Colorimetric supplement for DIN 33872-1 to -6

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Version 2.1: 2008-06-01

See also the web site of the BAM project group VIII.34, “Visual methods and colour reproduction”

http://www.ps.bam.de

For the German and English version of this paper see (1,4 MByte, 42 pages)

http://www.ps.bam.de/D33872-A.PDF

http://www.ps.bam.de/D33872-AE.PDF

For the *PS* and *PDF* test files in German and English of DIN 33872-1 to -6, see

http://www.ps.bam.de/33872

http://www.ps.bam.de/33872E

NOTE: There is an older German version of this paper belonging to the draft E DIN 33872-1 to 6:2007, see

http://www.ps.bam.de/D33872_A.PDF

and *PS* and *PDF* test files belonging to the draft (D) E DIN 33872-1 to -6, see

http://www.ps.bam.de/33872D

http://www.ps.bam.de/33872DE

About the content of the standard series DIN 33872-1 to -6

The multipage standard series DIN 33872 “Information technology – Office machines – Method of specifying relative colour reproduction with YES/NO criteria" is valid for colour reproduction systems "digital – analog". The colour reproduction systems include for example the computer operating system and the application software for the analog output of a digital file with the devices

– printers;
– multifunctional devices;
– displays;
– data projectors.

The standard series describes a method for the specification of the relative colour reproduction properties of the output according to defined YES/NO criteria. The aim is the visual assessment of the output properties of these colour reproduction systems.

The standard series DIN 33872 "Information technology – Office machines – Method of specifying relative colour reproduction with YES/NO criteria" consists of:

– *Part 1: Classification, terms and principles*
– *Part 2: Test charts for output properties – Testing of the discriminability of 5 and 16 step colour series*
– *Part 3: Test charts for output properties – Testing of equality for four equivalent grey definitions and discriminability of the 16 grey steps*
– *Part 4: Test charts for output properties – Testing of equality for two equivalent colour definitions with 5 and 16 step colour series*
– *Part 5: Test charts for output properties – Testing of elementary hue agreement and hue discriminability*
– *Part 6: Test charts for output properties – Testing of equivalent spacing and of the regular chromatic spacing*

Part 1 includes *Classification, terms and principles* which are extended by this colorimetric amendment to DIN 33872-1 to -6. The DIN standard committee Information Technology “Office Systems” has decided at their meeting in March 2007 to publish this suplement in a separate paper in the internet on the above BAM server.

This allows a distribution free of charge and a long term presentation of this colorimetric suplement with many colour figures in the PDF file format and with low file size. With this method all colour figures can be presented in scalable vector graphics. Direct links in the PDF file to the test charts and to the colorimetric example calculations are realised and presented on the above BAM server.

Abstract

The colorimetric amendment for DIN 33872-1 to -6 includes in section 1 references to other standard documents of the section colour reproduction. In the sections 2 and 3 the colours are described which are used in television and in print as well as in colour order systems and in information technology. In the sections 4 to 6 the colorimetric *standard-, adapted* and *relative* CIELAB data of these fields are given. Section 7 shows calculation examples. Section 8 to 10 includes basic descriptions of the CIELAB colour system, of the colours of equal blackness and the four visual elementary hues *RJGB* as well as their special importance for user friendly coordinates in information technology. Section 11 shows in hue triangles the *affine* output strategy of DIN 33872 and other possibilities for the reproduction of the 16 step colour series with different output devices. Section 12 adds the colorimetric specifications of many outputs in DIN 33872 by an additional specification for the illuminants D50 and D65 with *cmy* input data of the same printer.

1. Standard documents in the field of colour image reproduction

Table 1 – Standard series DIN 33866 and application field

Input	**Output**	**Input and output media and applications**			**Standard**
		Input media	**Output media**	**Application**	
–	–	–	–	**Basis**	**DIN 33866−1**
analog	**analog**	DIN-test chart (hardcopy)	Hardcopy	**Copier**	**DIN 33866−2**
analog	**digital**	DIN-test chart (hardcopy)	File	**Scanner**	**DIN 33866−4**
digital	**analog**	DIN-test chart (file)	Hardcopy Softcopy	**Printer** **Monitor**	**DIN 33866−3** **DIN 33866−5**

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Table 2 – International standards and technical reports which correspond to DIN 33866

Input	**Output**	**Input and output media and applications**			**Technical Report (TR) or Standard**
		Input media	**Output media**	**Application**	
–	–	–	–	**Basis**	**ISO/IEC TR 24705**
analog	**analog**	ISO/IEC-test chart (hardcopy)	Hardcopy	**Copier**	**ISO/IEC 15775**
analog	**digital**	ISO/IEC-test chart (hardcopy)	File	**Scanner**	**ISO/IEC TR 24705**
digital	**analog**	ISO/IEC-test chart (file)	Hardcopy Softcopy	**Printer** **Monitor**	**ISO/IEC TR 24705** **ISO/IEC TR 24705**

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The tables 1 and 2 show Standards and Technical Reports in which the equally spaced visual and colorimetric output is defined for equally spaced digital input data. The tables 1 and 2 show the relation between DIN 33866-1 to 5, ISO/IEC 15775, and ISO/IEC TR 24705.

DIN 33866 defines *PS* and *PDF* test files with digital *cmy0* input data for the printer output. However, increasingly *rgb* input data are used. The input data *cmy0* and *rgb* show increasingly different output as the software and hardware not any more uses the ”1-minus-relations“ between *cmy0* and *rgb* data. The ”1-minus-relations“, for example *c = 1 – r*, *m = 1 – g*, and*y= 1 – b,* are visually required and are defined linearly in *PostScript*. The outputs of these equivalent digital colour data show on some devices colorimetric differences of up to 30 CIELAB which may be compared with the difference of 75 CIELAB between black and white.

In ISO/IEC 15775 a colorimetric output tolerance of 3 CIELAB is defined. With many output devices a standard deviation of 1 CIELAB is reached. This corresponds to the visual discriminability. Output differences of up to 30 CIELAB for example can occur by the following sources: computer operating system, application software, device driver, printer software and illuminant used for the colour assessment.

The output of the test files according to DIN 33872-1 to -6 results in device system colours which are created by the summary of the above five sources. The output is visually assessed and/or specified by colorimetry. This test is of advantage for both the user and the device manufacturer. If the device system colours are changed by the output then often the possible source of the change can be detected.

The test may help that user can produce a long term storage and output of digital colour data. The user gets a help to choose according to his wishes the appropriate computer operating system, application software, device driver, device system and illumination. A linearised output is a basis for the security of discriminabillity of defined colour differences which are included in the *rgb* or *cmy* test files according to DIN 33872-2 to -6.

2. Colours in print and television

This section describes the colours which are used in print and television.

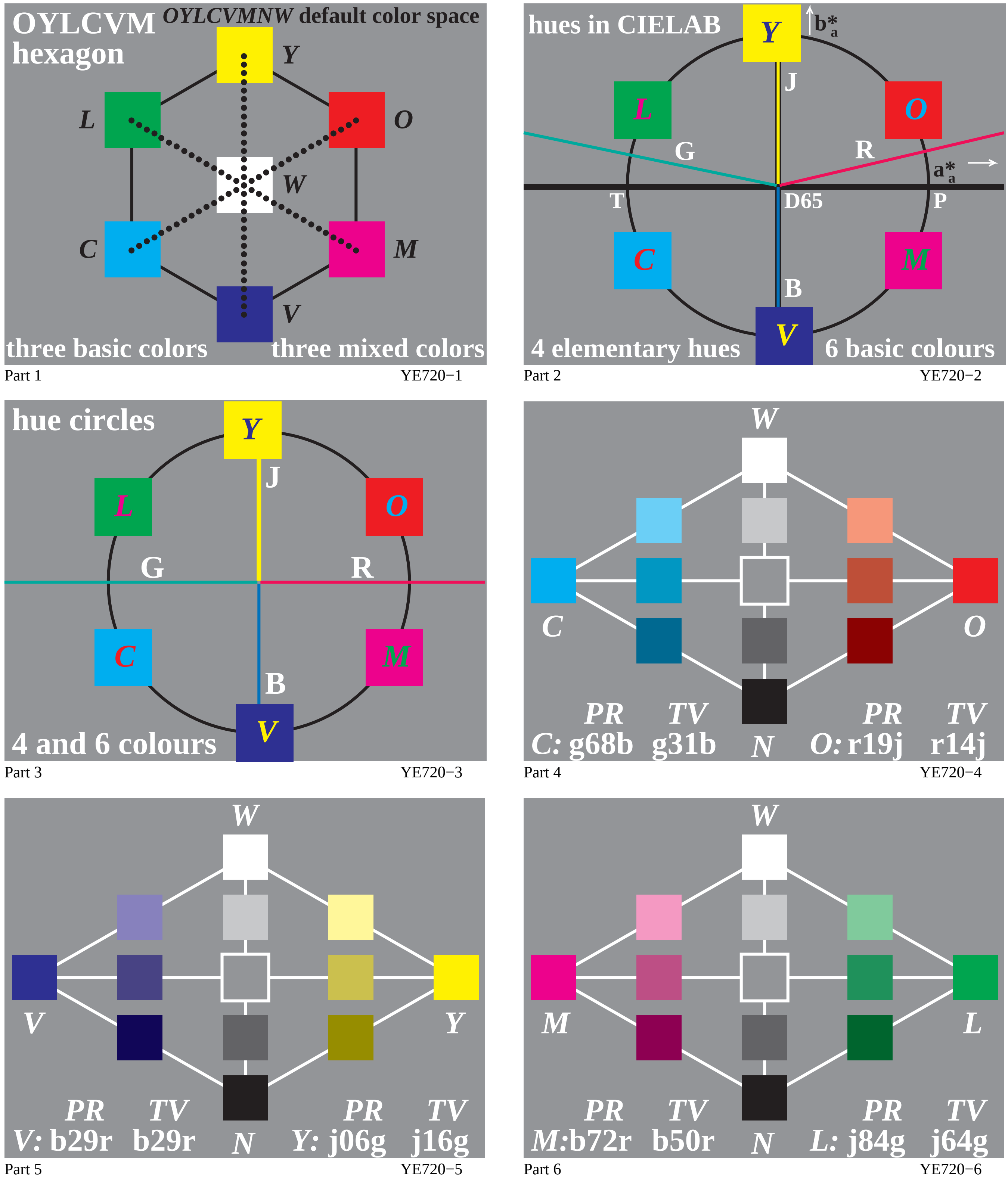

Figure 1 – Colours *OLV* and *CMY* of television (TV) and print (PR) and elementary colours *RJGB*

Figure 1 part 1 *(top left)* shows the six colours *OYLCVM* of television (TV) and print (PR) according to ISO/IEC 15775. The hue series of the six chromatic colours *X* = *OYLCVM* is either shown in a colour hexagon or in a hue circle. For a constant hue the colours are usually shown in a *colour triangle* with black N and white W on the vertical axis. The presentation in a colour triangle corresponds to the experience of using colorants. In the application often the colours are mixed proportional with black *N* and white *W*, or with chromatic *X* and white *W*, or with chromatic *X* and black *N*. The hue angles are shifted here between the six colours by 60 degree starting with 30 degree. The hue angles are assumed here to be equal for TV (Television) and PR (Print). The angles are in applications different depending on the monitor and print device. In figure 7 of section 6 the angles are shown for the standard processes CRT monitor (TV) and offset printing (PR).

