Smooth change of color difference metric

between large CIELAB color differences and threshold color differences

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Background

At the CIE Division 1 meeting in Veszprem (Hungary) in 2002 the CIE decided a Reportership:

R1-31:
Smooth change of color difference metric between large CIELAB color differences and threshold color differences

Terms of reference:
*To bridge the gap for color differences between large CIELAB color differences, e.g. DE*ab=20, and threshold differences for office conditions*

Reporter:
Klaus Richter (DE)

This Reporter is to investigate the relation between a large color difference and a small one at the threshold level with a possible method to develop a continuous metric to connect them, and to report Division 1 how we work on this issue in the future

To start work and discussions the reporter has written a paper with the title

Basic data, methods and formula to bridge the gap for color differences between large CIELAB color differences, e.g. DE*ab=20, and threshold differences for office conditions
see for the content at the internet

http://www.ps.bam.de/CIE6303B.PDF

1.0 Basic papers, methods and data to solve the term of reference

A paper "Cube root color spaces and chromatic adaptation" published in 1980 in Color, Research and Application by K. Richter and new experimental data and models are useful to bridge the gap between large CIELAB colour differences and threshold differences for office conditions.

The office conditions are e. g. specified in ISO/IEC 15775 produced by ISO/IEC SC28 "Office equipment". This standard specifies the standard observation and illumination conditions for the comparison of the colour copy with the original ISO/IEC-test chart.The following specifications are given:

Illumination: 1000 lux, Illuminant: Daylight D65, CIE 2 degree observer, CIE 45/0 visual and measurement geometry, 5fold back packing of the white paper for the visual comparison and the CIE measurement.

It is the aim here not to change the main properties of the CIELAB colour space which is recommended and used for large colour differences. e. g. DE*ab=20, in the colour reproduction area.

There are CIE colour difference formulas for industrial application, e. g. CIE2000, which are recommended for up to 5 CIELAB units. These industrial colour differences are designed for above threshold (pass-fail) experiments.

It is expected that the new model which will bridge the experimental threshold results of e. g. of MacAdam, Richter, Holtsmark-Valberg and others and the large CIELAB colour differences. This bridge may produce a continuous metric for the range between threshold and 5 CIELAB up to large CIELAB colour differences.

The metric for the description of both the large CIELAB colour differences and the threshold differences will be very different, e. g. more different compared to the CIE colour difference formulas CIELAB 1976 and CIE2000.

2.0 Basic properties of the metric

We have to define a "line element" with metric quantities (e. g. threshold, above threshold up to about 3 threshold, medium and large color differences) as function of the stimuli properties (e. g. luminance, chromaticity) for the standard office condition. Richter has developed separate formulas for threshold differences and CIELAB differences (1976, 1985, 1996). It is expected that a trial to combine both formulas and the use of many known properties of vision may help to solve the term of reference.

A opponent system which models opponent physiological responses will be used in the following.This model adds two "threshold (th)" processes a White process W and a Black process N to a "scaling (sc)" process NW. In many cases a slope 1 for the threshold process W is given by experimental results. In this case the equations are much simpler compared to the processes N and NW with slopes different to 1. Therefore we start to model this process.

3.0 Modelling the threshold and scaling data as function of sample luminance

We construct a simple model for the physiological signals and look how this changes as function of chromaticity and luminance of the stimuli. A special case is the discussion of the physiological signals for the complementary optimal colours which produce according to the Holtsmark-Valberg experiments (1969) equal thresholds. Therefore the model must have a high degree of symmetry for complementary optimal colours and the following has such properties.

