Linear Relationship between CIELAB and Device Coordinates for a new Colorimetric Image Technology (CIT)

Prof. Dr. Klaus Richter, BAM and TU Berlin

Federal Institute for Materials Research and Testing (BAM)
Project Group VIII.34, Visual Methods and Image Reproduction

Unter den Eichen 87, D-12205 Berlin

Tel. +49 30 8104 1834; Fax +49 30 8104 1807

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klaus.richter@bam.de

http://www.ps.bam.de

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http://www.ps.bam.de/CIE05.PDF

Abstract

The *Ostwald* and the *Natural Colour System* (*NCS*) use as colour space a three dimensional **double cone with a circular base.** In this paper simple equations connect the coordinates of the **device dependent colorimetric colour space** with this classical double cone. The **Natural Colour Connection Space (NCCS) is device independent** because it appears as the same double cone for all devices if for example the coordinates relative triangle lightness ***t****, relative chromaticness ***c**** and relative hue ***h**** are used. In every hue triangle ***c**** and ***t**** change between 0 and 1 both in the horizontal and vertical direction with the values ***(c*, t*)*** = (1, 0.5) for the colour Ma of maximum chroma. For the colour Ma the relative CIE lightness ***l**** is hue dependent and ***t**** is not hue dependent. The coordinates of the NCCS connect the device dependent colorimetric coordinates of input and output devices.

In the CIELAB space there are equivalent coordinates *L*a*b* (LAB*)* and $L^*C^*_{ab}H^*$ *(LCH*)* for the same colours which are connected by equations. In the NCCS there are many more equivalent coordinates for example ***nch*, tch**** and ***nce****. Some of the NCCS coordinates are simple *relative CIELAB coordinates* for example relative CIELAB lightness ***l****, relative CIELAB chromaticness ***c**** and relative CIELAB hue ***h**** all with values between 0 and 1. The other NCCS coordinates are based on ***l*, c**** and ***h**** for example the relative triangle lightness ***t*,*** the relative blackness ***n*,*** the relative whiteness ***w*,*** and the relative elementary hue ***e****. The NCCS coordinates have linear relationships to the device dependent colorimetric coordinates ***olv**** and ***cmy**** used in ISO/IEC 15775, ISO/IEC TR 19797 and ISO/IEC TR 24705 (under publication). Additionally the coordinates ***olv**** and ***cmy**** have a defined relationship to the CIELAB coordinates which is a basic for a new Colorimetric Image Technology (CIT).

1. Introduction

The NCCS coordinates may form a new basis for the long term storage of colours in documents (ISO TC 171) and for the viewing of colours on displays (ISO TC 159) at daylight work places. The encoding and decoding of the space coordinates is of interest for digital cameras and in photography (ISO TC42). The simple NCCS has many advantages for the colour workflow between colour input and output in the office area (ISO/IEC JTC1/SC28). The new coordinates may have also advantages for the area of professional printing (ISO TC 130).

A key topic is the definition of the **device dependent colorimetric coordinates**. There are the **colour coordinates of CIEXYZ** (type 1) **and of CIELAB or CIELUV** (type 2) which are connected by non linear (cube root) equations. In image technology type 1 relations are less important compared to **linear** relationships to CIELAB (type 2). Only in this case there is a **linear** relationship between device coordinate differences, CIELAB differences and visual differences. Fifty years ago colour television has started to use coordinates of the device type 2 (using $R^*=G^*=B^*=Y^{1/2}$) which shows approximately a linear relationship between the television coordinates and visual differences for achromatic colours. This is now similar for achromatic colours in the sRGB and scRGB colour spaces used in the International Standard Series IEC 61966-2-1 and -2. For the 16 step series White – Cyan of ISO/IEC 15775 which are equally spaced in CIELAB and defined by equal changes of the ***o**** values the scRGB coordinate differences vary by a factor 8.

In this paper the **device dependent colorimetric coordinates** which are in use in image technology for many years now are defined by a set of equations which shows the **linear** relationships to the CIELAB coordinates.

2. Definition of device dependent colorimetric coordinates

Device dependent colorimetric coordinates,
have a defined relationship to the CIEXYZ (type 1) or the CIELAB or CIELUV (type 2) coordinates of the colour and some device specific colours.

NOTE 1: In image technology device dependent colorimetric coordinates with **linear** relationships to the type 2 coordinates are more important. Only in that case there is a **linear** relationship between device coordinate differences, CIELAB differences and visual differences.
NOTE 2: Device specific colours are for example six chromatic colours and black and white of the device.