The six chromatic device colours *X = OYLCVM* are *device dependent*. The visual human colour system uses four *device independent* elementary colours *RJGB* for the description of all hues. Many users wish a *device independent* hue output which limits the increasing device dependent colour output. These users expect that for the *rgb* input data with the values (1,0,0), (0,1,0,) and (0,0,1) the three elementary colours red *R*, green *G* and blue *B* are produced in the device output. The elementary colour yellow *J* shall be produced with the colour values (1,1,0). Therefore it is tested in DIN 33872 part 5 if a hue agreement with the elementary hue is reached.

In figure 1 *(top right)* the hue angle of the four elementary colours red *R*, yellow *J*, green *G,* and blue *B* is shown in the *adapted* CIELAB hue diagram (a^*_a, b^*_a) in comparison to the chromatic device colours *X = OYLCVM.* Often the elementary colours *R – G* and *J – B* are shown on the vertical and horizontal axis (see figure 1, part 3). The visual location is often given in percent, for example orange red *R* (86%) and Yellow *J* (14%) which may be specified by the abbreviation r14j, see figure 1, part 4. Figure 1 shows in part 3 to 6 equal hue cuts of the device colour pairs *O – C*, *Y – V ,* and *L – M* and includes the elementary hue text *u** for the standard offset printing process (PR) and the standard television monitor (TV).

The orange red colour *O* of the printing process is located according to figure 1, part 4, for PR at r19j and for TV near r14j. Orange red *O* of PR includes therefore 81% red *R* and 19% yellow *J*. Similar orange red *O* of TV includes 86% red *R* and 14% yellow *J*. Larger differences occur for cyan blue *C* with g68b for PR and g31b for TV. The difference in the adapted CIELAB hue diagram (a^*_a, b^*_a) corresponds to a hue angle shift of about 30 degrees. A smaller hue difference (20 degree) between PR and TV is given for the colour magenta red *M* (see Figure 1, part 6). The hue differences between PR and TV for the colours *C* and *M* are much larger compared to the visual standard deviation of the elementary hue discriminability of real hue circles of colour order systems which is about 4 degrees.

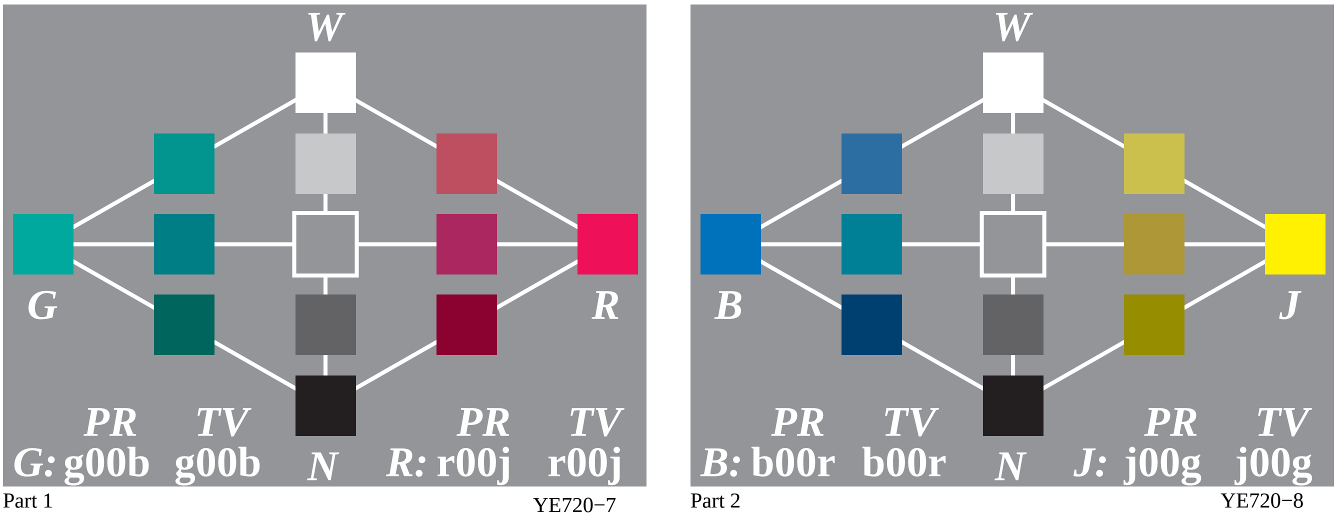

Figure 2 – Elementary colours red *R* and green *G* as well as yellow *J* and blue *B*

Figure 2 shows the hue planes of the elementary colours red R and green G as well as yellow J and blue B together with the achromatic axis *N – W.*

The elementary colours play a special role in the visual human system. Every observer can easily name the elementary hues and identify them **without** any comparison colour, for example elementary Yellow *J* as neither reddish nor greenish. The *Swedish Natural Colour System* NCS (compare the Swedish standards SS 01 91 00 to 03) uses the hue specification elementary hue text ***u**** (for example *r14j*) with the four elementary colours as well as the colour attributes relative blackness ***n**** and relative chroma ***c**** for the defined specifications of colours. The NCS colour order system includes about 1.500 colour samples. For their coordinates relative blackness ***n****, relative chroma ***c**** and elementary hue text ***u**** there are defined relations to the standard CIE system CIEXYZ in the Swedish Standards.

The most important and relevant colour attribute of the three colour attributes of NCS is the elementary hue. A symmetric hue circle with 20 steps and with the elementary hues on the horizontal and vertical axis is used in DIN 33872-5 for the test if there is elementary hue agreement in the output.

This property is of special importance for the users. During the last years more and more colour devices with different primary and secondary colours are produced and sold in the market. For CRT monitors the standard primary colour violet blue V is located at 305 degree instead of 272 for elementary blue in the adapted CIELAB hue diagram (a^*_a, b^*_a). This hue appears visually as strongly reddish blue. For newer LCD monitors the primary colour violet blue V is located more near the elementary blue. For new OLED monitors there may be a hue shift towards 240 degree which appear greenish blue. The hue angle difference between CRT and OLED monitors (305 and 240 degree) is 65 degree. This hue shift is similar as between elementary red and yellow (26 and 92 degree) and about 15 times larger compared to the standard deviation for the visual determination of the four elementary hues which is around 4 degrees.

3. User friendly colour image technology

Increasingly the colour image technology must consider the visual properties of colour vision. User friendliness is for example reached if the CIELAB hue angles of the elementary colours are considered. This is further reached by a simple and efficient coding with linear relationships between the colorimetric coordinates ***rgb**** and ***L*, a*, b**** or ***L*, C*$_{ab}$, h$_{ab}$***, of CIELAB.

Application of colour in daily life or in Colour Information Technology (IT)	
Design, architecture, art, industrial products **Measured for CIE standard illuminant D65**	**Colour Information Technology** **Measured for CIE illuminants D65 and D50**
colour order system; name and coordinates: ***RAL Design System (CIELAB)*** $L^*C^*_{ab}h_{ab}$, lightness, chroma, hue angle ***Munsell Colour System*** ***VCH***, lightness (Value), Chroma, Hue text ***Natural Colour System (NCS)*** ***ncu****: relative blackness, relative chroma relative elementary hue text	**Device system name and coordinates:** **Printer system (illuminants D50 or D65):** ***cmy***, content of "cyan", "magenta", "yellow" **Display system (standard illuminant D65):** ***rgb/sRGB***, content of "red", "green", "blue" *No user friendly colour coordinates* *Nearly no connection to colour order systems*
Aim: define user friendly connection **New: Interpretation of the *rgb* colour data in the range 0 to 1 as elementary colour data rgb^*_3** ***Linear*** relations between ***relative*** and ***absolute*** coordinates ***lab* − LAB**** $rgb^*_3 - L^*a^*b^*C^*_{ab}h_{ab}$ (CIELAB) $rgb - cmy$, $rgb^*_3 - cmy^*_3$ ("1-minus"-relation) $rgb^*_3 - nce^*$, $rgb^*_3 - ncu^*$ ***relative*** coordinates ***lab****: elementary redness r^*_3, greenness g^*_3, blueness b^*_3, blackness n^* chroma c^*, elementary hue e^*, elementary hue text u^*	

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Figure 3 – Colour order systems and colour image technology

Figure 3 shows the application of colour in daily life and in colour image technology. There is approximately no connection between the coordinates *rgb* and *cmy* of colour image technology and the colour coordinates of the colour systems *RAL, Munsell* and *NCS*. In the area for image technology there are many *rgb* definitions which are oriented on device systems and not on colorimetric colour order systems. Also the *rgb* data of the colour spaces *sRGB* and *AdobeRGB* are especially based on properties of television monitors and only to a small part on the visual and colorimetric colour systems. In figure 3 the interpretation of the ***rgb*** colour data of information technology as elementary colour data rgb^*_3 defines a *linear* and therefore especially simple and efficient connection.

The colorimetric *relative* coordinates ***ncu**** (*relative* blackness ***n****, *relative* chroma ***c**** and the elementary hue text ***u****) are for every hue text defined as ***linear*** function (F_{lin} or f_{lin}) of the *adapted* CIELAB coordinates LCH^*_a (= L^*, $C^*_{ab,a}$, $h_{ab,a}$; lightness, *adapted* chroma and hue angle) and the *relative* CIELAB coordinates ***lch**** (= ***l*, c*, h****; *relative* lightness, *relative* chroma and *relative* hue angle $h^* = h_{ab,a} / 360$)

$$ncu^* = F_{lin}(L^*, C^*_{ab}, h_{ab,a}) \qquad (1)$$

$$ncu^* = f_{lin}(l^*, c^*, h^*) \qquad (2)$$

If the application program allows the input of the appropriate and user friendly colour coordinates ***ncu**** then the colour image technology is connected in a new way to the colour coordinates used in design, art and architecture. For every elementary hue text ***u**** the coordinates ***rgb*** of colour image technology are interpreted as elementary colour coordinates rgb^*_3 which are connected by the simple and linear relations

$\boldsymbol{n^*} = 1 - max\ (r^*_3, g^*_3, b^*_3)$ (3)

$\boldsymbol{c^*} = max\ (r^*_3, g^*_3, b^*_3) - min\ (r^*_3, g^*_3, b^*_3)$ (4)

with the new user friendly coordinates ***ncu****. Similar coordinates are used in the Natural Colour System *NCS* of the Swedish Standards SS 01 91 00 to 03. In agreement with *NCS* the relative blackness ***n**** and the relative chroma ***c**** have a colorimetric definition. These colour attributes are defined in the colour system *NCS device independent* by visual experiments.

