TUB registration: 20230701-eeo4/eeo4l0np.pdf /.ps
application for evaluation and measurement of display or print output
TUB material: code=rha4ta

Mathematical equations of hyperbel functions
See: *Papula, L., (2003), Mathematische Formelsammlung, Vieweg*

$$F(x)=\tanh(x)=\frac{e^x-e^{-x}}{e^x+e^{-x}}=\frac{u(x)}{v(x)} \quad \begin{array}{l}u'(x)=v(x)\\v'(x)=u(x)\end{array} \quad [1]$$

$$F'(x)=\frac{u'(x)\,v(x)-u(x)\,v'(x)}{v^2(x)}=\frac{v^2(x)-u^2(x)}{v^2(x)} \quad [2]$$

$$F'(x)=\frac{[e^x+e^{-x}][e^x+e^{-x}]-[e^x-e^{-x}][e^x-e^{-x}]}{[e^x+e^{-x}]^2} \quad [3]$$

$$F'(x)=\frac{4}{[e^x+e^{-x}]^2}=\frac{1}{\cosh^2(x)} \quad [4]$$

eeo40–1n eeo31–2n

Mathematical equations of hyperbel functions
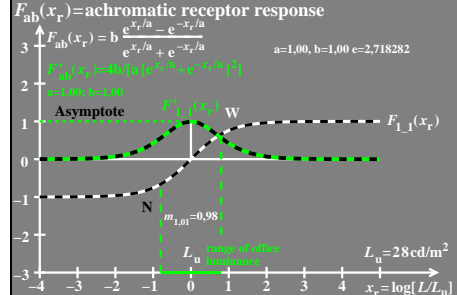See: *Papula, L., (2003), Mathematische Formelsammlung, Vieweg*

$$F(x/a)=\tanh(x/a)=\frac{e^{x/a}-e^{-x/a}}{e^{x/a}+e^{-x/a}}=\frac{u(x/a)}{v(x/a)} \quad [1]$$

$$F'(x/a)=\frac{u'(x/a)v(x/a)-u(x/a)v'(x/a)}{v^2(x/a)} \quad [2]$$

$$F'(x/a)=\frac{v^2(x/a)-u^2(x/a)}{a\,v^2(x/a)} \quad [3]$$

$$F'(x/a)=\frac{4}{a\,[e^{x/a}+e^{-x/a}]^2}=\frac{1}{a\,\cosh^2(x/a)} \quad [4]$$

eeo40–2n eeo31–4n

$F_{ab}(x_r)$=achromatic receptor response

$$F_{ab}(x_r)=\frac{e^{x_r/a}-e^{-x_r/a}}{e^{x_r/a}+e^{-x_r/a}} \qquad a=1,00,\ b=1,00\ e=2,718282$$

N $m_{1,01}=0,98$ L_u range of office luminance $L_u=28\,cd/m^2$ $x_r=\log[L/L_u]$

eeo41–1n eeo31–5n

Mathematical equations of hyperbel functions
See: *Papula, L., (2003), Mathematische Formelsammlung, Vieweg*

$$F_{ab}(x/a)=b\tanh(x/a)=b\frac{e^{x/a}-e^{-x/a}}{e^{x/a}+e^{-x/a}}=b\frac{u(x/a)}{v(x/a)} \quad [1]$$

$$F'_{ab}(x/a)=b\frac{u'(x/a)\,v(x/a)-u(x/a)\,v'(x/a)}{v^2(x/a)} \quad [2]$$

$$F'_{ab}(x/a)=b\frac{v^2(x/a)-u^2(x/a)}{a\,v^2(x/a)} \quad [3]$$

$$F'_{ab}(x/a)=\frac{4b}{a\,[e^{x/a}+e^{-x/a}]^2}=\frac{b}{a\,\cosh^2(x/a)} \quad [4]$$

eeo41–2n

Mathematical equations of hyperbel functions
See: *Papula, L., (2003), Mathematische Formelsammlung, Vieweg*

$$F_{1b}(x)=b\tanh(x)=b\frac{e^x-e^{-x}}{e^x+e^{-x}}=b\frac{u(x)}{v(x)} \quad [1]$$

