

**Achromatic colour vision with relative luminance**

**Mathematical equations with potential functions**

$$F_{ab}(L_T, m) = b \tanh(x_T/a) = b \frac{L^m - L^{-m}}{L^m + L^{-m}} \quad \frac{x_T = \log(L_T/a)}{L_T = L/a \quad L_T = L/a}$$

$$\frac{dF_{ab}(L_T, m)}{dL_T} = \frac{4bm}{L_T(L_T^m + L_T^{-m})^2} \frac{dx_T/dL_T = 1/(10 \log L_T)}{m=1/(10 \log L_T)} \quad [5]$$

$$\frac{dF_{ab}(L_T, m)}{dL} = \frac{4bmL_U}{L_T(L_T^m + L_T^{-m})^2} \frac{dL_T/dL = L_U/L}{dF_{ab}(L_T, m) = 1} \quad [6]$$

$$\frac{L}{dL} = \frac{4bmL_U L}{L_T(L_T^m + L_T^{-m})^2} \quad \frac{dL}{dL} = \frac{L_U(L_T^m + L_T^{-m})^2}{4bmL_U} \quad [7]$$

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TUB-test chart eerl; Model of normalized receptor-response functions  $F_{ab}(L_T)$  and  $F_{cb}(L_T)$

Calculation of derivations  $F'_{ab}(L_T)$ ,  $F'_{cb}(L_T)$ , of contrasts  $L/\Delta L$ , and discriminations  $(\Delta L)_{ab}$ ,  $(\Delta L)_{cb}$