In the following section 4 the *rgb* of colour image technology are either interpreted as device system data olv^*_3 or as elementary data rgb^*_3. In the following figure 5 part 1 to 4 of section 4 the two above equations are explained and show visually the calculation of *relative* blackness ***n**** and *relative* chroma ***c****.

4. Colorimetric *Standard, adapted* and *relative* CIELAB data

In section 3.1 of DIN 33872-1 many coordinates are given which are all related by *linear* relations for a given output device. These relations are explained in the following by some figures. As example especially for the elementary colour red *R* the $lab^*rgb^*_3$ coordinates are shown and the relation to the *relative* colour attributes whiteness ***w****, blackness ***n****, chroma ***c****, colour deepness ***d**** and brilliantness ***i**** is given.

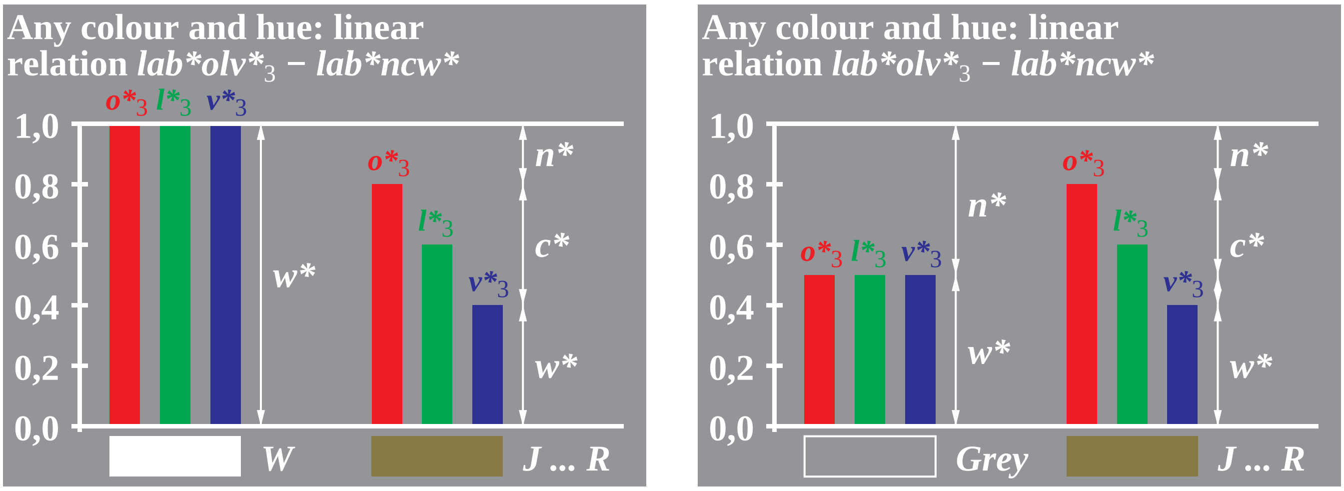


Figure 4 – Relative device coordinates olv^*_3 and the colour mixture of white *W*, grey *Z* and yellow red *J...R*.

Figure 4 shows the *relative* device coordinates olv^*_3 and the colour mixture of white *W*, central grey *Z* and yellow red *J...R* from these device coordinates olv^*_3. In this paper it is distinguished between *absolute* CIELAB coordinates *LAB** (three capital letters) and *relative* CIELAB coordinates *lab** (three small letters). The coordinates olv^*_3 are *relative* coordinates which are described by the addition $lab^*olv^*_3$. The value range is between 0 and 1. The index 3 limits the set of the coordinates to 3 values which may be compared for example with the 4 values of $cmyn^*_4$.

The star (*) defines a *linear* relation between $olv^*_3 = lab^*olv^*_3$ and $LCH^*_a = LAB^*_aLCH^*_a$ of CIELAB. The star therefore defines colorimetric coordinates in a similar way compared to the *rgb* coordinates of the colour space *sRGB* which have a defined relation to CIELAB (see IEC 61966-7-1:2006 for *RGB* colour printers).

The coordinates ***rgb*** of colour image technology are usually interpreted as device coordinates $\boldsymbol{olv^*_3}$ which are connected by the simple and linear relations

$\boldsymbol{n^*} = 1 - max\ (o^*_3, l^*_3, v^*_3)$ (1)

$\boldsymbol{c^*} = max\ (o^*_3, l^*_3, v^*_3) - min\ (o^*_3, l^*_3, v^*_3)$ (2)

with the user friendly coordinates *relative* blackness ***n**** and *relative* chroma ***c*.*** This relation is shown in figure 4 by arrows. In the case of equality of the three coordinates $o^*_3 = l^*_3 = v^*_3$ or *r=g=b* then in colour image technology the maximum and minimum is identical. Then the *relative* chroma ***c**** is cero. The *relative* blackness ***n**** decreases from the value 1 at black *N* to the value 0 at white *W*. The user expects after this that the output for all device systems in the special case of *r=g=b* (0<= *r, g, b* <= 1) leads to the chroma *c** = 0 and appears therefore visually achromatic.

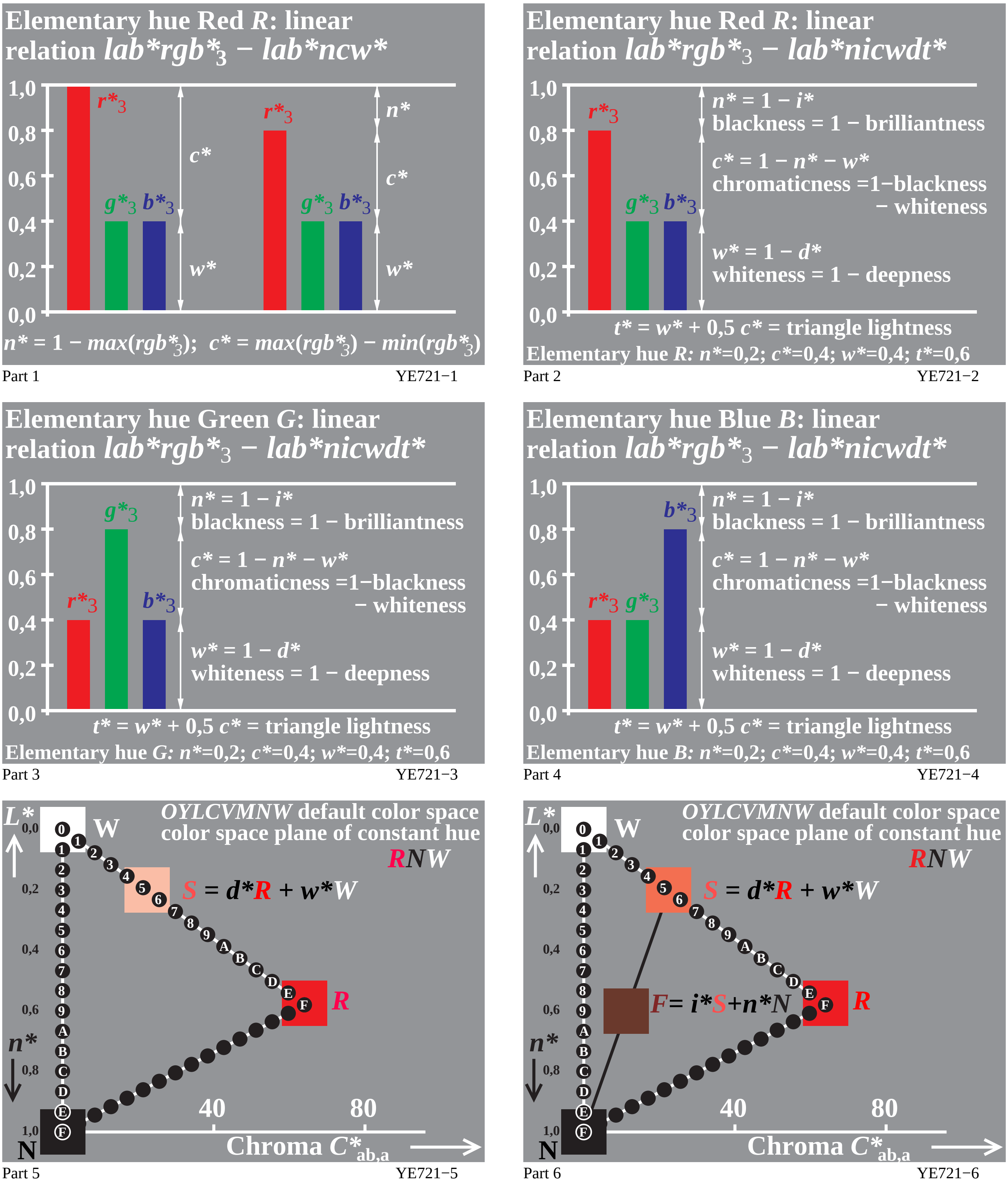


Figure 5 – Relations of the coordinates *rgb$_3$ and coordinates *w*, d*, n*, i** and of the *relative* chroma *c***

Many software products of image technology require up to now that the user adapts to special device properties. Many software products use the special property of standard offset printing that for example the standard colorants *C, M,* and *Y* produce a mean grey with a *CMY* mixture in the relation 30 : 30 : 45. This property is used by many *Windows* and *Mac* versions of *Adobe Photoshop*. The user must input the data *cmy* = (0,3 0,3, 0,45) to produce a mean grey in the output on monitors and printers instead of *cmy* = (0,5, 0,5, 0,5). The user therefore must learn a special property of a special device process. Increasingly the users don’t like to learn the many device properties and appreciate versions of *Adobe Photoshop* which for example produce an achromatic mean grey for the values *cmy* = *rgb* = (0,5, 0,5, 0,5) both for the display and printer output with some *Unix* and *Display PostScript* versions.

More and more the user wishes software with this natural and user friendly property.

Figure 5 parts 1 to 4 shows the relations of the elementary colour coordinates rgb^*_3 and the coordinates *relative* whiteness ***w****, *relative* colour deepness ***d****, *relative* blackness ***n****, *relative* brilliantness ***i**** and *relative* chroma ***c****.

In Figure 5 parts 1 to 4 he largest of the three coordinates r^*_3, g^*_3, b^*_3 determines the hue. In the case of part 1 to 4 the other two coordinates are equal and therefore only the hues of the elementary colours appear, for example red *R* (figure 5, parts 1 and 2) and green *G* and blue *B* (figure 5, parts 3 and 4).