$$F'_{1b}(x)=b\frac{u'(x)\,v(x)-u(x)\,v'(x)}{v^2(x)} \quad [2]$$

$$F'_{1b}(x)=b\frac{v^2(x)-u^2(x)}{a\,v^2(x)} \quad [3]$$

$$F'_{1b}(x)=\frac{4b}{[e^x+e^{-x}]^2}=\frac{b}{\cosh^2(x)} \quad [4]$$

eeo40–3n eeo31–6n

Mathematical equations of hyperbel functions
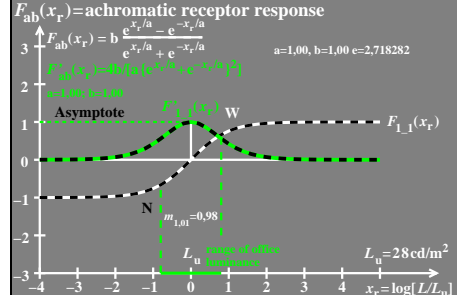See: *Papula, L., (2003), Mathematische Formelsammlung, Vieweg*

$$F_{ab}(x/a)=b\tanh(x/a)=b\frac{e^{x/a}-e^{-x/a}}{e^{x/a}+e^{-x/a}}=b\frac{u(x/a)}{v(x/a)} \quad [1]$$

$$F'_{ab}(x/a)=b\frac{u'(x/a)\,v(x/a)-u(x/a)\,v'(x/a)}{v^2(x/a)} \quad [2]$$

$$F'_{ab}(x/a)=b\frac{v^2(x/a)-u^2(x/a)}{a\,v^2(x/a)} \quad [3]$$

$$F'_{ab}(x/a)=\frac{4b}{a\,[e^{x/a}+e^{-x/a}]^2}=\frac{b}{a\,\cosh^2(x/a)} \quad [4]$$

eeo40–4n eeo31–8n

$F_{ab}(x_r)$=achromatic receptor response

$$F_{ab}(x_r)=b\frac{e^{x_r/a}-e^{-x_r/a}}{e^{x_r/a}+e^{-x_r/a}} \qquad a=1,00,\ b=1,00\ e=2,718282$$

N $m_{1,01}=0,98$ L_u range of office luminance $L_u=28\,cd/m^2$ $x_r=\log[L/L_u]$

eeo41–3n eeo31–5n

Mathematical equations of hyperbel functions
See: *Papula, L., (2003), Mathematische Formelsammlung, Vieweg*
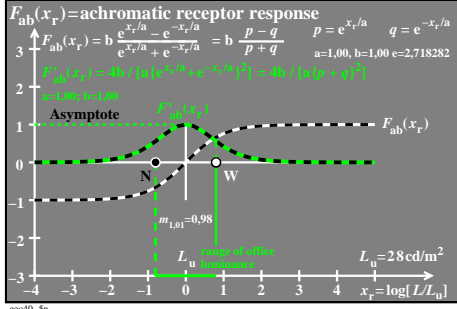
$$F_{ab}(x/a)=b\tanh(x/a)=b\frac{e^{x/a}-e^{-x/a}}{e^{x/a}+e^{-x/a}}=b\frac{u(x/a)}{v(x/a)} \quad [1]$$

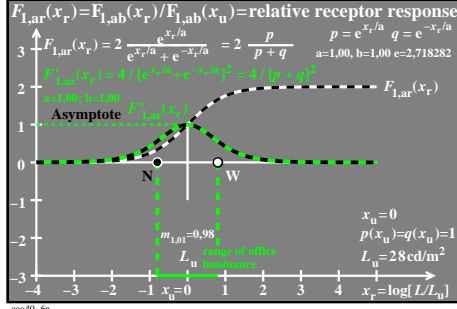$$F'_{ab}(x/a)=b\frac{u'(x/a)\,v(x/a)-u(x/a)\,v'(x/a)}{v^2(x/a)} \quad [2]$$

