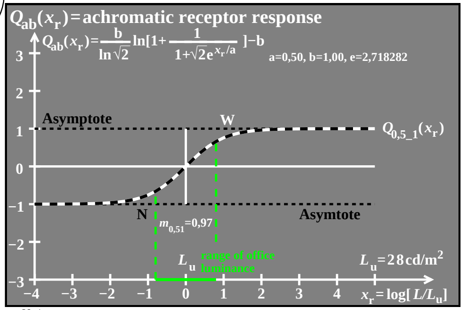


eer20−1n

Achromatic receptor-response function $Q_{ab}[x_r/a]$ for a=0,5 and b=1,0

with x_r=log [L/L_u] (L=test luminance)

L_u=surround luminance

$$Q_{ab}[x_r/a] = \frac{b}{\ln\sqrt{2}} \ln\left[\frac{1}{1+\sqrt{2}e^{(x_r/a)}}\right] - b$$

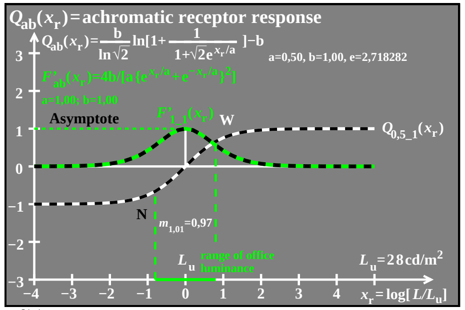
function values for b=1 and any a>0 :

$Q_{a1}[x_r/a \to -\infty]$ = −1 x=log$L$, u=logL_u

$Q_{a1}[x_r/a = 0]$ = 0 x_r=log [L/L_u]

$Q_{a1}[x_r/a \to +\infty]$ = +1 = $x - u$

eer20−2n



eer21−1n

Achromatic colour vision with relative luminance Mathematical equations with hyperbel functions

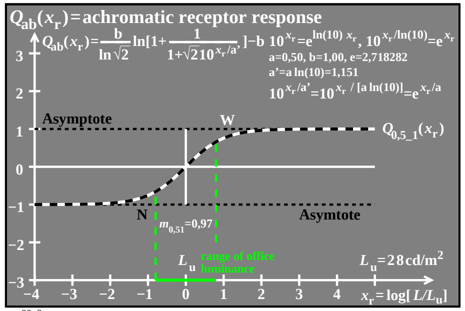
$$F(x) = \tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{u(x)}{v(x)} \quad u'(x)=v(x),\ v'(x)=u(x) \quad [1]$$

$$\frac{dF(x)}{dx} = \frac{u'(x)\,v(x) - u(x)\,v'(x)}{v^2(x)} = \frac{v^2(x) - u^2(x)}{v^2(x)} \quad [2]$$

$$\frac{dF(x)}{dx} = \frac{[e^x+e^{-x}][e^x+e^{-x}] - [e^x-e^{-x}][e^x-e^{-x}]}{[e^x+e^{-x}]^2} \quad [3]$$

$$\frac{dF(x)}{dx} = \frac{4}{[e^x+e^{-x}]^2} = \frac{1}{\cosh^2(x)} \quad [4]$$

eer21−2n



eer20−3n

Achromatic receptor-response function $Q_{ab}[x_r/a]$ for a=0,5 and b=1,0

with x_r=log [L/L_u] (L=test luminance)

L_u=surround luminance

$$Q_{ab}[x_r/a] = \frac{b}{\ln\sqrt{2}} \ln\left[\frac{1}{1+\sqrt{2}e^{(x_r/a)}}\right] - b$$

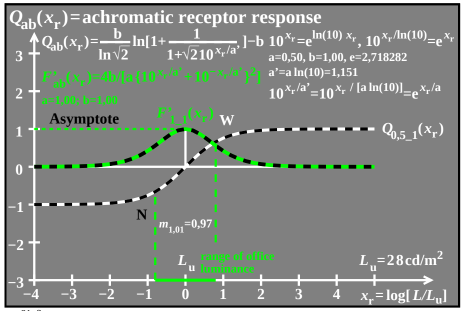
function values for b=1 and any a>0 :