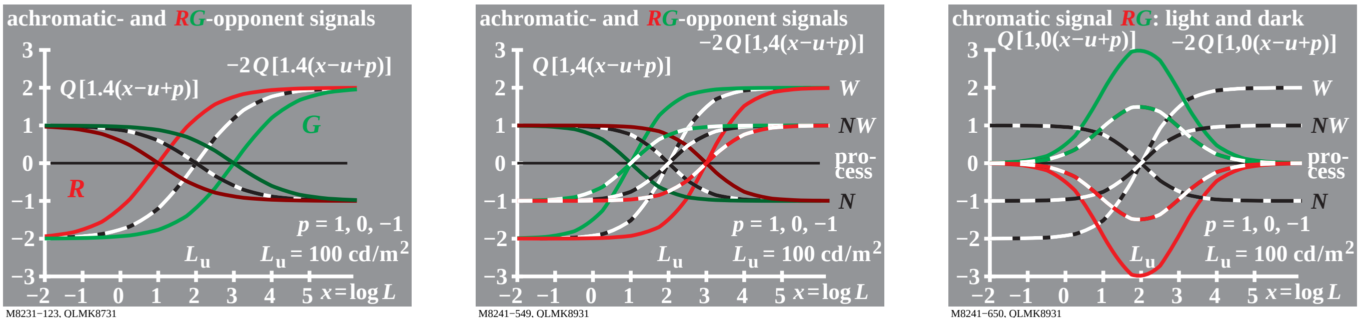

Figure 1: Processes White W and Black N and sum NW and high discrimination for threshold process W

Fig. 1 shows of the left side two opponent achromatic processes White W and Black N and two opponent chromatic processes Red and Green. Light and dark colours are used for the coding. The slope and amplitude in vision seems to change according to temporal, spacial and luminance properties of the stimuli. We have used an **amplitude and slope** of **one log unit and –0.5 for the Black process N** and **two log units and 1.0 for the White process W**.

> Remark: Normal colour vision in offices shows samples with a luminance factor between 2.5 and 90 in a grey background with the luminance factor of 20. If the black line is the basic line for the grey surround then this range is totally within the +/– one log unit range of the black process N and within the limit of all other processes.

The sum of the two processes N and W in Fig. XX (middle) is the scaling process NW with a slope of 0.5. This slope 0.5 is very different compared to the slope 1.0 of the threshold process White W. Exactly this is the main property of many experimental results on threshold and scaling.

It is assumed that at threshold only one process is active and describes threshold. The other starts to contribute. e. g. near two thresholds and contributes to the colour difference. In Fig. 3 (right) the green and green–white curves describe the ability to discriminate for the processes W and N. The process W can discriminate better by a factor of e. g. 2.

> Example: For a luminous factor of 0.5 the threshold for the White process W is at 0.005 (=0.5/100). The factor 100 is given by experience. At threshold the luminance factors are 0.500 and 0.505 for the two fields which we can just discriminate. For a luminous factor of 0.5 the threshold for the Black process N is at $0.07=(0.5/100)^{1/2}$. Therefore the Black process N starts to contribute to the colour difference above the threshold of the White process. The starting point of this contribution depends on the amplitude and slope of the two processes shown in Fig. 1.

4.0 CIE coordinates and normalization used in Image Technology (IT)

During the last years the reporter has worked a lot on International Standards and Technical Reports in Image Technology (IT). In IT the CIE coordinates must be normalized for the region between 0 and 1.

The Index 01 is used here for coordinates in the range 0 to 1 (This normalisation is already used in CIELAB but without a special abbreviation). We define

$$X_{01} = X / X_n \qquad Y_{01} = Y / Y_n \qquad Z_{01} = Z / Z_n \qquad (4; 1)$$

$$x_{01} = X_{01} / (X_{01} + Y_{01} + Z_{01}) \qquad x_{01} = Y_{01} / (X_{01} + Y_{01} + Z_{01}) \qquad (4; 2)$$

There are many advantages of these abbreviations. If the coordinate range is between 0 and 1 then for any transfer function (e. g. square root) the end points 0 and 1 remain unchanged. The luminance factor of a complementary (c) optimal colour is defined e. g. by $Y_{01c} = 1 - Y_{01}$.

