

Mathematical equations of hyperbel functions

See: Papula, L., (2003), *Mathematische Formelnammlung*, Vieweg

$$F(x) = \tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{u(x)}{v(x)} \quad u'(x) = v'(x) \quad (1)$$

$$F'(x) = \frac{u'(x)v(x) - u(x)v'(x)}{v^2(x)} = \frac{v^2(x) - u^2(x)}{v^2(x)} \quad (2)$$

$$F'(x) = \frac{[e^x + e^{-x}] [e^x + e^{-x}] - [e^x - e^{-x}] [e^x - e^{-x}]}{[e^x + e^{-x}]^2} \quad (3)$$

$$F'(x) = \frac{4}{[e^x + e^{-x}]^2} = \frac{1}{\cosh^2(x)} \quad (4)$$

aeo4b-2a aeo41-2a

Mathematical equations of hyperbel functions

See: Papula, L., (2003), *Mathematische Formelnammlung*, Vieweg

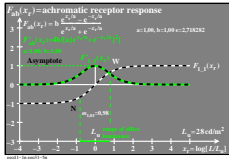
$$F(x/a) = \tanh(x/a) = \frac{e^{x/a} - e^{-x/a}}{e^{x/a} + e^{-x/a}} = \frac{u(x/a)}{v(x/a)} \quad (1)$$

$$F'(x/a) = \frac{u'(x/a)v(x/a) - u(x/a)v'(x/a)}{v^2(x/a)} \quad (2)$$

$$F'(x/a) = \frac{v^2(x/a) - u^2(x/a)}{a v^2(x/a)} \quad (3)$$

$$F'(x/a) = \frac{4}{a [e^{x/a} + e^{-x/a}]^2} = \frac{1}{a \cosh^2(x/a)} \quad (4)$$

aeo4b-2a aeo41-2a



Mathematical equations of hyperbel functions

See: Papula, L., (2003), *Mathematische Formelnammlung*, Vieweg

$$F_{ab}(x/a) = b \tanh(x/a) = b \frac{e^{x/a} - e^{-x/a}}{e^{x/a} + e^{-x/a}} = b \frac{u(x/a)}{v(x/a)} \quad (1)$$

$$F'_{ab}(x/a) = b \frac{u'(x/a)v(x/a) - u(x/a)v'(x/a)}{v^2(x/a)} \quad (2)$$

$$F'_{ab}(x/a) = b \frac{v^2(x/a) - u^2(x/a)}{a v^2(x/a)} \quad (3)$$

$$F'_{ab}(x/a) = \frac{4b}{a [e^{x/a} + e^{-x/a}]^2} = \frac{b}{a \cosh^2(x/a)} \quad (4)$$

aeo4b-2a

Mathematical equations of hyperbel functions

See: Papula, L., (2003), *Mathematische Formelnammlung*, Vieweg

$$F_{1b}(x) = b \tanh(x) = b \frac{e^x - e^{-x}}{e^x + e^{-x}} = b \frac{u(x)}{v(x)} \quad (1)$$

$$F'_{1b}(x) = b \frac{u'(x)v(x) - u(x)v'(x)}{v^2(x)} \quad (2)$$

$$F'_{1b}(x) = b \frac{v^2(x) - u^2(x)}{a v^2(x)} \quad (3)$$

$$F'_{1b}(x) = \frac{4b}{[e^x + e^{-x}]^2} = \frac{b}{\cosh^2(x)} \quad (4)$$

aeo4b-2a aeo41-2a

Mathematical equations of hyperbel functions

See: Papula, L., (2003), *Mathematische Formelnammlung*, Vieweg

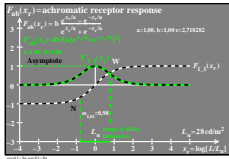
$$F_{1b}(x/a) = b \tanh(x/a) = b \frac{e^{x/a} - e^{-x/a}}{e^{x/a} + e^{-x/a}} = b \frac{u(x/a)}{v(x/a)} \quad (1)$$

$$F'_{1b}(x/a) = b \frac{u'(x/a)v(x/a) - u(x/a)v'(x/a)}{v^2(x/a)} \quad (2)$$

$$F'_{1b}(x/a) = b \frac{v^2(x/a) - u^2(x/a)}{a v^2(x/a)} \quad (3)$$

$$F'_{1b}(x/a) = \frac{4b}{a [e^{x/a} + e^{-x/a}]^2} = \frac{b}{a \cosh^2(x/a)} \quad (4)$$

aeo4b-2a aeo41-2a



Mathematical equations of hyperbel functions

See: Papula, L., (2003), *Mathematische Formelnammlung*, Vieweg

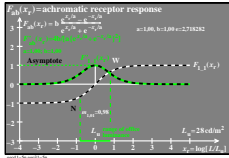
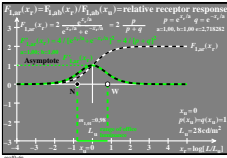
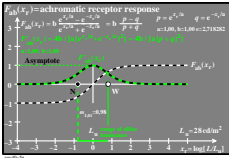
$$F_{ab}(x/a) = b \tanh(x/a) = b \frac{e^{x/a} - e^{-x/a}}{e^{x/a} + e^{-x/a}} = b \frac{u(x/a)}{v(x/a)} \quad (1)$$