Figure 5 parts 5 and 6 show colour triangles of the elementary colour red R which are defined by the colorimetric *adapted* CIELAB coordinates ($C^*_{ab,a}$, L^*). Both figure parts show always 16 steps between black *N* and white *W* and between black *N* and chromatic *X,* and between chromatic *X* and white W.

In figure 5 part 5 the colour *S* is mixed by the colour red *R* and white *W*. The mixture ratios are defined by the *relative* deepness ***d**** and the *relative* whiteness ***w**** with a value range between 0 and 1.

In figure 5 part 6 the colour *F* is additionally mixed by the colour *S* and black *N*. The mixture ratios are defined by the *relative* brilliantness ***i**** and the *relative* blackness ***n**** with a value range between 0 and 1.

Instead of the three coordinates r^*_3, g^*_3, b^*_3 a user may for example use the *relative* blackness ***n*,*** the *relative* chroma ***c*,*** and the elementary hue *R*. For example a user may specify the user friendly colour attributes *relative* blackness ***n*,*** *relative* chroma ***c*,*** and the elementary hue text ***u**** to create a CAD drawing or a colour design.

The coordinates rgb^*_3 have for example **four important advantages** compared to the *rgb c*oordinates of the colour space *sRGB*, see IEC 61966-2-1:

1. The coordinates rgb^*_3 are **device independent** and correspond to the visual system;
2. 16 step **equally spaced digital input data** produce visually **equidistant colour series** in CIELAB;
3. The coordinates rgb^*_3 have simple *linear* relations to visual colour attributes ***w*, d*, n*, i*,*** and ***c*;***
4. The coordinates rgb^*_3 have a simple *linear* relation to the elementary hue text ***u*.***

In figure 5 parts 5 and 6 the linear relation of these visual colour attributes to the *adapted* CIELAB data $L^*_a = L^*$ and $C^*_{ab,a}$ is shown. In the CIELAB colour space the colour *S* is a *linear* mixture between elementary red *R* and white *W* and the colour *F* is a *linear* mixture between the colour *S* and black *N*.

In figure 4 part 1 the colour white *W* has the three coordinates $olv^*_3 = rgb^*_3 = (1,1,1)$ and in figure 4 part 2 mean grey *Z* has the three coordinates $olv^*_3 = rgb^*_3 = (0{,}5,\ 0{,}5,\ 0{,}5)$. For example a colour *F* may therefore may be calculated from the *relative* whiteness ***w*,*** and the *relative* blackness ***n**** and the *relative* colour deepness ***d* = 1 – w**** and the *relative* brilliantness ***i* = 1 – n****. The relations of the coordinates rgb^*_3 and the different *relative* coordinates ***w*, d*, n*, i*,*** and the *relative* chroma ***c**** are linear and are shown in figure 5 parts 1 to 4.

In the standard series DIN 33872 the *rgb* input data may be interpreted as rgb^*_3 elementary colour data. In this case in the output the hues of the four elementary colours are produced. This interpretation is shown with an arrow in the following way ($rgb \rightarrow rgb^*_3$).

Another interpretation is shown by ($rgb \rightarrow olv^*_3$). In this case the *rgb* input data are interpreted as olv^*_3 device data and in this case the six chromatic device colours are produced.

In the case of the input data for blue *rgb* = (0, 0, 1) then in figure 9 in section 7 the maximum colour shows the hue angle 306 degree for the system TLS00 or the hue angle 305 degree for the system ORS18. These outputs lead in both cases to a reddish blue. However, the user appreciates a user friendly and device independent hue output of elementary blue with the hue angel 272 degree.

DIN 33872-2 to -6 tests by YES/NO criteria if the output properties according to user wishes are fulfilled or not. A quality assessment of the device system is not intended. For one user the output property to produce the elementary hue may be important and for another users unimportant.

If a user requires output properties which are defined in DIN 33872 parts 2 to 6 then there are many device systems which fulfill these properties or which may fulfill these output properties by appropriate default values of the device system.

In many cases a user wishes alternate possibilities for the output, for example either the output property “device system colour output” or “elementary colour output” by appropriate default parameters (see ORS18, TLS00, NRS18 and SRS18 in figure 9 of section 7).

In the field „Remarks to the test results“ descriptions of these alternate output possibilities are appreciated (see DIN 33872-2 to 6).

5. *Linear* colorimetry in hue triangle and *relative* lightness *l**

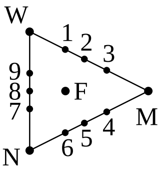
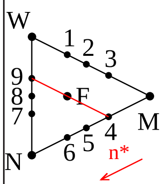
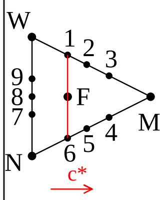
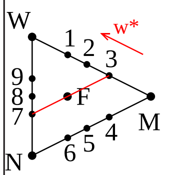
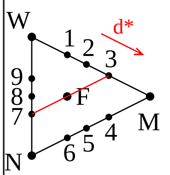
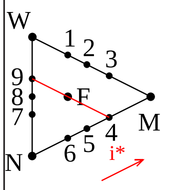
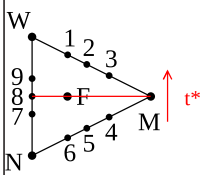
Colour F and 9 others	Colorimetric coordinates in colour triangle of CIELAB hue h_{ab} Formula use given data of *relative* chroma *c** and blackness *n**					
(c*,n*)=(0.3, 0.35) 	*blackness n** n* = 0.35 	*chromatic-ness c** c* = 0.30 	*whiteness w* = 1 − n* − c** w* = 0.35 	*deepness d* = 1 − w* = n* + c** d* = 0.65 	*brilliantness i* = 1 − n** i* = 0.70 	*triangle lightness t* = 1 − n* − 0.5 c** t* = 0.50 
Colour 1 *Colour 2=S* *Colour 3*	0 0 0	c* c*/(1−n*) n*+c*	1−c* 1−c*/(1−n*) 1−n*−c*	c* c*/(1−n*) n*+c*	1 1 1	1−0.5c* 1−0.5c*/(1−n*) 1−0.5(n*+c*)
Colour 4 *Colour 5=Q* *Colour 6*	n* n*/(n*+c*) 1−c*	1−n* c*/(n*+c*) c*	0 0 0	1 1 1	1−n* c*/(n*+c*) c*	0.5(1−n*) 0.5c*/(n*+c*) 0.5c*
Colour 7 *Colour 8* *Colour 9*	1−n* 1−n*−0.5c* 1−n*−c*	0 0 0	n* n*+0.5c* n*+c*	1−n* 1−n*−0.5c* 1−n*−c*	n* n*+0.5c* n*+c*	n* n*+0.5c* n*+c*

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Figure 6 – Colorimetric coordinates for given *relative* chroma *c and *relative* blackness *n****

Figure 6 shows the relation of colorimetric coordinates of a colour ***F*** if *relative* chroma ***c**** and *relative* blackness ***n**** are given. The ***c**** and ***n**** data are transferred to the *relative* whiteness ***w****, the *relative* colour deepness ***d****, the *relative* brilliantness ***i**** and the *relative* triangle lightness ***t**** in the first line of the figure. This line defines additionally 6 colours produced by red parallels compared to the three triangle lines trough F and three further colours by the connections 2–F–N, 5–F–W and 8–F–M. For these 9 colours the coordinates are calculated from the given coordinates ***c**** and ***n****.

In any application two equivalent of the 6 colour attributes can be used instead, for example the *relative* triangle lightness ***t****, and the *relative* chroma ***c****. Together with the hue angle ***h**** in this case the cylindric coordinates ***tch**** instead of triangle coordinates ***nch**** are calculated. The following page shows colorimetric coordinates of a colour ***F*** for given *relative* chroma ***c**** and *relative* triangle lightness ***t****.

http://www.ps.bam.de/YE76/10L/L76E00NP.PDF

In applications it is realized that the colour coordinates *relative* chroma ***c**** and *relative* blackness ***n**** can be evaluated or guessed visually more easily compared to the others. Therefore these two have been chosen in the Swedish *Natural Colour System* NCS, see for example *Hard* and *Sivik* (1980). The calculations from the *adapted* CIELAB data LCH^*_a require the lightness and chroma of the maximum colour *M* of the same hue compared to the given colour *F*. The maximum colour *M* has the chroma $\boldsymbol{C^*}_{ab,a,M}$ and the lightness $\boldsymbol{L^*}_M$. The colour *F* has the chroma $\boldsymbol{C^*}_{ab,a}$ and the lightness $\boldsymbol{L^*}$. Therefore the following equations are valid:

$$\boldsymbol{c^*} = \boldsymbol{C^*}_{ab,a} / \boldsymbol{C^*}_{ab,a,M} \tag{1}$$

$$\boldsymbol{l^*} = [\, \boldsymbol{L^*} - \boldsymbol{L^*}_N] \, / \, [\, \boldsymbol{L^*}_W - \boldsymbol{L^*}_N \,] \tag{2}$$

$$\boldsymbol{t^*} = \boldsymbol{l^*} - \boldsymbol{c^*} \{ [\, \boldsymbol{L^*}_M - \boldsymbol{L^*}_N \,] \, / \, [\, \boldsymbol{L^*}_W - \boldsymbol{L^*}_N \,] - 0.5 \} \tag{3}$$

$$\boldsymbol{n^*} = 1 - \boldsymbol{t^*} - 0.5\, \boldsymbol{c^*} \tag{4}$$

With these equation the *relative* chroma ***c****, the *relative* lightness ***l****, the *relative* triangle lightness ***t**** and the *relative* blackness ***n**** are calculated from the *adapted* CIELAB data of a colour *F* and their maximum colour *M* of the same hue. *Linear* equations serve for the determination of the CIELAB data of the maximum colour *M* from the *adapted* CIELAB data of a colour *F*. The CIELAB data of *M* depend on the CIELAB hue angle $h_{ab,a}$ of *F* and the *adapted* CIELAB data of the six chromatic device colours *X=OYLCVM*.

6. Colorimetric *adapted* CIELAB data in a hue hexagon

The colorimetric data of television, printing and of elementary colours in an *adapted* and *relative* CIELAB chroma diagram are of special importance for the output linearisation on an output device.

The six chromatic colours *X=OYLCVM* and their linear mixtures create the maximum colours *M.* For each *adapted* CIELAB hue angle $h_{ab,a}$ there is a maximum colour *M* with defined CIELAB data L^*, $C^*_{ab,a}$, and $h^*_{ab,a}$.The linearisation, for example according to ISO/IEC TR 19797, produces 16 step equidistant colour series in the output.