$Q_{a1}[x_r/a \to -\infty]$ = −1 x=log$L$, u=logL_u

$Q_{a1}[x_r/a = 0]$ = 0 x_r=log [L/L_u]

$Q_{a1}[x_r/a \to +\infty]$ = +1 = $x - u$

eer20−4n



eer21−3n

Achromatic colour vision with relative luminance Mathematical equations with hyperbel functions

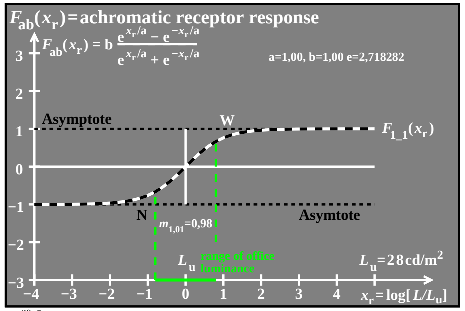
$$F(x,a) = \tanh(x/a) = \frac{e^{x/a} - e^{-x/a}}{e^{x/a} + e^{-x/a}} = \frac{u(x/a)}{v(x/a)} \quad [1]$$

$$\frac{dF(x,a)}{dx} = \frac{u'(x/a)v(x/a) - u(x/a)v'(x/a)}{v^2(x/a)} \quad [2]$$

$$\frac{dF(x,a)}{dx} = \frac{v^2(x/a) - u^2(x/a)}{a\,v^2(x/a)} \quad [3]$$

$$\frac{dF(x,a)}{dx} = \frac{4}{a\,[e^{x/a}+e^{-x/a}]^2} = \frac{1}{a\cosh^2(x/a)} \quad [4]$$

eer21−4n



eer20−5n

Mathematical equations of hyperbel functions

See: *Papula, L., (2003), Mathematische Formelsammlung, Vieweg*

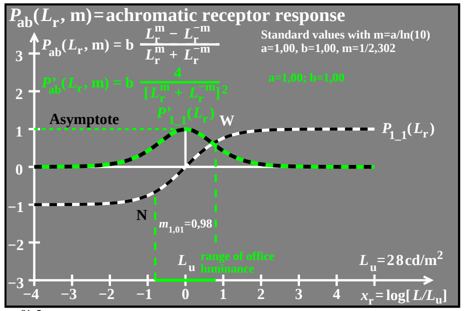
$$\sinh(x) = \frac{e^x - e^{-x}}{2} \quad [1], \quad \cosh(x) = \frac{e^x + e^{-x}}{2} \quad [2]$$

$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}} \quad [3]$$

$$\tanh(x/2) = \frac{\sinh(x)}{\cosh(x)+1} = \frac{\cosh(x)+1}{\sinh(x)} = \frac{e^{x/2} - e^{-x/2}}{e^{x/2} + e^{-x/2}} \quad [4]$$

$$\sinh^2(x) + \cosh^2(x) = 1 \quad [5]$$

eer20−6n



eer21−5n

Achromatic colour vision with relative luminance Mathematical equations with potential functions

$$F(L) = \frac{L^m - L^{-m}}{L^m + L^{-m}} = \frac{u(L)}{v(L)} \quad u'(L)=m[L^{m-1}+L^{-m-1}],\ v'(L)=m[L^{m-1}-L^{-m-1}] \quad [1]$$