In this paper the **device dependent colorimetric coordinates of type 2** have a defined **linear** relationship to the CIELAB colour coordinates *LAB** or *LCH** of the colour and of the six chromatic device colours *CMYOLV* and black *N* and White *W*. Equations (2) to (18) shows the **linear** relationships between the device and CIELAB coordinates.

3. Device independent and dependent colorimetric colour space of type 2

Both the spaces CIEXYZ and CIELAB are device independent. The CIELAB coordinates are defined by non linear (cube root) equations of the CIEXYZ coordinates and will not be repeated here, see CIE Publ. 15.3:2004. Within a CIELAB plane (compare Fig. 1) of a constant hue a **device dependent** colorimetric coordinate is defined by the CIELAB coordinates of the colour (Fa) **and** the CIELAB coordinates of Black (Na), White (Wa), and the colour (Ma). The colour Ma has the maximum chroma which a device produces for that hue. All maximum colours (Ma) are defined by six chromatic colours for example *CMY* and *OLV* of standard printing, see ISO/IEC 15775:1999. For constant hue similar as in the *Ostwald* and the *NCS* colour space the three colours (Na, Wa, Ma) are located on a triangle. The colour Fa is usually located inside the colour triangle. Such a hue triangle (including other special colours Sa, Qa, Xa) is plotted in Fig. 1 together with 12 device dependent equivalent colorimetric coordinates (definition see later).

It is important to realize that a mixture (additive or subtractive) of two colours with CIELAB coordinates L^*_1, a^*_1, b^*_1 and L^*_2, a^{**}_2, b^*_2 usually produce only approximately the intended colour L^*_{1+2}, a^*_{1+2}, b^*_{1+2} which is located in the middle between both mixture colours 1 and 2. The device dependent colorimetric colour space of type 2 **defined here** assumes the production of this intended colours. There is no discussion here how to reach this goal. The International Technical Report ISO/IEC TR 19797:2004-09 describes a method for output linearization in CIELAB which reduces the output differences between actual and intended colours in CIELAB often by a factor 3 to 5 for colour printers and other devices.

4. Connection equations between the NCCS and the CIELAB space

Within a hue triangle (CIELAB hue angle H^* = const.) there is the classical Ostwald equation:

relative blackness + relative chromaticness + relative whiteness equals 1

or $$\boldsymbol{n^*} + \boldsymbol{c^*} + \boldsymbol{w^*} = 1 \qquad (0 <= \boldsymbol{n^*}, \boldsymbol{c^*}, \boldsymbol{w^*} <= 1) \qquad (1)$$

A hue triangle in the CIELAB space is defined by the CIELAB coordinates of the given colour (Fa), the colours Black (Na), White (Wa) and the colour of maximum chroma (Ma). For the colour Fa (a=adapted see below) the **relative** coordinates chromaticness $\boldsymbol{c^*}$, lightness $\boldsymbol{l^*}$, triangle lightness $\boldsymbol{t^*}$, whiteness $\boldsymbol{w^*}$ and blackness $\boldsymbol{n^*}$ may be calculated in the following sequence of the equations (2) to (6).

$$\boldsymbol{c^*}\,(\text{Fa}) = C^*ab\,(\text{Fa}) \,/\, C^*ab\,(\text{Ma}) \qquad (2)$$

$$\boldsymbol{l^*}\,(\text{Fa}) = [\, L^*\,(\text{Fa}) - L^*\,(\text{Na})\,] \,/\, [\, L^*\,(\text{Wa}) - L^*\,(\text{Na})\,] \qquad (3)$$

$$\boldsymbol{t^*}\,(\text{Fa}) = \boldsymbol{l^*}\,(\text{Fa}) - \boldsymbol{c^*}\,\{\, [\, L^*\,(\text{Ma}) - L^*\,(\text{Na})\,] \,/\, [\, L^*\,(\text{Wa}) - L^*\,(\text{Na})\,] - 0.5 \,\} \qquad (4)$$

$$\boldsymbol{w^*}\,(\text{Fa}) = \boldsymbol{t^*}\,(\text{Fa}) - 0.5\, \boldsymbol{c^*}\,(\text{Fa}) \qquad (5)$$

$$\boldsymbol{n^*}\,(\text{Fa}) = 1 - \boldsymbol{c^*}\,(\text{Fa}) - \boldsymbol{w^*}\,(\text{Fa}) \qquad (6)$$

