

Achromatic colour vision with relative luminance Mathematical equations with hyperbel functions

$$F_{ab}(x_r, a) = b \tanh(x_r/a) = b \frac{e^{x_r/a} - e^{-x_r/a}}{e^{x_r/a} + e^{-x_r/a}} \quad x_r = \log(L_r)$$

$$\frac{dF_{ab}(x_r, a)}{dx_r} = \frac{4b}{a[e^{x_r/a} + e^{-x_r/a}]^2} \quad x_r = \ln L_r / \ln(10)$$

$$\frac{dF_{ab}(x_r, a)}{dx_r} = \frac{4b}{a[e^{x_r/a} + e^{-x_r/a}]^2} \quad x_r = \ln L_r / \ln(10) \quad m = 1 / (\ln(10)a) \quad [5]$$

$$\frac{dF_{ab}(x_r, a)}{dx_r} = \frac{4bm}{[e^{x_r/a} + e^{-x_r/a}]^2 L_r} \quad L_r / dL_r = L / dL \quad F_{ab}(x_r, a) = 1 \quad [6]$$

$$\frac{L}{dL} = \frac{4bm}{[e^{x_r/a} + e^{-x_r/a}]^2} \quad dL = \frac{[e^{x_r/a} + e^{-x_r/a}]^2 L}{4bm} \quad [7]$$

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$$\frac{dF_{ab}(x_r, a)}{dx_r} = \frac{4b}{a[e^{x_r/a} + e^{-x_r/a}]^2} \quad x_r = \ln L_r / \ln(10) \quad dx_r/dL_r = 1/(ln(10)L_r) \quad [5]$$

$$\frac{L}{dL} = \frac{4bm}{[e^{x_r/a} + e^{-x_r/a}]^2} \quad dL = \frac{a[e^{x_r/a} + e^{-x_r/a}]^2 L}{4bm} \quad [7]$$

$$\frac{L/dL}{(D/L)_u} = \frac{4}{[e^{x_r/a} + e^{-x_r/a}]^2}; \quad \frac{dL}{dL_u} = \frac{[e^{x_r/a} + e^{-x_r/a}]^2 L}{4L_u} \quad [8]$$

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$$\frac{dL}{dL_u} = \frac{4}{[e^{x_r/a} + e^{-x_r/a}]^2}; \quad \frac{dL}{dL_u} = \frac{[e^{x_r/a} + e^{-x_r/a}]^2 L}{4 L_u} \quad m = 1 / (\ln(10) a) \quad [5]$$

$$\frac{L/dL}{(dL/dL_u)} = 1 \text{ for } \begin{cases} L=L_u \\ x_r=0 \end{cases} \quad \frac{dL}{dL_u} = 1 \text{ for } \begin{cases} L=L_u \\ x_r=0 \end{cases} \quad [9]$$

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$$\frac{dF_{ab}(x_r, a)}{dx_r} = \frac{4b}{a[e^{x_r/a} + e^{-x_r/a}]^2} \quad x_r = \ln L_r / \ln(10) \quad dx_r/dL_r = 1 / (\ln(10)L_r) \quad m = 1 / (\ln(10)a) \quad [5]$$

$$\frac{L/dL}{(L/dL)_u} = \frac{4}{[e^{x_r/a} + e^{-x_r/a}]^2} \quad \frac{dL}{dL_u} = \frac{[e^{x_r/a} + e^{-x_r/a}]^2 L}{4L_u} \quad [8]$$

$$\frac{L/dL}{(L/dL)_u} = 1 \text{ for } \begin{cases} L_r = 1 \\ x_r = 0 \end{cases} \quad \frac{dL}{dL_u} = 1 \text{ for } \begin{cases} L_r = 1 \\ x_r = 0 \end{cases} \quad [9]$$