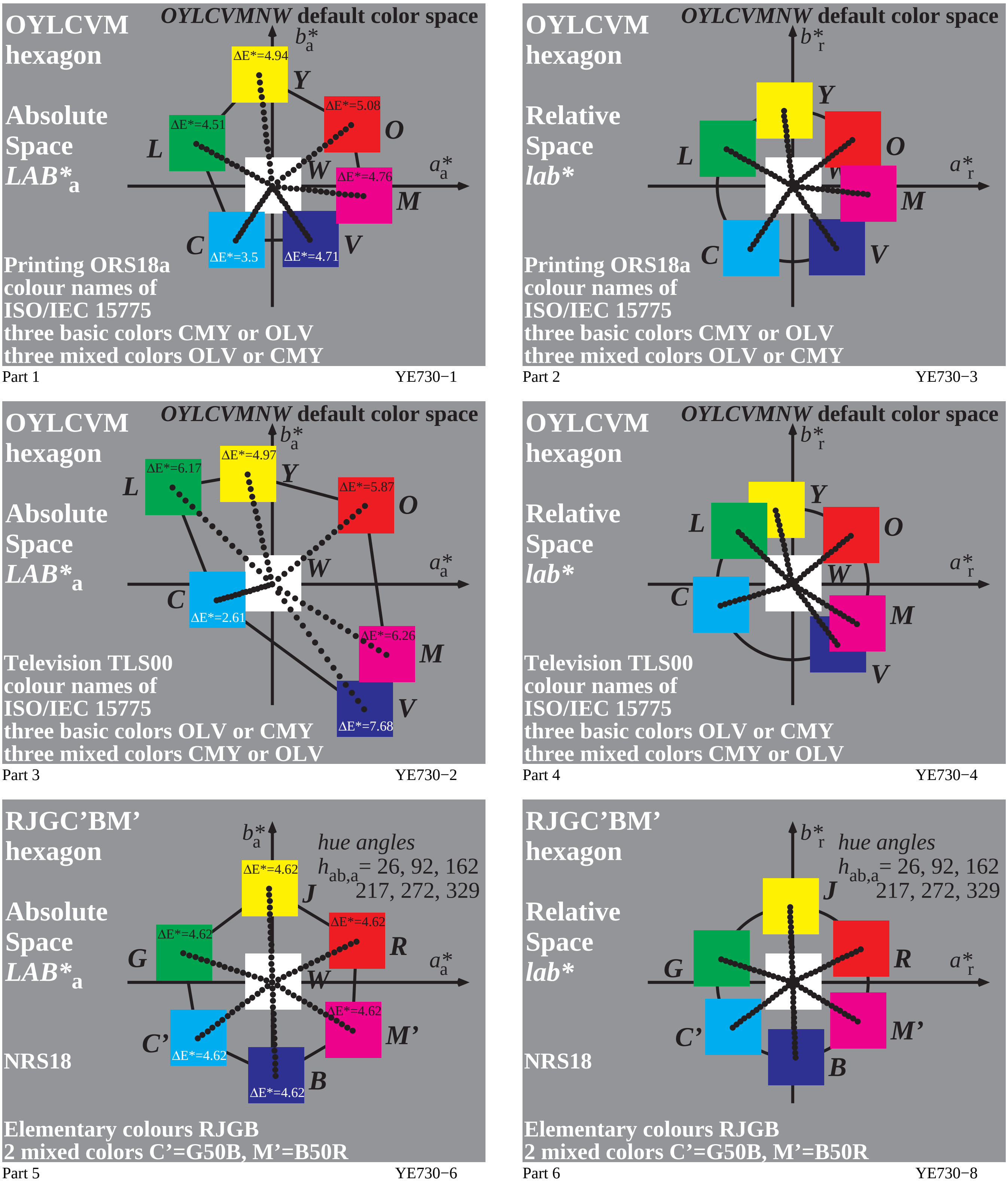

Figure 7 – Television and printing colours in the *adapted* and *relative* CIELAB chroma diagram

Figure 7 parts 1, 3, and 5 *(all left)* show the standard offset printing colours ORS18 (Offset Reflective System with lightness L^*=18 for black *N*), the standard television colours TLS00 (Television Luminous System with lightness L^* = 0 for black *N)*, and a colour elementary system NRS18 (Natural Reflective System with lightness L^* =18 for black *N*) in the colorimetric *adapted* CIELAB chroma diagram (a^*_a, b^*_a). For the chroma components (a^*_a, b^*_a) it is valid $a^*_a = LAB^*_a a^*_a$ and $b^*_a = LAB_a{}^*b^*_a$.

Figure 7 parts 2, 4, and 6 *(all right)* shows the same colours in the colorimetric *relative* CIELAB chroma diagram (a^*_r, b^*_r). For the relative chroma components (a^*_r, b^*_r) it is valid $a^*_r = lab^*a^*_r$ and $b^*_r = lab^*b^*_r$.

The hue angle difference between cyan blue C and violet blue V is for the system TLS00 approximately 110 (= 306 – 196) degree and for the system OLS18 approximately 68 (= 304 – 236) degree. The systems TLS00 and OLS18 produce hue angle differences of 183% and 113% compared to the mean hue angle difference of 60 degree (100%) for the six chromatic device colours *OYLCVM.* According to this view the basic colours of the system ORS18 are much more regular compared to the basic colours of TSL00 which are used today mostly for the coding and the transmittance of the colour information, for example in the colour space *sRGB* (see IEC 61966-2-1).

The colorimetric *adapted* CIELAB hue angles in the system NRS18 are still more regular compared to TLS00 and ORS18. The system NRS18 is additionally **device independent** and is based on the elementary colours *RJGB* which include many properties of the visual human colour vision system. The colorimetric *adapted* CIELAB hue angle are located for the four elementary colours *RJGB* in a rough approximation near 30, 90, 150, and 270 degree. Between red *R,* yellow *J*, and green *G* the shift is in both cases 60 degree and between green *G*, blue *B,* and red *R* the shift is in both cases 120 degree. The exact location in the *relative* CIELAB chroma diagram (a^*_r, b^*_r) is also included in figure 7, and in figure 8 of section 7. With the definition of the two "mean hues" cyan blue C_{gb}=G50B, and magenta red M_{br}=B50R which are located in both cases at the mean between *G – B,* and *B – R* there is created again a hexagon. If one looks at the six colours $X=RJGC_{gb}BM_{br}$ which include the four elementary colours *RJGB* and the intermediate hues $C_{gb}M_{br}$ then the hue angle difference is approximately regular and is about 60 degree between any two neighboring hues.

The colorimetric connection between the *rgb* coordinates of the information technology with 16 step colour series and the $L^*a^*b^*$ coordinates of CIELAB is studied in many papers. Some of these papers study the connection on a colorimetric and visual basis and therefore some are listed in the following.

A multispectral CIELAB camera which can measure the output colours of the information technology and also retro-reflective and fluorescent colours has been described by *Stephan Jaeger* (2005, 2006).

The CIELAB data for the CIE standard illuminant D65, and the CIE illuminant D50 of output colours of the information technology has been used by *Hans Wagenknecht* (2005, 2006) for the calculation of *rgb* data and their special transformations to calibrate scanners and printers.

New methods for the production of colour test charts according to DIN 33866, ISO/IEC 15775, and ISO/IEC TR 24705 with colorimetric equidistant 16 step output colour series has been developed by *Jens Witt* (2005, 2006).

All papers serve for the purpose to connect the coordinates of colour image technology in an improved way with the colour coordinates used in every day life, compare also figure 3 in section 3.

7. Colorimetric calculation examples for four device systems

The six colours $X=OYLCVM$ or $X=RJGC_{gb}BM_{br}$ of the different device systems of section 6 have different hue angles. Four device systems will be viewed in the following: ORS18, TLS18, NRS18, and SRS18. The first two are real standard device systems, and the two others are ideal (artificial) device systems. NRS18 is based on the hue angles of the elementary colours in CIELAB, and SRS18 uses the hue angles of the six device colours with 60 degree difference of any two neighboring hues.

The lightness difference $\Delta L^* = L^*_W - L^*_N$ (= 95 –18 = 77) between white and black of ORS18 is also chosen for NRS18 and SRS18. The chroma C^*_{ab} of the six (ideal) basic colours $X=OYLCVM$ has also the value 77. Because of this symmetric structure of the colour space and its LAB^* and RGB^* colour coordinates one may expect equal colour discrimination for the complementary colours of the monitor, for example *O* and *C*. This property has been verified for example by *Holtsmark* and *Valberg* (1971) by experimental measurements with complementary optimal colours. The CIELAB colour metric is in parts in opposition to this experimental results. However, a new hexagon metric which has to be developed further includes a structure which may directly describe the experimental results, see also *Richter* (2007).

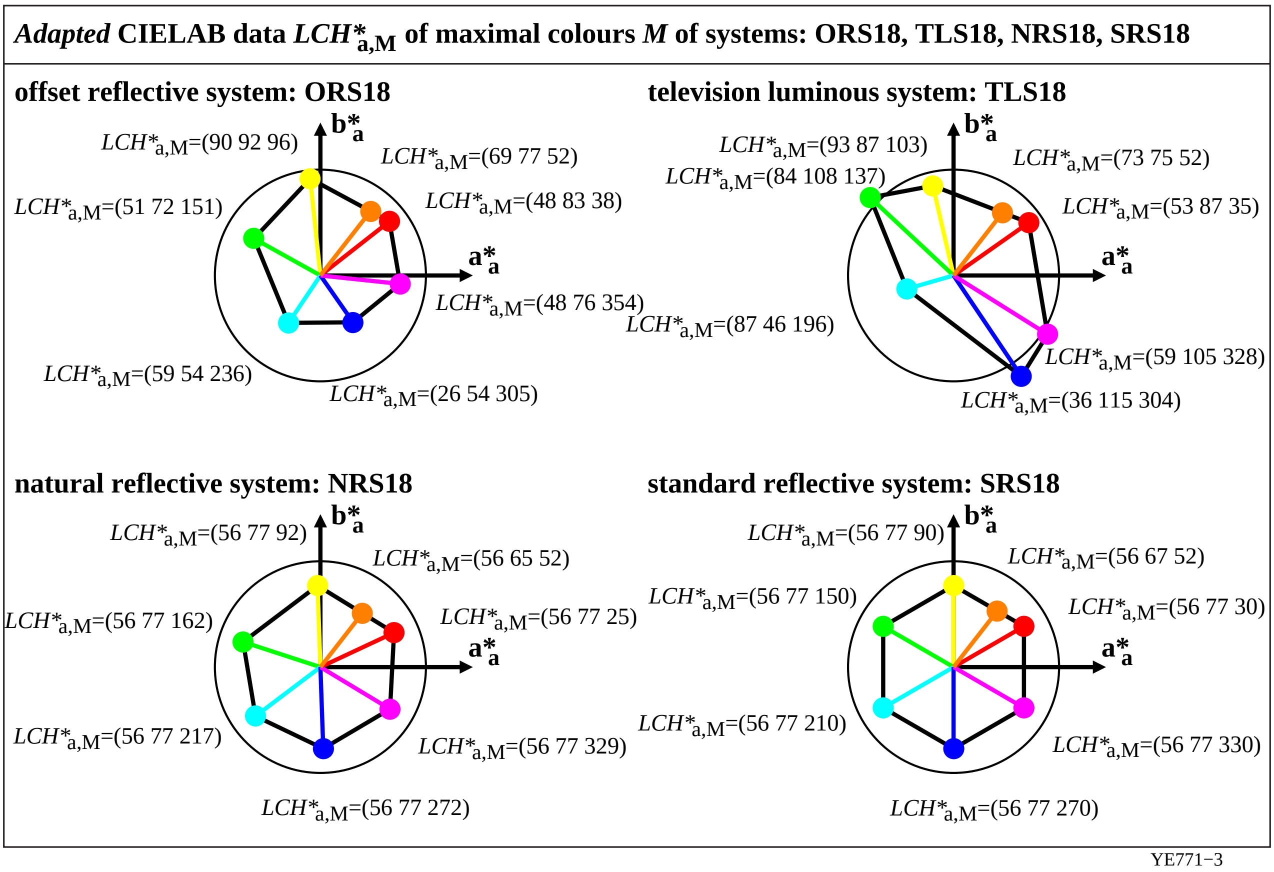

Figure 8 – *Adapted* CIELAB data LAB^*_{a} of four device systems ORS18, TLS18, NRS18, and SRS18