The coordinates of the new relative device dependent space NCCS (small letters) are given in **bold** and *italics* and the CIELAB coordinates (capital letters) are given only in *italics* for easy identification.The coordinates are completed by the relative hue ($0 <= \boldsymbol{h^*} <= 1$) and two cartesian components $-1 <= \boldsymbol{a^*}, \boldsymbol{b^*} <= 1$

$$\boldsymbol{h^*}\,(\text{Fa}) = H^*\,(\text{Fa}) \,/\, 360 \qquad (7)$$

$$\boldsymbol{a^*}\,(\text{Fa}) = \boldsymbol{c^*}\,(\text{Fa}) \cos\,(H^*\,(\text{Fa})) \qquad (8)$$

$$\boldsymbol{b^*}\,(\text{Fa}) = \boldsymbol{c^*}\,(\text{Fa}) \sin\,(H^*\,(\text{Fa})) \qquad (9)$$

The values of the device dependent coordinates are usually between 0 and 1 for equations (2) to (7) and different compared to CIELAB coordinates (usually between 0 and 100). Exceptions may appear for luminous and fluorescent colours. In Image Technology for different applications (printers and displays) complementary coordinates are used for example blackness $\boldsymbol{n^*}$ for printers and whiteness $\boldsymbol{w^*} = 1 - \boldsymbol{n^*}$ for displays and $\boldsymbol{c^*}, \boldsymbol{m^*},$ and $\boldsymbol{y^*}$ for printers and $\boldsymbol{o^*} = 1 - \boldsymbol{c^*}$, $\boldsymbol{l^*} = 1 - \boldsymbol{m^*}$, $\boldsymbol{v^*} = 1 - \boldsymbol{y^*}$ for displays. The letters $\boldsymbol{olv^*}$ with o = orange red, l = leaf green, and v = violett blue are used instead of *rgb.* The elementary colours use *rjgb*; compare ISO/IEC 15775:1999. For the calculation of these coordinates (compare $\boldsymbol{cmyn^*}$ and $\boldsymbol{olvi^*}$ in Fig. 1 with *three* and *four* components) additionally the CIELAB data of **six** colours *CMY* and *OLV* are necessary. The CIELAB data of the six colours allow to calculate the colour of maximum chroma for every hue $\boldsymbol{h^*}$, compare ISO/IEC TR 19797:2004-09

Figure 1: 12 device dependent colorimetric coordinates for the standard Offset device space