5.0 Basic properties of a threshold metric

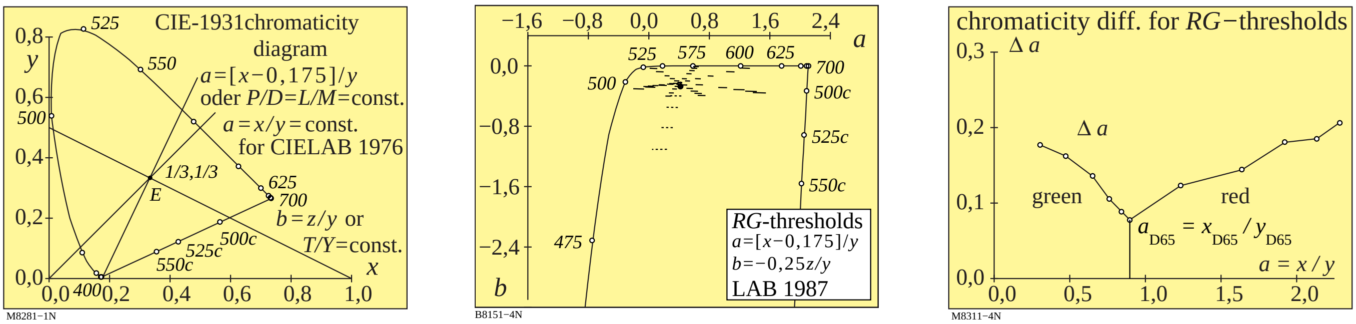

Figure 2: Analysis of threshold axis and increase of threshold in chromaticity diagram (*a, b*)

In Fig. 2 the main axis to describe scaling and threshold data are shown in the (*x, y*) chromaticity diagram (left). Threshold data along the axis Purple–Turquoise are shown in an (*a, b*) chromaticity diagram. The chromaticity difference (delta *a*) increases both to the red and green side. The break point indicates that two visual processes are active which are called red–green and green–red process.

The coordinates of two chromaticity diagrams (a_{01}, b_{01}) are useful to calculate the threshold and large colour differences. The a axis is slightly different for both cases. The coordinates of the (a_{01}, b_{01}) chromaticity diagrams are for the description of threshold data

$$a_{0,.th} = (x_{01} - 0.175) / y_{01} \quad \text{and} \quad b_{01,th} = -0.25\, z_{01} / y_{01} \qquad (5; 1)$$

and for the description of **scaling data**

$$a_{01,sc} = x_{01} / y_{01} \quad \text{and} \quad b_{01,sc} = -0.40\, z_{01} / y_{01} \qquad (5; 2)$$

Remark 1: The coordinates of some figures are the CIE standard coordinates. They are not normalized for the range between 0 and 1. The conclusion will be the same if the 01-coordinates will be used.

Remark 2: The distance in the chromaticity diagrams (a_{01},b_{01}) describe approximately e. g. the *Evans* G_0-colours of equal greyness and the *Evans* chromatic threshold. *Evans* called the amount of the G_0-phenomena which is highly correlated to the distance between the spectral colours and the achromatic point in the (a_{01}, b_{01}) chromaticity diagram the “chromatic strength” of the spectral colours.

6.0 Threshold metric for the Holtsmark–Valberg threshold of complementary optimal colours

Holtsmark and Valberg published in 1969 and 1971 the experimental result that complementary optimal colours have the same threshold.

Richter (2003) developed a threshold formula which is in full agreement with the Holtsmark-Valberg results. A linear colour space ABY is used in this formula

$$\text{delta } E^*_{ABY,th} = \text{const} \{ [(\text{delta } A_{01}) / A_{01}]^2 + [(\text{delta } B_{01}) / B_{01}]^2 + [(\text{delta } Y_{01}) / Y_{01}]^2\}^{1/2} \qquad (6; 1)$$

We have to show that the threshold difference is equal for complementary optimal colours. With the definitions of IT

$$A_{01} = (a_{01} - a_{01n}) Y_{01} = (x_{01} / y_{01} - 1) Y_{01} = (X_{01} / Y_{01} - 1) Y_{01} = X_{01} - Y_{01} \qquad (6; 2)$$