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$$I_{cb}(x_r, c) = b \tanh(x_r/c) = b \frac{e^{x_r/c} - e^{-x_r/c}}{e^{x_r/c} + e^{-x_r/c}} \quad x_r = \log(L_r/L_u) \quad L_r = L L_u$$

$$\frac{dI_{cb}(x_r, c)}{dx_r} = \frac{4b}{c[e^{x_r/c} + e^{-x_r/c}]^2} \quad x_r = \ln(L_r/\ln(10)) \quad dx_r/dL_r = 1/(ln(10)L_r) \quad n = 1/(ln(10)c) \quad [5]$$

$$\frac{dI_{cb}(x_r, c)}{dL_r} = \frac{4bm}{[e^{x_r/c} + e^{-x_r/c}]^2 L_r} \quad L_r/dL_r = L/dL \quad dI_{cb}(x_r, c) = 1 \quad [6]$$

$$\frac{L}{IL} = \frac{4bm}{[e^{x_r/c} + e^{-x_r/c}]^2} \quad dL = \frac{[e^{x_r/c} + e^{-x_r/c}]^2 L}{4bm} \quad [7]$$

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$$\frac{IF_{cb}(x_r, c)}{dx_r} = \frac{4b}{c[e^{x_r/c} + e^{-x_r/c}]^2} \quad x_r = \ln L_r / \ln(10) \quad dx_r/dL_r = 1 / (\ln(10) L_r) \quad [5]$$

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$$\frac{dI_{cb}(x_r, c)}{dx_r} = \frac{4b}{c[e^{x_r/c} + e^{-x_r/c}]^2} \quad x_r = \ln L / \ln(10) \quad dx_r / dL_r = 1 / (\ln(10) L_r) \quad n = 1 / (\ln(10) c) \quad [5]$$

$$\frac{dL/dL_u}{dL_u} = \frac{4}{[e^{x_r/c} + e^{-x_r/c}]^2}; \quad \frac{dL}{dL_u} = \frac{[e^{x_r/c} + e^{-x_r/c}]^2 L}{4 L_u} \quad [8]$$

$$\frac{dL/dL_u}{dL_u} = 1 \text{ for } \begin{cases} L = L_u \\ x_r = 0 \end{cases} \quad \frac{dL}{dL_u} = 1 \text{ for } \begin{cases} L = L_u \\ x_r = 0 \end{cases} \quad [9]$$

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 Mathematical equations with hyperbel functions

$$F_{cb}(x_r, c) = b \tanh(x_r/c) = b \frac{e^{x_r/c} - e^{-x_r/c}}{e^{x_r/c} + e^{-x_r/c}} \quad x_r > 0 \quad [1]$$

$$\frac{IF_{cb}(x_r, c)}{dx_r} = \frac{4b}{c[e^{x_r/c} + e^{-x_r/c}]^2} \quad x_r = \ln L_r / \ln(10) \quad [5]$$

$$\frac{L/dL}{L/D_u} = \frac{4}{[e^{x_r/c} + e^{-x_r/c}]^2}; \quad \frac{dL}{dL_u} = \frac{[e^{x_r/c} + e^{-x_r/c}]^2 L}{4L_u} \quad [8]$$

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$$\frac{dF_{ab}(x_r, a)}{dx_r} = \frac{4b}{a[e^{x_r/a} + e^{-x_r/a}]^2} \quad x_r = \ln L_r / \ln(10)$$

$$\frac{dL}{dL_r} = \frac{4bm}{[e^{x_r/a} + e^{-x_r/a}]^2} \quad dL = \frac{[e^{x_r/a} + e^{-x_r/a}]^2 L}{4bm} \quad [7]$$

$$\frac{dL}{dL_r^m} = \frac{4bm}{[L_r^m + L_r^{-m}]^2} \quad dL = \frac{[L_r^m + L_r^{-m}]^2 L}{4bm} \quad [8]$$