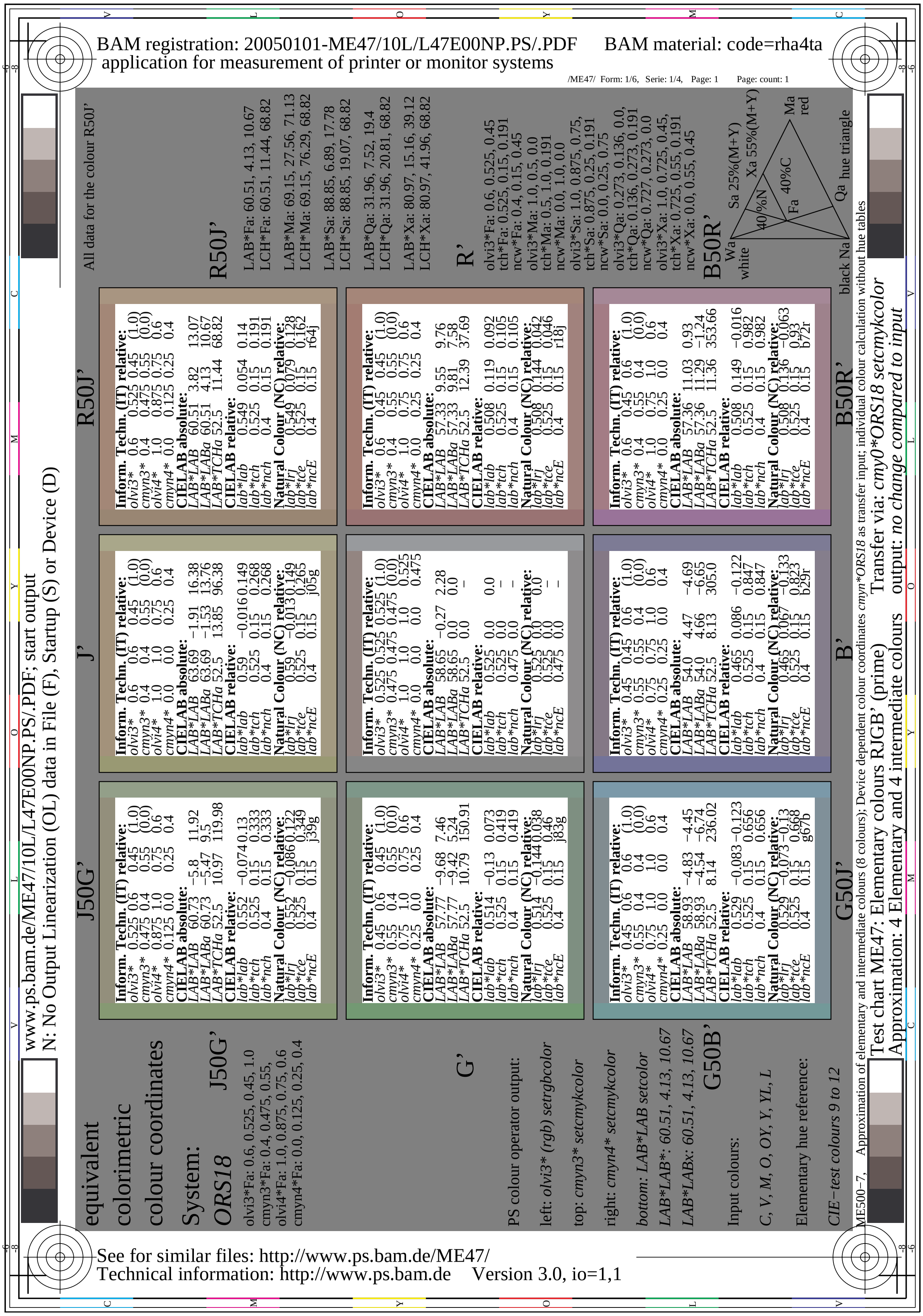


Fig 1 shows the CIELAB and 12 device dependent colorimetric coordinates for the standard Offset device space (ORS18) according to ISO/IEC 15775:1999. Rough approximations of the elementary and intermediate hues R’, R45J’, J’, J50G’, G’, G50B’, B50R’ are shown. For the hue R50J’ some intermediate data are calculated on the right side, which shows also the hue triangle with the achromatic colours (Na, Wa) and the colour of maximum chroma Ma.

For any real device the red-green and yellow-blue chroma coordinates of the CIELAB space are usually only approximately cero. Therefore an appropriate **adaptation** (F**a**) instead of (F) is necessary which tilt the achromatic colours of the device exactly on the achromatic vertical axis. An inverse adaptation after the calculations will produce again measured CIELAB data (compare ISO/IEC TR 19797:2004-09).

The inverse coordinates are completed if instead of relative blackness ***n**** the inverse coordinates brilliantness ***i**** and instead of whiteness ***w**** the inverse coordinate deepness ***d**** is used, see Richter (1980).

i* (Fa) = 1 – ***n**** (Fa) (10)

d* (Fa) = 1 – ***w**** (Fa) (11)

5. Definition of different equivalent colorimetric coordinates

Device independent equivalent colorimetric coordinates,
are transformed by equations to the same CIELAB coordinates for example to *LCH**.

NOTE 1: The device independent equivalent colorimetric coordinates for example *LAB** or *LUV** or *XYZ* are transformed to the same CIELAB coordinates for example *LCH**.

NOTE 2: A compatibility test may show if in image technology the same output colours appear if the data *LAB*, LUV** and *XYZ* are used in the file.

Device dependent equivalent colorimetric coordinates,
are transformed by equations to the same CIELAB coordinates for example *LCH**.

NOTE 1: The device dependent equivalent colorimetric coordinates for example *nch*, tch*, nce*, cmy*, olv** are transformed to the same CIELAB coordinates for example *LCH**.

NOTE 2: A compatibility test may show if in image technology the same output colours appear if equivalent device dependent colorimetric coordinates are used in the file.

6. Device dependent equivalent colorimetric coordinates

Fig. 1 shows 12 different device dependent equivalent colorimetric coordinates and the CIELAB coordinates *(LAB** and *LCH*)*. There are much more possibilities if in addition the inverse NCCS coordinates are used. The user friendly coordinates ***nce**** are similar to the coordinates blackness ***n****, chromaticness ***c**** and elementary hue ***e**** of the *Natural Colour System NCS*. In future for many applications in design and architecture the coordinates ***nce**** may be preferred compared to ***olvi**** or ***cmyn****. For the definition of the elementary hues ***e**** of Red, Yellow, Green and Blue (*RJGB)* the CIELAB data of the CIE-test colours no. 9 to 12 are used. The CIE-test colours no. 9 to 12 are within 3% identical to the *Miescher* and *NCS* elementary hues, see Richter (1969).