For the complementary colours it is always valid $X_{01c} = 1 - X_{01}$, $Y_{01c} = 1 - Y_{01}$, $Z_{01c} = 1 - Z_{01}$. Therefore

$$A_{01c} = X_{01c} - Y_{01c} = 1 - X_{01} - (1 - Y_{01}) = Y_{01} - X_{01} = - A_{01} \qquad (6; 3)$$

and similar

$$B_{01c} = - B_{01} \qquad (6; 4)$$

According to the Weber-Fechner law it is valid at threshold for the luminance factor Y_{01}

$$(\text{delta } Y_{01}) / Y_{01} = \text{const} \qquad 6; 5)$$

If the basic colour is dark then the complementary is light and vice versa then the Weber-Fechner law lead to the same constant for complementary optimal colours. Therefore

$$(\text{delta } Y_{01}) / Y_{01} = (\text{delta } Y_{01c}) / Y_{01c} \qquad (6; 6)$$

Therefore the three components of equation (6; 1) are equal and we can write the following combined result

$$\text{delta } E^*_{ABY,th,basic\ colour} = \text{delta } E^*_{ABY,th,complementary\ colour} \qquad (6;7)$$

The development of the equation (6; 7) is the proof of the Holtsmark–Valberg threshold results for complementary optimal colours

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7.0 The threshold metric of Richter

There are some equations of the threshold metrics of Richter (1985) in Appendix A

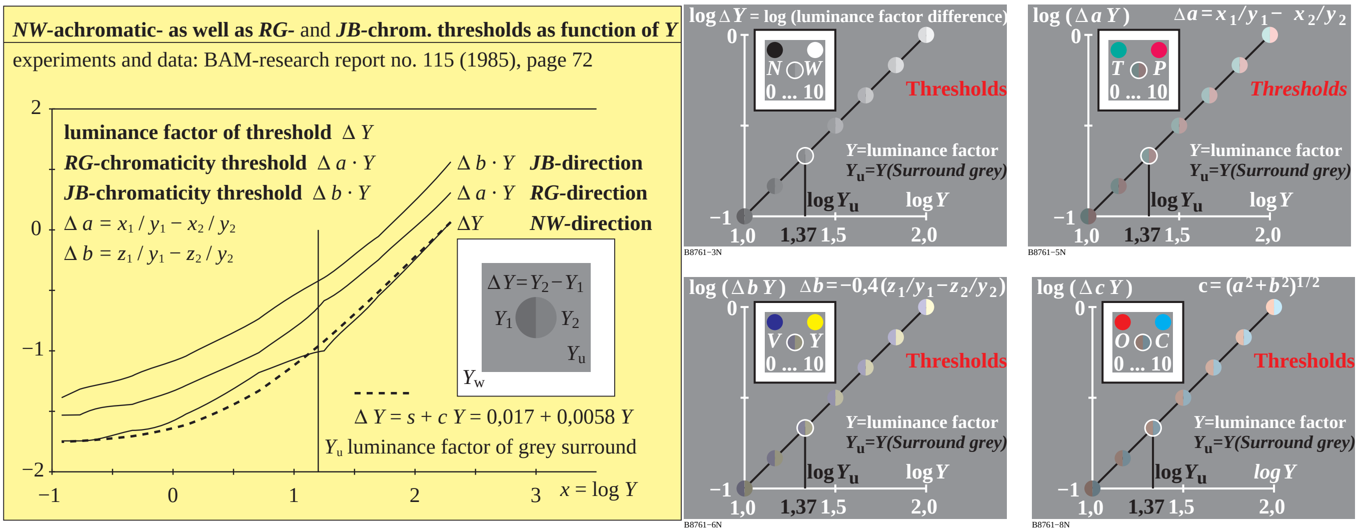

Figure 3: Analysis of threshold change as function of luminance and chromaticity (*a, b*)

Fig. 3 describes for **achromatic** colours **the luminance factor** threshold (delta *Y*) as function of luminance factor *Y*

$$(\text{delta } Y_{01,th}) = \text{const } ln\,(1 + c_Y\, Y_{01}) \quad \text{with} \quad c_Y = c2 / c1 = 0.0058 / 1.7 \qquad (7; 1)$$