$$F'_{ab}(x/a) = b \frac{u'(x/a)v(x/a) - u(x/a)v'(x/a)}{v^2(x/a)} \quad (2)$$

$$F'_{ab}(x/a) = b \frac{v^2(x/a) - u^2(x/a)}{a v^2(x/a)} \quad (3)$$

$$F'_{ab}(x/a) = \frac{4b}{a [e^{x/a} + e^{-x/a}]^2} = \frac{b}{a \cosh^2(x/a)} \quad (4)$$

aeo4b-2a



Mathematical equations of hyperbel functions

See: Papula, L., (2003), *Mathematische Formelnammlung*, Vieweg

$$F_{ab}(x/a) = b \tanh(x/a) = b \frac{e^{x/a} - e^{-x/a}}{e^{x/a} + e^{-x/a}} = \frac{b}{a} \frac{u(x/a)}{v(x/a)} \quad (1)$$

$$\frac{dF_{ab}(x_r/a)}{dx_r} = \frac{4b}{a [e^{x_r/a} + e^{-x_r/a}]^2} = \frac{b}{a \cosh^2(x_r/a)} \quad (4)$$

$$\frac{dF_{ab}(x_r/a)}{dx_r} = \frac{4b}{a [e^{x_r/a} + e^{-x_r/a}]^2} \quad x_r = \log(L/L_0) \quad (5)$$

$$\frac{dF_{ab}(x_r/a)}{dx_r} \frac{dx_r}{dL} = \frac{4b}{a [e^{x_r/a} + e^{-x_r/a}]^2} \frac{\ln(10)}{L} \quad (6)$$

$$\frac{dF_{ab}(x_r/a)}{dL} = \frac{4b \ln(10)}{a [e^{x_r/a} + e^{-x_r/a}]^2 L} \quad (6)$$

$$\frac{dF_{ab}(x_r/a)}{dL} = \frac{4b \ln(10)}{a [e^{x_r/a} + e^{-x_r/a}]^2 L} \quad (6)$$

$$\frac{dF_{ab}(x_r/a)}{dL} = \frac{4b \ln(10)}{a [e^{x_r/a} + e^{-x_r/a}]^2 L} \quad (6)$$

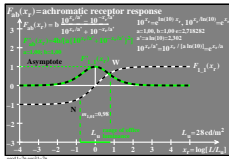
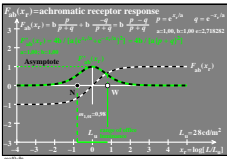
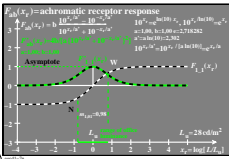
$$\frac{dF_{ab}(x_r/a)}{dL} = \frac{4b \ln(10)}{a [e^{x_r/a} + e^{-x_r/a}]^2 L} \quad (6)$$

$$\frac{dF_{ab}(x_r/a)}{dL} = \frac{4b \ln(10)}{a [e^{x_r/a} + e^{-x_r/a}]^2 L} \quad (6)$$

$$\frac{dF_{ab}(x_r/a)}{dL} = \frac{4b \ln(10)}{a [e^{x_r/a} + e^{-x_r/a}]^2 L} \quad (6)$$

$$\frac{dF_{ab}(x_r/a)}{dL} = \frac{4b \ln(10)}{a [e^{x_r/a} + e^{-x_r/a}]^2 L} \quad (6)$$

$$\frac{dF_{ab}(x_r/a)}{dL} = \frac{4b \ln(10)}{a [e^{x_r/a} + e^{-x_r/a}]^2 L} \quad (6)$$



Mathematical equations of hyperbel functions

See: Papula, L., (2003), *Mathematische Formelnammlung*, Vieweg

$$F_{ab}(x_r/a) = b \tanh(x_r/a) = b \frac{e^{x_r/a} - e^{-x_r/a}}{e^{x_r/a} + e^{-x_r/a}} = \frac{b}{a} \frac{u(x_r/a)}{v(x_r/a)} \quad (1)$$

$$\frac{dF_{ab}(x_r/a)}{dx_r} = \frac{4b}{a [e^{x_r/a} + e^{-x_r/a}]^2} \quad x_r = \log(L/L_0) \quad (5)$$

$$\frac{dF_{ab}(x_r/a)}{dx_r} \frac{dx_r}{dL} = \frac{4b}{a [e^{x_r/a} + e^{-x_r/a}]^2} \frac{\ln(10)}{L} \quad (6)$$

$$\frac{dF_{ab}(x_r/a)}{dL} = \frac{4b \ln(10)}{a [e^{x_r/a} + e^{-x_r/a}]^2 L} \quad (6)$$

$$\frac{dF_{ab}(x_r/a)}{dL} = \frac{4b \ln(10)}{a [e^{x_r/a} + e^{-x_r/a}]^2 L} \quad (6)$$

$$\frac{dF_{ab}(x_r/a)}{dL} = \frac{4b \ln(10)}{a [e^{x_r/a} + e^{-x_r/a}]^2 L} \quad (6)$$

$$\frac{dF_{ab}(x_r/a)}{dL} = \frac{4b \ln(10)}{a [e^{x_r/a} + e^{-x_r/a}]^2 L} \quad (6)$$

TUB-test chart eo4; Model of normalized response function $F_{ab}(x_r)$ and derivation $F'_{ab}(x_r)$
 Mathematical calculation of the derivation $F'_{ab}(x_r)$, of the contrast $\Delta L/\Delta L$, and the discrimination ΔL