The coordinates ***cmyn**** with three, four *(using black generation)* and five coordinates *(using under colour removal, not shown here)* are necessary for **Material efficiency** of printer output. Some printers produce achromatic colours by an overprint of three chromatic colours instead of using only one achromatic colour black. For the output intend with black generation four components ***cmyn3**** are necessary. The coordinates ***cmyn3**** include no use of black for output and ***cmyn4**** includes black generation for output. Using only the three components ***olv* (rgb*)*** will often not lead to Material efficiency.

7. Device dependent colorimetric coordinates *cmy** and *olv**

There is an everyday question how to calculate the ***cmy**** or ***olv* (rgb*)*** data for a given *LCH** colour of the CIELAB space. Can we solve this question by the equations (2) to (11)? The answer is “YES” but only if we know the *LAB** and ***cmy**** data of the Ma colour with maximum chroma.

How to get the Ma colour with maximum chroma of the device for that hue? For any given colour Fa (compare Fig. 1) there are methods to calculate the *LAB** and ***cmy**** data for the colour Ma. Additionally there are tables with the *LAB** and ***cmy**** data for 360 CIELAB hue steps.

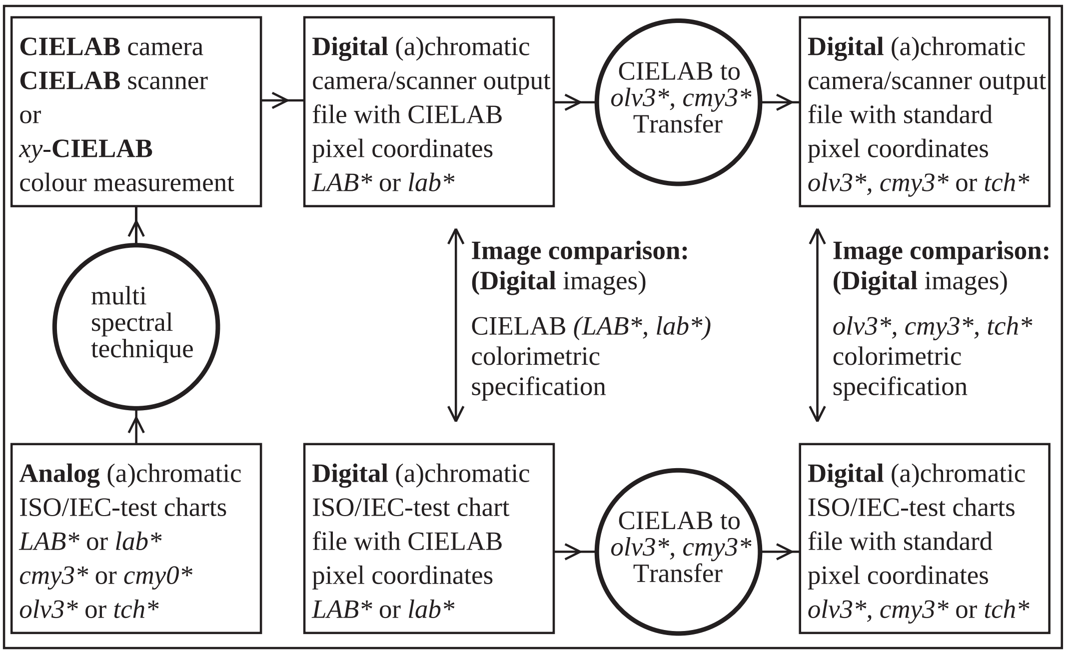
Program subroutines in the PostScript programming language in source code are included in the PS and PDF file format version of Fig. 1. For the files with the extensions PS, PDF or DAT (only subroutines) see the URL

http://www.ps.bam.de/ME47

For some standard devices the *LAB** data of the 8 device colours CMYOLVNW and a given colour Fa are used to calculate the *LAB** and ***cmy**** data of the colours Ma, Sa, Qa, Xa. For special devices one must determine the *LAB** data of the 8 device colours CMYOLVNW by colour measurement or by visual comparison with colours of the *RAL Design Atlas* which includes 1688 colour samples with the CIELAB *LCH** data for CIE standard illuminant D65.