$$F'_{ab}(x/a)=b\frac{v^2(x/a)-u^2(x/a)}{a\,v^2(x/a)} \quad [3]$$

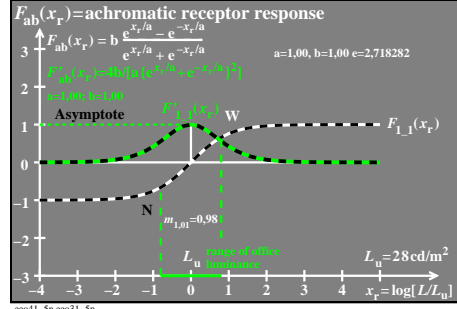$$F'_{ab}(x/a)=\frac{4b}{a\,[e^{x/a}+e^{-x/a}]^2}=\frac{b}{a\,\cosh^2(x/a)} \quad [4]$$

eeo41–4n

$F_{ab}(x_r)$=achromatic receptor response

$$F_{ab}(x_r)=b\frac{e^{x_r/a}-e^{-x_r/a}}{e^{x_r/a}+e^{-x_r/a}}=b\frac{p-q}{p+q} \quad p=e^{x_r/a} \quad q=e^{-x_r/a} \qquad a=1,00,\ b=1,00\ e=2,718282$$

Asymptote $F_{ab}(x_r)$ N   W $m_{1,01}=0,98$ L_u range of office luminance $L_u=28\,cd/m^2$ $x_r=\log[L/L_u]$

eeo40–5n

$F_{1,ar}(x_r)=F_{1,ab}(x_r)/F_{1,ab}(x_u)$=relative receptor response

$$F_{1,ar}(x_r)=2\frac{e^{x_r/a}}{e^{x_r/a}+e^{-x_r/a}}=2\frac{p}{p+q} \quad p=e^{x_r/a} \quad q=e^{-x_r/a} \qquad a=1,00,\ b=1,00\ e=2,718282$$

Asymptote $F_{1,ar}(x_r)$ N   W $x_u=0$ $p(x_u)=q(x_u)=1$ $m_{1,01}=0,98$ L_u range of office luminance $L_u=28\,cd/m^2$ $x_r=\log[L/L_u]$

eeo40–6n

$F_{ab}(x_r)$=achromatic receptor response

$$F_{ab}(x_r)=b\frac{e^{x_r/a}-e^{-x_r/a}}{e^{x_r/a}+e^{-x_r/a}} \qquad a=1,00,\ b=1,00\ e=2,718282$$

Asymptote $F_{ab}(x_r)$ W   N $m_{1,01}=0,98$ L_u $L_u=28\,cd/m^2$ $x_r=\log[L/L_u]$

eeo41–5n eeo31–5n

Mathematical equations of hyperbel functions
See: *Papula, L., (2003), Mathematische Formelsammlung, Vieweg*
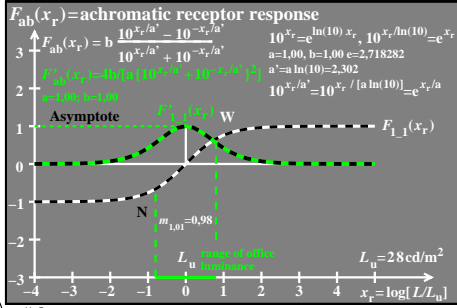
$$F_{ab}(x/a)=b\tanh(x/a)=b\frac{e^{x/a}-e^{-x/a}}{e^{x/a}+e^{-x/a}} \quad [1]$$

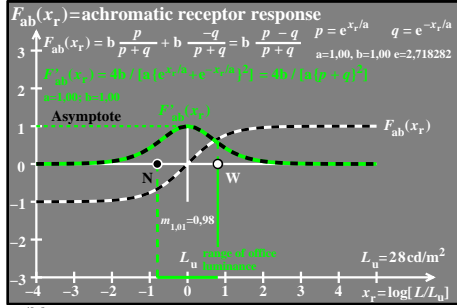$$\frac{dF_{ab}(x/a)}{dx}=\frac{4b}{a[e^{x/a}+e^{-x/a}]^2}=\frac{b}{a\,\cosh^2(x/a)} \quad [4]$$

$$\frac{dF_{ab}(x_r/a)}{dx_r}=\frac{4b}{a[e^{x_r/a}+e^{-x_r/a}]^2}\frac{x_r=\log(L/L_u)}{dx_r/dL=\ln(10)/L} \quad [5]$$