Figure 8 shows the *adapted* CIELAB data L^*_{M}, $C^*_{ab,a,M}$, and $H^*_{a,M}$ of six basic colours X=*OYLCVM*. The four device systems have different lightness, chroma, and hue angle. For ORS18 and TLS18 the *adapted* CIELAB data L^*_{M} and $C^*_{ab,a,M}$ are very different and for the (theoretical) systems SRS18 and NRS18 they are equal. The hue angles are for TLS18 very unequal and for SRS18 exact regular and shifted by 60 degree between neighboring hues. The systems SRS18 and NRS18 have the same lightness and chroma difference with the value 77. The system NRS18 has the three visual elementary hue angles 25,162, and 272 degree, and SRS18 has the hue angles 30, 150, and 270 degree with a regular difference of 120 degree. Elementary yellow *J* is located at the hue angle 93 [= (162+25)/2] degree or at 90 [=(30+150)/2] degree at the intermediate point between the hue angle of red *R* and green *G*. Similar the hue angle of cyan blue C_{gb} is located at the intermediate point between green *G* and blue *B,* and magenta red M_{br} is located at the intermediate point between blue *B* and red *R*.

Figure 8 shows the maximum colour *M* with the CIELAB hue angle h_{ab} = 52 degree for the four device systems ORS18, TLS18, NRS18, and SRS18. For the calculations the following equations are valid:

$$b^*_{a} = a^*_{a} \tan(h_{ab,a}) \qquad (1)$$

$$b^*_{a} = b^*_{a,i0} + m\,[a^*_{a} - a^*_{a,i0}] \qquad (2)$$

with

$$m = [b^*_{a,i0+1} - b^*_{a,i0}] / [a^*_{a,i0+1} - a^*_{a,i0}] \qquad (3) \qquad (i0 = 0, 1, .., 5 \text{ for } OYLCVM)$$

For example in figure 8 the equation (1) with the *adapted* CIELAB hue angle $h_{ab,a}$ describes the *(orange)* line trough the origin with the angle $h_{ab,a}$ = 52 degree. The second equation describes a line trough the points of the device colours *O* and *Y*. The cut point between both lines (ball of the colour orange in figure 8) leads to the chroma components ($a^*_{a,M}$, $b^*_{a,M}$) of the maximum colour *M* which are calculated from the *adapted* CIELAB chroma of the colours *O* and *Y*. It is unknown how the visual system determines these calculations.

Probably the visual system determines the data with the *relative* chroma and lightness of *O* and *Y*. The calculations determine first the *relative* CIELAB hue angle ratio α_{M} between the two neighboring maximum colours *O* and *Y*. The chroma $a^*_{a,M}$ and $b^*_{a,M}$, and the lightness L^*_{M} of the mixed maximum colour *M* are all determined by the *relative* hue angle ratio α_{M} of *O* and *Y*.

It is valid:

$$\alpha_M = [\, h_{ab,a,M} - h_{ab,a,O} \,] \,/\, [\, h_{ab,a,Y} - h_{ab,a,O} \,] \qquad (4)$$

$$a^*_{a,M} = \alpha_M \, a^*_{a,Y} + (\, 1 - \alpha_M \,) \, a^*_{a,O} \qquad (5)$$

$$b^*_{a,M} = \alpha_M \, b^*_{a,Y} + (\, 1 - \alpha_M \,) \, b^*_{a,O} \qquad (6)$$

$$L^*_M = \alpha_M \, L^*_Y + (\, 1 - \alpha_M \,) \, L^*_O \qquad (7)$$

$$C^*_{ab,a,M} = [\, a^{*2}_{a,M} + b^{*2}_{a,M} \,]^{1/2} \qquad (8)$$

One may consider that the *adapted* CIELAB chroma $C^*_{ab,a,M}$ has to be calculated from the components $a^*_{a,M}$ and $b^*_{a,M}$ of the maximum colour *M*. In figure 8 and for ORS18 for example the value (=77) of the *adapted* CIELAB chroma $C^*_{ab,a,M}$ is less then both the chroma of *O* and *Y* (value = 83 and value = 92) and can not be calculated direct from α_M.

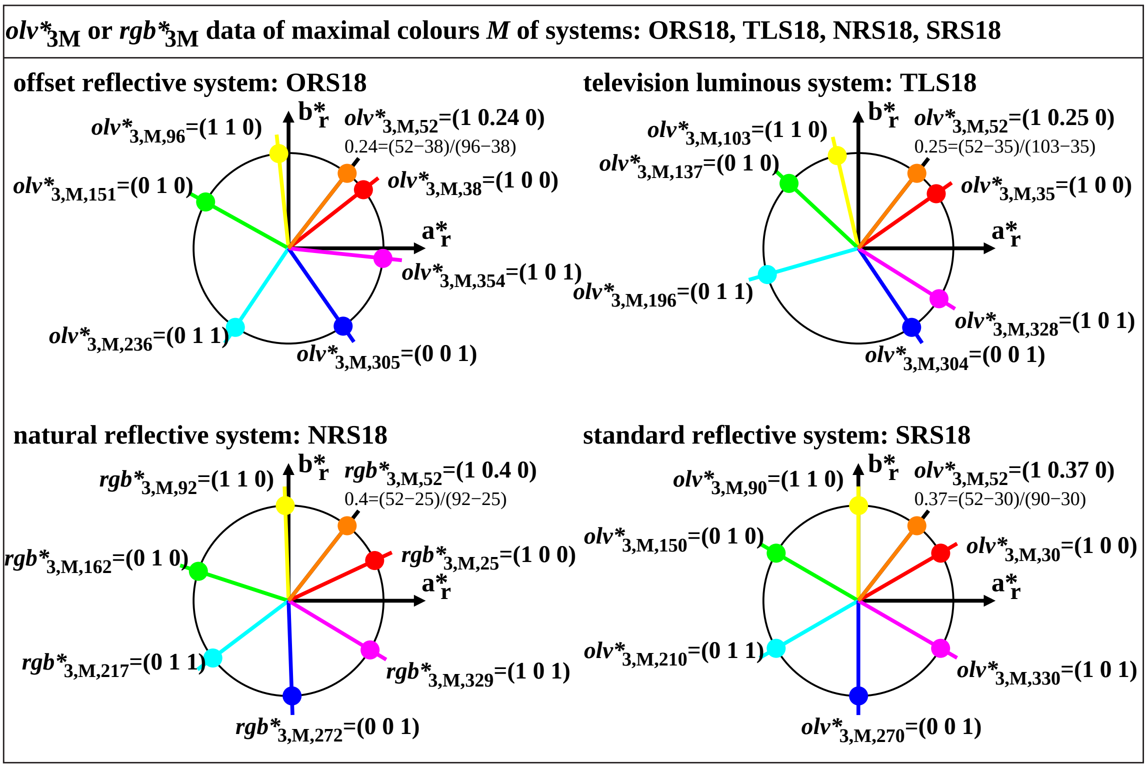

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Figure 9 – *Relative* CIELAB data $lab^*olv^*_{3M}$ of four device systems ORS18, TLS18, NRS18, and SRS18

Figure 9 shows the *relative* CIELAB data olv^*_{3M} and rgb^*_{3M} of six device colours *X=OYLCVWM,* and of a maximum colour *M.* In the sector *O–Y* the second coordinate of olv^*_{3M} changes *linear* with the *relative* hue angle α_M between the values 0 and 1.

The calculation of the *relative* hue angle α_M is given in each of the four parts for the same CIELAB hue angle $h_{ab,a,M}$ = 52 degree. In figure 9 by use of equation (4) and the relative hue angle α_M the following three equations for the *relative* CIELAB data $lab^*olv^*_{3,M}$ are valid for the maximum colours between *O* and *Y.*

$$o^*_{3,M} = \alpha_M \, o^*_{3,Y} + (\, 1 - \alpha_M \,) \, o^*_{3,O} \qquad (9)$$

$$l^*_{3,M} = \alpha_M \, l^*_{3,Y} + (\, 1 - \alpha_M \,) \, l^*_{3,O} \qquad (10)$$

$$v^*_{3,M} = \alpha_M \, v^*_{3,Y} + (\, 1 - \alpha_M \,) \, v^*_{3,O} \qquad (11)$$

In the special case of the Natural Reflective system NRS18 these CIELAB data are called $lab^*rgb^*_3$ instead of $lab^*olv^*_3$, see figure 9 *(bottom left)*. The letters *rgb** include the definition according to three visual elementary colours *RGB* which are independent of the many definitions of device colours *OLV.*