New CIELAB camera and scanners are on the market which measure the spectral reflectance at 31 wavelength between 400nm and 700nm. The software calculates the CIELAB *LCH** data for CIE standard illuminant D65.

Figure 2: New CIELAB cameras and scanners and transfer from *LAB to *cmy***



LE430−72, *LAB* olv3*, cmy3* and tch**

Fig. 2 shows new CIELAB cameras and scanners. The output needs a transfer from *LAB** to ***cmy3*.*** Equations (2) to (18) of this paper give a solution for this topic.

The ***cmy3**** and the inverse ***olv3**** device dependent colorimetric coordinates are connected by the so called "1 minus relation". If for example all three components of ***cmy3**** are equal one then the three components of ***olv3**** are all cero. Because there are ***cmy3*, cmy4**** and ***cmy5**** data the number of coordinates is added here.

For the colours Ma located in the hue planes C, M and Y one component of ***cmy3**** has a value 1 and the other two are cero. In the hue planes O, L and V two components of ***cmy3**** have a value 1 and one is cero.

We order the value according to the maximum and minimum value of ***cmy3**** for example

$$\boldsymbol{u^*} = \max(c3^*, m3^*, y3^*) \quad (12)$$

$$\boldsymbol{v^*} = \min(c3^*, m3^*, y3^*) \quad (13)$$

Then there are three relations for any colour (Fa, Ma, Sa, Qa, Xa) of Fig. 1

$$\boldsymbol{n^*} = \boldsymbol{v^*} \quad (14)$$

$$\boldsymbol{w^*} = 1 - \boldsymbol{u^*} \quad (15)$$

$$\boldsymbol{c^*} = 1 - \boldsymbol{w^*} - \boldsymbol{n^*} = \boldsymbol{u^*} - \boldsymbol{v^*} \quad (16)$$

Equation (16) is identical to the *Ostwald* equation (1).

There are further linear relations which connect the ***cmy3**** values of Fa, Sa, Ma, Wa, Na

$$\begin{aligned} \boldsymbol{cmy3^*}(\mathrm{Sa}) &= \boldsymbol{cmy3^*}(\mathrm{Ma})(1 - \boldsymbol{w^*}) + \boldsymbol{cmy3^*}(\mathrm{Wa})\,\boldsymbol{w^*} \\ &= \boldsymbol{cmy3^*}(\mathrm{Ma})(1 - \boldsymbol{w^*}) \end{aligned} \quad (17)$$

$$\begin{aligned} \boldsymbol{cmy3^*}(\mathrm{Fa}) &= \boldsymbol{cmy3^*}(\mathrm{Sa})(1 - \boldsymbol{n^*}) + \boldsymbol{cmy3^*}(\mathrm{Na})\,\boldsymbol{n^*} \\ &= \boldsymbol{cmy3^*}(\mathrm{Sa})(1 - \boldsymbol{n^*}) + \boldsymbol{n^*} \end{aligned} \quad (18)$$

As a result for any colour Fa in the plane of the hue triangle (inside and outside) the three coordinates ***cmy3**** can be calculated if the three coordinates ***cmy3**** of the colour Ma are given. The equations (17) and (18) allow the inverse calculation from the ***cmy3**** data of a colour Fa to the ***cmy3**** data of the colour Ma.

If the CIELAB data are needed then for example only the first coordinate of ***cmy3**** may have a value 1 and the other two may have a value cero. Then the colour Ma has the CIELAB data of the Cyan device colour.

8. Summary

All NCCS coordinates are simple **linear** transformations of the CIELAB coordinates, compare equations (2) to (18). Therefore for example an equal 16 step spacing of a colour series between white and cyan in CIELAB space will transform to 16 step equally spaced digital values of the cyan coordinate ***c3**** and vice versa. The 16 equally spaced digital values are used to define the 16 step colour series in the file.

There are many sets of device dependent equivalent colorimetric coordinates. A colour engine uses the equations (2) to (18) to transform from all equivalent NCCS coordinates to the same *cmyn4** or *olvi3** (and the same CIELAB) coordinates. The *cmyn4** coordinates are useful for many printer applications. There is an appropriate setup interpretation by nearly all software products which can be improved by the output linearization method of ISO/IEC TR 19797:2004. In cases where it is possible to use directly the CIELAB coordinates in the file then the results are often not appropriate and this workflow is therefore not recommended here.

9. References

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Remark: The International Technical Report ISO/IEC TR 24705:2005-XX corresponds to DIN 33866-1 and -3 to -5:2000

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http://www.lichttechnik.tu-berlin.de

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