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Mathematical hyperbel and potential functions

$$F_{ab}(x_r, a) = b \tanh(x_r/a) = b \frac{e^{x_r/a} - e^{-x_r/a}}{e^{x_r/a} + e^{-x_r/a}} \quad x_r = \log(L_r) \quad L_r = dL_u / L_u \quad [1]$$

$$\frac{dF_{ab}(x_r, a)}{dx_r} = \frac{4b}{a[e^{x_r/a} + e^{-x_r/a}]^2} \quad x_r = \ln L_r / \ln(10) \quad dx_r / dL_r = 1 / (\ln(10) a) \quad [5]$$

$$\frac{L}{dL} = \frac{4bm}{[e^{x_r/a} + e^{-x_r/a}]^2} \quad dL = \frac{[e^{x_r/a} + e^{-x_r/a}]^2 L}{4bm} \quad [7]$$

$$\frac{L}{dL} = \frac{4bm}{[L_r^{2m} + 2 + L_r^{-2m}]} \quad dL = \frac{[L_r^{2m} + 2 + L_r^{-2m}] L}{4bm} \quad [8]$$

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 Mathematical hyperbel and potential functions

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$$\frac{dF_{ab}(x_r, a)}{dx_r} = \frac{4b}{a[e^{x_r/a} + e^{-x_r/a}]^2}$$

$$\frac{L/dL}{(L/dL)_u} = \frac{4}{[e^{x_r/a} + e^{-x_r/a}]^2}; \frac{dL}{dL_u} = \frac{[e^{x_r/a} + e^{-x_r/a}]^2 L}{4 L_u}$$

$$\frac{L/dL}{(L/dL)_u} = \frac{4}{[L^m_u + L^{-m}_u]^2}; \frac{dL}{dL_u} = \frac{[L^m_u + L^{-m}_u]^2 L}{4 L_u}$$

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$$\frac{L/dL}{(L/dL)_u} = \frac{4}{L_u^{2m} + 2 + L_r^{-2m}}; \frac{dL}{dL_u} = \frac{(L_r^{2m} + 2 + L_r^{-2m})L}{4L_u} \quad [9]$$

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$$\frac{dF_{cb}(x_r, c)}{dx_r} = \frac{4b}{c[e^{x_r/c} + e^{-x_r/c}]^2} \quad \begin{aligned} x_r &= \ln L_r / \ln(10) \\ \frac{dx_r}{dL_r} &= 1 / (\ln(10) L_r) \end{aligned} \quad [5]$$

$$\frac{L}{dL} = \frac{4bn}{[e^{x_r/c} + e^{-x_r/c}]^2} \quad dL = \frac{[e^{x_r/c} + e^{-x_r/c}]^2 L}{4bn} \quad [7]$$

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$$F_{cb}(x_r, c) = b \tanh(x_r/c) - b \frac{e^{x_r/c} - e^{-x_r/c}}{e^{x_r/c} + e^{-x_r/c}} \quad x_r = \ln L_r / \ln(10)$$

$$\frac{dF_{cb}(x_r, c)}{dx_r} = \frac{4b}{c[e^{x_r/c} + e^{-x_r/c}]^2} \quad x_r > 0 \quad [5]$$

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$$\frac{L/dL}{(L/dL)_u} = \frac{4}{[L_u^n + L_r^{-n}]^2}; \quad \frac{dL}{dL_u} = \frac{[L_u^n + L_r^{-n}]^2 L}{4 L_u} \quad [9]$$

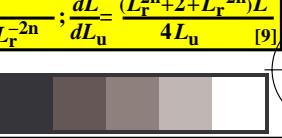
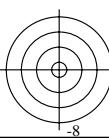
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$$\frac{L/dL}{(L/dL)_u} = \frac{4}{L_u^{2n+2} + L_r^{-2n}}, \quad \frac{dL}{dL_u} = \frac{(L_r^{2n+2} + L_r^{-2n})L}{4L_u} \quad [9]$$



TUB-test chart eer0; Model of normalized receptor-response functions $F_{ab}(x_r)$ and $F_{cb}(x_r)$

Calculation of derivations $F'_{ab}(x_r)$, $F'_{cb}(x_r)$, of contrasts $L/\Delta L$, and discriminations $(\Delta L)_{ab}$, $(\Delta L)_{cb}$