Fig. 3 describes for **achromatic** colours the **chromaticity** threshold (delta *a*) as function of luminance factor *Y*

$$log\,[(\text{delta } a)\, Y\,] = log\, Y \qquad (7; 2)$$

This leads to (compare Fig. 3, the slope is one for $Y > Y_u$)

$$(\text{delta } a) = \text{const.} \qquad (7; 3)$$

The **threshold metric** (Index th) for **chromatic** colours of **equal** luminance factor in the red–green direction is given by the equation (see Fig. 2, right)

$$a_{01,th} = const\ (\,a_{01} - 1) / (1 + 0.5\,|\,a_{01} - 1\,|\,) \qquad (7; 4)$$

The following equation describes the threshold for **achromatic** samples of **equal luminance** factor for small $a_{01,th} - 1$ by

$$(\text{delta } a_{01,th}) = const \qquad (7; 5)$$

This is the same as equation (7; 3) with the coordinates of IT. The complete line element must allow to calculate all the threshold properties of the Holtsmark–Valberg and Richter threshold experiments in one equation.

Remark: The intended complete equations with all the properties of chapter 6.0 and 7.0 is not known yet. One must study more carefully the scaling of A_{01} in equation (6; 1) and of $a_{01} = A_{01} / Y_{01}$ in (7; 4). The scaling of B_{01} and b_{01} is expected to be similar.

8.0 Scaling metric of the CIELAB color space

The scaling metric of the CIELAB space is different but connected to the threshold metric.

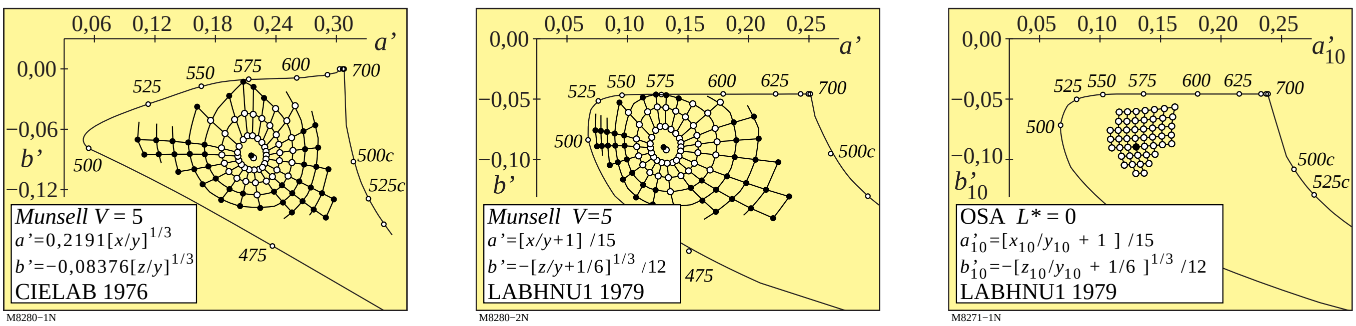

Figure 4: Munsell and OSA colours of mean lightness in diagrams (*a',b'*).

Fig. 4 shows colours of equal luminance factor in the diagrams (*a',b'*). The middle and right diagram uses the **linear** chromaticity $a = x / y$ in red–green direction. For surface colours (open circles in Fig. 3 middle) the spacing is not very different compared to the left, so a linear coordinate can be used in red–green direction as a good approximation. The extrapolation of the spacing of the Munsell system in the saturated green (filled circles) is wrong and calculate to high differences according to the experimental results of Richter (1985).

The **CIELAB metric** (scaling metric, index sc) in the red–green direction of the CIELAB colour space is given by

$$a^*_{sc} = const\ [\ (X/X_n)^{1/3} - (Y/Y_n)^{1/3}] \quad (8;\ 1))$$

The normalisation in the range between 0 and 1 and the use of the chromaticity coordinate $a_{01} = x_{01} / y_{01}$ leads to