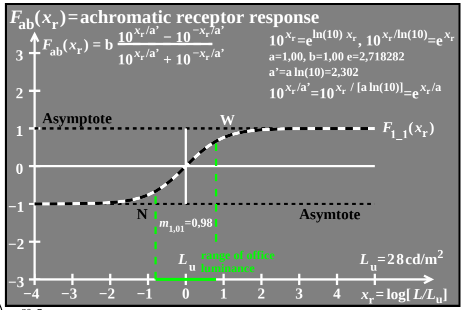
$$\frac{dF(L)}{dL} = \frac{u'(L)v(L) - u(L)v'(L)}{v^2(L)} \quad [2]$$

$$u'(L)v(L) - v'(L)u(L) = m\{[L^{m-1}+L^{-m-1}][L^m+L^{-m}] - [L^{m-1}-L^{-m-1}][L^m-L^{-m}]\} \quad [3]$$

$$= m\{L^{2m-1}+L^{-1}+L^{-1}+L^{-2m-1} - L^{2m-1}+L^{-1}+L^{-1}-L^{-2m-1}\} = 4m/L$$

$$\frac{dF(L)}{dL} = \frac{4m}{L[L^m+L^{-m}]^2} \quad [4]$$

eer21−6n



eer20−7n

Mathematical equations of hyperbel functions

See: *Papula, L., (2003), Mathematische Formelsammlung, Vieweg*

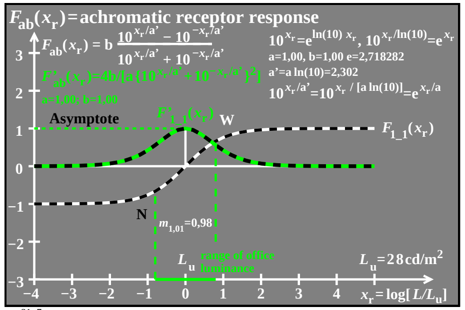
$$\sinh(x) = \frac{10^{x_r/a'} - 10^{-x_r/a'}}{2} \quad [1], \quad \cosh(x) = \frac{10^{x_r/a'} + 10^{-x_r/a'}}{2} \quad [2]$$

$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{10^{x_r/a'} - 10^{-x_r/a'}}{10^{x_r/a'} + 10^{-x_r/a'}} \quad [3]$$

$$\tanh(x/2) = \frac{\sinh(x)}{\cosh(x)+1} = \frac{\cosh(x)+1}{\sinh(x)} = \frac{10^{x_r/2a'} - 10^{-x_r/2a'}}{10^{x_r/2a'} + 10^{-x_r/2a'}} \quad [4]$$

$$\sinh^2(x) + \cosh^2(x) = 1 \quad [5]$$

eer20−8n



eer21−7n

Achromatic colour vision with relative luminance Equations with hyperbel and potential functions

$$F_{ab}(x_r,a) = b\tanh(x_r/a) = b\frac{e^{x_r/a} - e^{-x_r/a}}{e^{x_r/a} + e^{-x_r/a}} \quad x_r=\log(L_r),\ L_r=L/L_u,\ x_r<=0 \quad [1a]$$

$$\frac{dF_{ab}(x_r,a)}{dx_r} = \frac{4b}{a[e^{x_r/a}+e^{-x_r/a}]^2} \quad x_r=\ln L_r/\ln(10),\ dx_r/dL=1/(\ln(10)L),\ m=1/(\ln(10)a) \quad [5a]$$

$$F_{ab}(L_r,m) = b\tanh(x_r/a) = b\frac{L_r^m - L_r^{-m}}{L_r^m + L_r^{-m}} \quad x_r=\log(L_r),\ L_r=L/L_u,\ L_r<=1 \quad [1b]$$

$$\frac{dF_{ab}(L_r,m)}{dL_r} = \frac{4bm}{L_r[L_r^m + L_r^{-m}]^2} \quad x_r=\ln L_r/\ln(10),\ dx_r/dL=1/(\ln(10)L),\ m=1/(\ln(10)a) \quad [5b]$$

eer21−8n

TUB-test chart eer2; Model of normalized response functions $F_{ab}(x_r)$, $Q_{ab}(x_r)$ & $P_{ab}(L_r)$, and derivation, Tangens hyperbolicus $\tanh(x_r)$ and modified functions with e^{x_r}, 10^{x_r}, and L_r

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TUB registration: 20230701-eer2/eer2l0np.pdf /.ps
application for evaluation and measurement of display or print output

TUB material: code=rha4ta