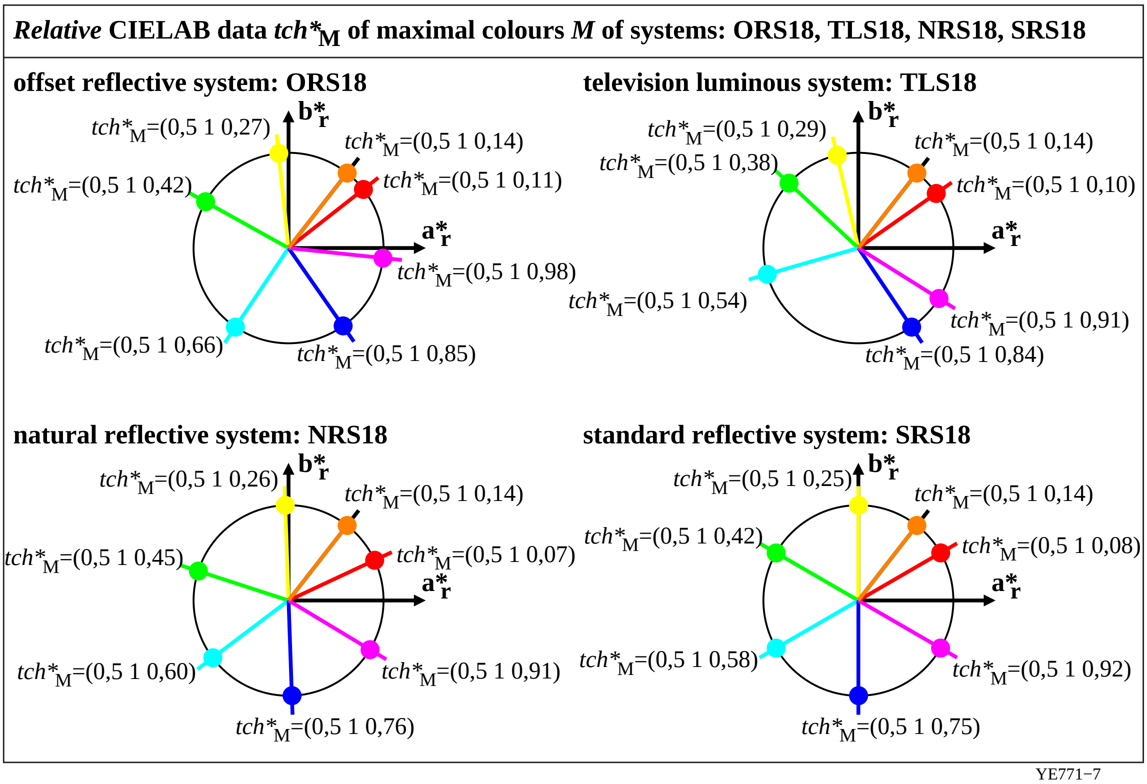


Figure 10 – *Relative* CIELAB data *lab*tch of four device systems ORS18, TLS18, NRS18, and SRS18**

Figure 10 shows the *relative* CIELAB data $\boldsymbol{t^*}_M$ and $\boldsymbol{c^*}_M$ of six basic colours *X=OYLCVWM*, and a maximum colour *M*. For all six basic colours and the maximum colour *M* the values for the *relative* triangle lightness $\boldsymbol{t^*}$ = 0,5, and the *relative* chroma $\boldsymbol{c^*}$ = 1 are constant. Colours with the *relative* chroma $\boldsymbol{c^*}$ = *1* always have the property that at least one of the three components of $lab^*olv^*_{3M}$ has the value 1, and at least one other has the value 0. This is fulfilled in figure 10 for all six device colours, and the mixture colour of the hue angle 52 degree. Then for the *relative* blackness $\boldsymbol{n^*}$ and the *relative* chroma $\boldsymbol{c^*}$ it is valid.

$$\boldsymbol{n^*} = 1 - max\,(o^*_3, l^*_3, v^*_3) \qquad (12)$$

$$\boldsymbol{c^*} = max\,(o^*_3, l^*_3, v^*_3) - min\,(o^*_3, l^*_3, v^*_3) \qquad (13)$$

$$\boldsymbol{t^*} = 1 - \boldsymbol{n^*} - 0{,}5\,\boldsymbol{c^*} \qquad (14)$$

$$\boldsymbol{w^*} = 1 - \boldsymbol{n^*} - \boldsymbol{c^*} \qquad (15)$$

The *relative* triangle lightness $\boldsymbol{t^*}$ and the *relative* whiteness $\boldsymbol{w^*}$ are calculated according to equations (14) and (15) from the *relative* blackness $\boldsymbol{n^*}$ and the *relative* chroma $\boldsymbol{c^*}$.

Up to now the case was studied that the hue angle $h_{ab,a,M}$ of a maximum colour *M* was given. In general all other colour coordinates are needed of the four device systems, if **any** set of three coordinates of **any** colour F is given. We will study in the following the case that one of the data sets $LAB^*LCH^*_a$, $lab^*olv^*_3$, and *lab*nce** is given. Therefore in the following if one of these data sets is given the other two shall be calculated.

Given data set:	data set to be calculated:	
$LAB^*LCH^*_a$	$lab^*olv^*_3$	*lab*nce**
$lab^*olv^*_3$	$LAB^*LCH^*_a$	*lab*nce**
*lab*nce**	$LAB^*LCH^*_a$	$lab^*olv^*_3$

In the general case for the solution of this problem the equations (1) to (15) and the inverse equations are used. Figure 11 calculates the transformations according to the equations (1) to (15). If the following tables 3 and 4 are used then the solution is faster and this is shown in the following figures 12 to 14.

Table 3 serves for the determination of $lab^*olv^*_{3,M}$ and $LAB^*LCH^*_{aM}$ of the maximum colour *M* as function of the CIELAB hue angle $H^*_a = h_{ab,a}$ with differences ΔH^*_a = 10 degree. The hue angle difference ΔH^*_a = 10 degree is for many applications to rough and therefore a table with the difference ΔH^*_a = 1 degree is recommended, see the data for 8 device systems (64 pages, 2,1 Mbyte) at the URL:

http://www.ps.bam.de/YE00/10L/L00E00NP.PDF

Table 3 – Table for the determination of *lab*olv$_{3,M}$ and *LAB*LCH**$_{a,M}$ as function of $H^*_a = h_{ab,a}$**

Table: data α_{aM}, olv^*_{3M}, LCH^*_{aM}, a^*_{aM}, b^*_{aM} of System NRS18 for CIELAB hue angle $h_{ab,a}$ and transfer to the standard (s) and elementary (e) hue

$h_{ab,a}$	$h_{ab,s}$	$h_{ab,e}$	h^*	h^*_s	$h^*_e = e^*$	α_{aM}	o^*_{3M}	l^*_{3M}	v^*_{3M}	L^*_M	$C^*_{ab,aM}$	H^*_{aM}	a^*_{aM}	b^*_{aM}
0	3	340	0.0	0.009	0.944	0.552	1.0	0.0	0.448	56.71	68.15	0.0	68.15	0.0
10	14	348	0.028	0.038	0.966	0.728	1.0	0.0	0.272	56.71	69.84	10.0	68.78	12.13
20	24	356	0.056	0.067	0.988	0.904	1.0	0.0	0.096	56.71	73.91	20.0	69.46	25.28
30	34	6	0.083	0.095	0.016	0.068	1.0	0.068	0.0	56.71	73.79	30.0	63.9	36.89
40	43	19	0.111	0.12	0.054	0.217	1.0	0.217	0.0	56.71	68.28	40.0	52.31	43.89
50	52	33	0.139	0.144	0.091	0.367	1.0	0.367	0.0	56.71	65.39	50.0	42.03	50.09
60	61	46	0.167	0.169	0.129	0.517	1.0	0.517	0.0	56.71	64.62	60.0	32.31	55.96
70	70	60	0.194	0.194	0.166	0.666	1.0	0.666	0.0	56.71	65.84	70.0	22.52	61.87
80	79	73	0.222	0.219	0.204	0.816	1.0	0.816	0.0	56.71	69.25	80.0	12.03	68.2
90	88	87	0.25	0.244	0.241	0.966	1.0	0.966	0.0	56.71	75.46	90.0	0.0	75.46
100	97	100	0.278	0.268	0.277	0.11	0.89	1.0	0.0	56.71	71.36	100.0	−12.38	70.28
110	105	113	0.306	0.292	0.313	0.253	0.747	1.0	0.0	56.71	66.43	110.0	−22.71	62.42
120	114	126	0.333	0.316	0.349	0.396	0.604	1.0	0.0	56.71	63.95	120.0	−31.96	55.38
130	122	139	0.361	0.34	0.385	0.539	0.461	1.0	0.0	56.71	63.51	130.0	−40.81	48.65
140	131	151	0.389	0.364	0.421	0.682	0.318	1.0	0.0	56.71	65.04	140.0	−49.81	41.8
150	140	164	0.417	0.388	0.456	0.825	0.175	1.0	0.0	56.71	68.78	150.0	−59.56	34.39
160	148	177	0.444	0.411	0.492	0.968	0.032	1.0	0.0	56.71	75.42	160.0	−70.86	25.79
170	159	186	0.472	0.44	0.518	0.142	0.0	1.0	0.142	56.71	72.94	170.0	−71.83	12.67
180	169	195	0.5	0.471	0.541	0.325	0.0	1.0	0.325	56.71	69.69	180.0	−69.68	0.0
190	180	203	0.528	0.501	0.564	0.507	0.0	1.0	0.507	56.71	68.72	190.0	−67.66	−11.92
200	191	211	0.556	0.532	0.587	0.69	0.0	1.0	0.69	56.71	69.86	200.0	−65.64	−23.89
210	202	219	0.583	0.562	0.609	0.873	0.0	1.0	0.873	56.71	73.32	210.0	−63.48	−36.65
220	213	228	0.611	0.593	0.632	0.055	0.0	0.945	1.0	56.71	75.43	220.0	−57.77	−48.48
230	224	236	0.639	0.623	0.655	0.238	0.0	0.762	1.0	56.71	70.93	230.0	−45.58	−54.33
240	235	244	0.667	0.653	0.678	0.42	0.0	0.58	1.0	56.71	68.92	240.0	−34.45	−59.67
250	246	252	0.694	0.684	0.701	0.603	0.0	0.397	1.0	56.71	69.05	250.0	−23.61	−64.88
260	257	261	0.722	0.714	0.724	0.786	0.0	0.214	1.0	56.71	71.36	260.0	−12.38	−70.26
270	268	269	0.75	0.745	0.747	0.968	0.0	0.032	1.0	56.71	76.22	270.0	0.0	−76.21
280	279	277	0.778	0.774	0.769	0.145	0.145	0.0	1.0	56.71	72.5	280.0	12.59	−71.39
290	289	285	0.806	0.804	0.791	0.321	0.321	0.0	1.0	56.71	69.14	290.0	23.65	−64.96
300	300	292	0.833	0.833	0.812	0.497	0.497	0.0	1.0	56.71	68.05	300.0	34.02	−58.92
310	310	300	0.861	0.862	0.834	0.673	0.673	0.0	1.0	56.71	69.06	310.0	44.39	−52.89
320	321	308	0.889	0.891	0.856	0.849	0.849	0.0	1.0	56.71	72.34	320.0	55.41	−46.49
330	331	316	0.917	0.921	0.878	0.024	1.0	0.0	0.976	56.71	76.42	330.0	66.18	−38.2
340	342	324	0.944	0.95	0.9	0.2	1.0	0.0	0.8	56.71	71.19	340.0	66.89	−24.34
350	353	332	0.972	0.979	0.922	0.376	1.0	0.0	0.624	56.71	68.58	350.0	67.54	−11.9
0	3	340	0.0	0.009	0.944	0.552	1.0	0.0	0.448	56.71	68.15	0.0	68.15	0.0

ZE130−7

Table 3 serves for the determination of the colour data *lab*olv**$_{3,M}$ and *LAB*LCH**$_{aM}$ of the maximum colour *M* as function of the CIELAB hue angle $H^*_a = h_{ab,a}$ for the hue angle difference of ΔH^*_a = 10 degree.

