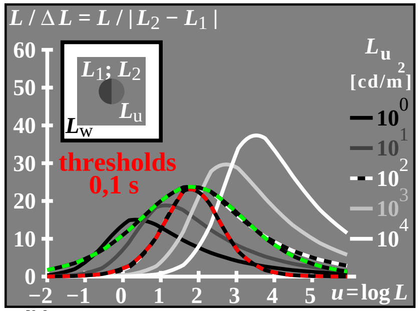
$$\frac{L}{dL} = \text{const}\; H/[(1+\sqrt{2}H)(2+\sqrt{2}H)]$$

$Q'[k(u-u_0) \to +\infty] = 0$
$Q'[k(u-u_0) = 0]$ = maximum
$Q'[k(u-u_0) \to -\infty] = 0$

een31−6n




een30−7n




een30−8n

**double line element of *Richter* (1987) for the lighting technology with the luminance $L = f(L_P, M_D, S_T)$**

$$F(L) = \int_{-\infty}^{L} (L/\Delta L)\, dL \quad \text{(relative } L, M, S\text{?)}$$
$$F(L) = i\,Q(H) = \begin{cases} \underline{i}\; Q(\underline{H}) & (u<u_0) \\ \bar{i}\; Q(\bar{H}) & (u \geq u_0) \end{cases}$$

with: $\underline{k}=1{,}4$ $\bar{k}=1$ $\underline{i}=1$ $\bar{i}=-2$
$u = \log L$ $u_0 = \log L_u$
$H = e^{k(u-u_0)}$, $\underline{H} = e^{\underline{k}(u-u_0)}$, $\bar{H} = e^{\bar{k}(u-u_0)}$

een31−7n

**double line element of *Richter* (1987) for the lighting technology with the luminance $L = F(L_P, M_D, S_T)$**

$$F(L) = \int_{-\infty}^{L} (L/\Delta L)\, dL \quad \text{(relative } L, M, S\text{?)}$$
$F(L) = i\,Q(H)$ $H = e^{k(u-u_0)}$
$Q(H) = [\ln\{1+1/(1+\sqrt{2}H)\}]/\ln\sqrt{2} - 1$

*Taylor*-derivations:

$$\Delta F(L) = \frac{dF}{dL}\Delta L = i\,\frac{dQ}{dH}\Delta H$$
$$= -i\sqrt{2}\,\Delta H/[\ln\sqrt{2}\,(1+\sqrt{2}H)(2+\sqrt{2}H)]$$

een31−8n

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technical information: http://farbe.li.tu-berlin.de or http://color.li.tu-berlin.de

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application for evaluation and measurement of display or print output
TUB material: code=rha4ta