$$a^*_{01,sc} = const\ \ [\ X_{01}^{1/3} - Y_{01}^{1/3}] \quad (8;\ 2)$$

$$= const\ \ [\ (X_{01}/Y_{01})^{1/3} - 1\ ]\ Y_{01}^{1/3} \quad (8;\ 3)$$

$$= const\ [\ a_{01}^{1/3} - 1\ ]\ Y_{01}^{1/3} \quad (8;\ 4)$$

We can use the definition $a_{01}' = a_{01}^{1/3}$ to get

$$a^*_{01,sc} = const\ [\ a_{01}' - 1\ ]\ Y_{01}^{1/3} \quad (8;5)$$

Result 1: The **CIELAB metric in red-green direction** is a **nonlinear (cubic) metric** as cube root function of the chromaticity $a_{10} = x_{01} / y_{01}$ and a **nonlinear (cubic) metric** as cube root function of the luminance factor *Y*

Additionally we can calculate *approximately* CIELAB chroma according to Fig. 4 by using $a_{01} = x_{01} / y_{01}$

$$a^*_{01,sc} = const\ [\ a'_{01} - a'_{01,n}\ ]\ Y_{01}^{1/3} \quad (8;\ 6)$$

$$= const\ (1/15)\ [\ x_{01} / y_{01} - 1\ ]\ Y_{01}^{1/3} \quad (8;\ 7)$$

$$= const\ [\ a_{01}\ \ - 1\ ]\ Y_{01}^{1/3} \quad (8;\ 8)$$

Result 2: The **CIELAB metric in red-green direction** is approximately a **linear metric** as function of the chromaticity $a_{01} = x_{01} / y_{01}$ and a **nonlinear (cube root) metric** as function of the luminance factor *Y.* Equations (8; 5) and (8; 8) seem to be very different. But in the limited range of the chromaticity a_{01} for the surface colours of e.g. the Munsell and OSA scaling data both equations describe the scaling well (see Fig. 4)

The chroma difference of $a^*_{01,sc}$ in equation (8; 5) may be calculated as

$$(\text{delta } a^*_{01,sc}) = const\ (\text{delta } a_{01}')\ Y_{01}^{1/3} + const\ [\ a_{01}' - 1\ ](1/3\ (\text{delta } Y\ )\ /Y_{01}^{2/3}) \quad (8;\ 9)$$

The chroma difference of $a^*_{01,sc}$ in equation (8; 8) may be calculated (without the ' (prime)) as

$$(\text{delta } a^*_{01,sc}) = const\ (\text{delta } a_{01})\ Y_{01}^{1/3} + const\ [\ a_{01} - 1\ ](1/3\ (\text{delta } Y\ )\ /Y_{01}^{2/3}) \quad (8;10)$$

The two equations (8; 4) and (8; 8) are based on the chromaticity coordinate $a_{01} = x_{01} / y_{01}$

If $a_{01} = 1$ then $x_{01} = y_{01}$. This is the diagonal in the CIE (*x, y*) chromaticity diagram which is normalized to D65. In this diagram the achromatic point of D65 is at (x_{01}=1/3, y_{01}=1/3).

It has been shown that for thresholds the coordinate which cuts the x_{01} chromaticity axis at 0.175

$$a_{01} = (x_{01} - 0.175)\ /\ y_{01} \quad (8;12)$$

gives a better description for the threshold. This axis describes the P/D=L/M ratio and cuts the spectral locus near the wavelength lambda = 400 nm.

9.0 Summary

Equations which describe the threshold as function of luminance factor and chromaticity have been developed. The equations can describe the Holtsmark–Valberg (1969, 1971) threshold experiments for complementary optimal colours and many experimental results of Richter (1979, 1980, 1985, 1996) and others.

Similar chromaticity coordinates describe the Munsell and OSA scaling. Therefore a smooth change of the color difference metric between large CIELAB color differences and threshold color differences is expected in not to far future. The metric seems well defined for luminance factors below and above 0.5 log units of the surround luminance factor. For the larger luminance factor range a more complex S-shaped function of a physiological model (see Fig. 1) is necessary. The slopes of these functions are 0.5 and 1 similar as in a recent paper of Vienot (2003). The luminance range is by a factor two different for these functions. This seem similar in the spatial and temporal range discussed recently by Martinez-Uriegas (2003).

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