$$\frac{dF_{ab}(x_r/a)}{dx_r}\frac{dx_r}{dL}=\frac{4b}{a[e^{x_r/a}+e^{-x_r/a}]^2}\frac{\ln(10)}{L} \quad [6]$$

eeo41–6n

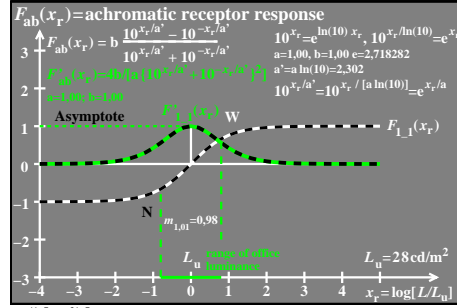## $F_{ab}(x_r)$=achromatic receptor response

$$F_{ab}(x_r)=b\frac{10^{x_r/a'}-10^{-x_r/a'}}{10^{x_r/a'}+10^{-x_r/a'}} \quad 10^{x_r}=e^{\ln(10)\,x_r},\ 10^{x_r/\ln(10)}=e^{x_r} \quad a=1,00,\ b=1,00\ e=2,718282 \quad a'=a\ln(10)=2,302 \quad 10^{x_r/a'}=10^{x_r/[a\ln(10)]}=e^{x_r/a}$$

Asymptote   N   W $m_{1,01}=0,98$ L_u range of office luminance $L_u=28\,cd/m^2$ $x_r=\log[L/L_u]$ $F_{1\_1}(x_r)$

eeo41–7n

$F_{ab}(x_r)$=achromatic receptor response

$$F_{ab}(x_r)=b\frac{p}{p+q}+b\frac{-q}{p+q}=b\frac{p-q}{p+q} \quad p=e^{x_r/a} \quad q=e^{-x_r/a} \qquad a=1,00,\ b=1,00\ e=2,718282$$

Asymptote   N   W $m_{1,01}=0,98$ L_u range of office luminance $L_u=28\,cd/m^2$ $x_r=\log[L/L_u]$

eeo40–8n

$F_{ab}(x_r)$=achromatic receptor response

$$F_{ab}(x_r)=b\frac{10^{x_r/a'}-10^{-x_r/a'}}{10^{x_r/a'}+10^{-x_r/a'}} \quad 10^{x_r}=e^{\ln(10)\,x_r},\ 10^{x_r/\ln(10)}=e^{x_r} \quad a=1,00,\ b=1,00\ e=2,718282 \quad a'=a\ln(10)=2,302 \quad 10^{x_r/a'}=10^{x_r/[a\ln(10)]}=e^{x_r/a}$$

Asymptote   N   W $m_{1,01}=0,98$ L_u range of office luminance $L_u=28\,cd/m^2$ $x_r=\log[L/L_u]$ $F_{1\_1}(x_r)$

eeo41–7n eeo31–7n

Mathematical equations of hyperbel functions
See: *Papula, L., (2003), Mathematische Formelsammlung, Vieweg*

$$F_{ab}(x_r/a)=b\tanh(x_r/a)=b\frac{e^{x_r/a}-e^{-x_r/a}}{e^{x_r/a}+e^{-x_r/a}} \quad [1]$$

$$\frac{dF_{ab}(x_r/a)}{dx_r}=\frac{4b}{a[e^{x_r/a}+e^{-x_r/a}]^2}\frac{x_r=\log(L/L_u)}{dx_r/dL=\ln(10)/L} \quad [5]$$

$$\frac{dF_{ab}(x_r/a)}{dx_r}\frac{dx_r}{dL}=\frac{4b}{a[e^{x_r/a}+e^{-x_r/a}]^2}\frac{\ln(10)}{L} \quad [6]$$

$$\frac{L}{dL}=\frac{4b\ln(10)}{a[e^{x_r/a}+e^{-x_r/a}]^2} \qquad dL=\frac{a[e^{x_r/a}+e^{-x_r/a}]^2L}{4b\ln(10)} \quad [7]$$

eeo41–8n

TUB-test chart eeo4; Model of normalized response function $F_{ab}(x_r)$ and derivation $F'_{ab}(x_r)$
Mathematical calculation of the derivation $F'_{ab}(x_r)$, of the contrast $L/\Delta L$, and the discrimination ΔL