Tabelle 4 – Relation between the elementary hue angle $H^*_e = h_{ab,e}$ and $H^*_a = h_{ab,a}$, and vice versa.

Table: CIELAB hue angle $h_{ab,a}$ of the System NRS18 and transfer to hue angles in the standard (s) or elementary (e) colour system																	
$h_{ab,a}$	$h_{ab,s}$	$h_{ab,e}$	h^*	h^*_s	$h^*_e=e^*$	$h_{ab,s}$	$h_{ab,a}$	$h_{ab,e}$	h^*_s	h^*	$h^*_e=e^*$	$h_{ab,e}$	$h_{ab,a}$	$h_{ab,s}$	$h^*_e=e^*$	h^*	h^*_s
0	3	340	0.0	0.009	0.944	0	357	337	0.0	0.992	0.937	0	26	30	0.0	0.071	0.001
10	14	348	0.028	0.038	0.966	10	7	345	0.028	0.018	0.959	10	33	37	0.028	0.092	0.028
20	24	356	0.056	0.067	0.988	20	16	352	0.056	0.044	0.979	20	40	43	0.056	0.112	0.054
30	34	6	0.083	0.095	0.016	30	25	359	0.083	0.071	0.999	30	48	50	0.083	0.133	0.084
40	43	19	0.111	0.12	0.054	40	37	15	0.111	0.102	0.043	40	55	57	0.111	0.154	0.11
50	52	33	0.139	0.144	0.091	50	48	30	0.139	0.133	0.084	50	63	64	0.139	0.174	0.14
60	61	46	0.167	0.169	0.129	60	59	45	0.167	0.164	0.125	60	70	70	0.167	0.195	0.166
70	70	60	0.194	0.194	0.166	70	70	60	0.194	0.195	0.166	70	78	77	0.194	0.215	0.196
80	79	73	0.222	0.219	0.204	80	81	75	0.222	0.225	0.207	80	85	83	0.222	0.236	0.222
90	88	87	0.25	0.244	0.241	90	92	89	0.25	0.256	0.249	90	92	90	0.25	0.257	0.249
100	97	100	0.278	0.268	0.277	100	104	105	0.278	0.289	0.292	100	100	97	0.278	0.278	0.277
110	105	113	0.306	0.292	0.313	110	116	120	0.306	0.321	0.335	110	108	103	0.306	0.3	0.306
120	114	126	0.333	0.316	0.349	120	127	135	0.333	0.354	0.374	120	116	110	0.333	0.321	0.335
130	122	139	0.361	0.34	0.385	130	139	150	0.361	0.386	0.417	130	123	116	0.361	0.343	0.36
140	131	151	0.389	0.364	0.421	140	151	166	0.389	0.418	0.46	140	131	123	0.389	0.364	0.388
150	140	164	0.417	0.388	0.456	150	162	180	0.417	0.451	0.499	150	139	130	0.417	0.386	0.417
160	148	177	0.444	0.411	0.492	160	171	187	0.444	0.476	0.52	160	147	137	0.444	0.407	0.446
170	159	186	0.472	0.44	0.518	170	180	195	0.472	0.501	0.541	170	154	143	0.472	0.429	0.471
180	169	195	0.5	0.471	0.541	180	190	203	0.5	0.527	0.564	180	162	150	0.5	0.45	0.499
190	180	203	0.528	0.501	0.564	190	199	210	0.528	0.552	0.584	190	174	163	0.528	0.484	0.527
200	191	211	0.556	0.532	0.587	200	208	218	0.556	0.577	0.605	200	186	176	0.556	0.518	0.554
210	202	219	0.583	0.562	0.609	210	217	225	0.583	0.603	0.625	210	199	190	0.583	0.552	0.584
220	213	228	0.611	0.593	0.632	220	226	233	0.611	0.628	0.646	220	211	203	0.611	0.585	0.612
230	224	236	0.639	0.623	0.655	230	235	240	0.639	0.653	0.667	230	223	217	0.639	0.619	0.639
240	235	244	0.667	0.653	0.678	240	244	247	0.667	0.679	0.687	240	235	230	0.667	0.653	0.667
250	246	252	0.694	0.684	0.701	250	253	255	0.694	0.704	0.708	250	247	243	0.694	0.687	0.694
260	257	261	0.722	0.714	0.724	260	263	263	0.722	0.729	0.731	260	259	256	0.722	0.72	0.721
270	268	269	0.75	0.745	0.747	270	272	270	0.75	0.755	0.751	270	271	269	0.75	0.754	0.749
280	279	277	0.778	0.774	0.769	280	281	277	0.778	0.781	0.771	280	284	283	0.778	0.789	0.777
290	289	285	0.806	0.804	0.791	290	291	285	0.806	0.807	0.793	290	297	297	0.806	0.825	0.806
300	300	292	0.833	0.833	0.812	300	300	292	0.833	0.834	0.812	300	310	310	0.833	0.86	0.834
310	310	300	0.861	0.862	0.834	310	310	300	0.861	0.86	0.834	310	322	323	0.861	0.895	0.861
320	321	308	0.889	0.891	0.856	320	319	307	0.889	0.886	0.854	320	335	337	0.889	0.93	0.889
330	331	316	0.917	0.921	0.878	330	329	315	0.917	0.913	0.876	330	348	350	0.917	0.966	0.918
340	342	324	0.944	0.95	0.9	340	338	322	0.944	0.939	0.896	340	0	3	0.944	0.001	0.944
350	353	332	0.972	0.979	0.922	350	348	330	0.972	0.965	0.918	350	13	17	0.972	0.036	0.972
0	3	340	0.0	0.009	0.944	0	357	337	0.0	0.992	0.937	0	26	30	0.0	0.071	0.001

ZE131−7

Table 4 serves for the determination of the standard hue angle $H^*_s = h_{ab,s}$ and the elementary hue angle $H^*_e = h_{ab,e}$ as function of the adapted CIELAB hue angle $H^*_a = h_{ab,a}$ for the hue angle difference of ΔH^*_a = 10 degree and in the inverse direction.

The hue angle difference ΔH^*_a = 10 degree is for many applications to rough and therefore a table with the difference ΔH^*_a = 1 degree is recommended.

Especially in the case with pixel graphics of for example 1 million pixels (1000 x 1000 image matrix) the transformation with table data is much faster compared to the method in figure 11, see also figure 12 to 14. In application for each pixel about 5 to 20 mathematical operations are necessary.

NOTE: In the following for any colour *F* **no** index is used but for the maximum colour *M* the index *M* is always used.

Equations: colorimetric data transfer from LCH^*_a (CIELAB) to *nce and olv^*_3**		
Given: CIELAB data of any colour L^*, $C^*_{ab,a}$, $h_{ab,a}$ = LCH^*_a = $LAB^*LCH^*_a$ or L^*, a^*_a, b^*_a		
CIELAB data L^*, $C^*_{ab,a}$, $h_{ab,a}$, a^*_a, b^*_a of eigth basic colours *X = OYLCVMNW*		
Aim: *nce and *rgb* device data olv^*_3 of the given colour** (in example *M* located between *O* and *Y*)		
CIELAB Hue angle of colour and maximum colour *M*	$h_{ab,a} = h_{ab,a,M}$ $(0 <= h_{ab,a} <= 360)$	(1)
Relative device hue angle ratio of *M*	$\alpha_{a,M} = [h_{ab,a,M} - h_{ab,a,O}] / [h_{ab,a,Y} - h_{ab,a,O}]$	(2)
CIELAB lightness of *M*	$L^*_M = \alpha_{a,M} L^*_{a,Y} + (1 - \alpha_{a,M}) L^*_{a,O}$	(3)
CIELAB red-green chroma of *M*	$a^*_{a,M} = \alpha_{a,M} a^*_{a,Y} + (1 - \alpha_{a,M}) a^*_{a,O}$	(4)
CIELAB yellow-blue chroma of *M*	$b^*_{a,M} = \alpha_{a,M} b^*_{a,Y} + (1 - \alpha_{a,M}) b^*_{a,O}$	(5)
radial CIELAB chroma of *M*	$C^*_{ab,a,M} = [a^{*2}_{a,M} + b^{*2}_{a,M}]^{1/2}$	(6)
relative lightness of the given colour	$l^* = [L^* - L^*_N] / [L^*_W - L^*_N]$	(7)
relative chroma of the given colour	$c^* = C^*_{ab,a} / C^*_{ab,a,M}$	(8)
relative triangle lightness of the given colour	$t^* = l^* - [L^*_M - L^*_N] / [L^*_W - L^*_N] c^* + 0{,}5 c^*$	(9)
relative blackness of the given colour	$n^* = 1 - t^* - 0{,}5 c^*$	(10)
relative whiteness of the given colour	$w^* = 1 - n^* - c^*$	(11)
elementary hue angle of the given colour	e^* = function $[h_{ab,a}]$ (with table/equation)	(12)
relative $olv^*_{3,M}$ data of *M*	$o^*_{3,M} = \alpha_{a,M} o^*_{3,Y} + (1 - \alpha_{a,M}) o^*_{3,O}$	(13)
	$l^*_{3,M} = \alpha_{a,M} l^*_{3,Y} + (1 - \alpha_{a,M}) l^*_{3,O}$	(14)
	$v^*_{3,M} = \alpha_{a,M} v^*_{3,Y} + (1 - \alpha_{a,M}) v^*_{3,O}$	(15)
relative olv^*_3 data of the given colour	$o^*_3 = w^* + c^* o^*_{3,M}$	(16)
	$l^*_3 = w^* + c^* l^*_{3,M}$	(17)
	$v^*_3 = w^* + c^* v^*_{3,M}$	(18)

ZE120−3

Figure 11 – Transformation of given data $LAB^*LCH^*_a$ to $lab^*olv^*_3$ and *lab*nce*

Figure 11 shows the transformations of the given data $LAB^*LCH^*_a$ to $lab^*olv^*_3$ and *lab*nce**.* For this transformation except for the calculation of *e** the equations (1) to (15) are used. In the example it is assumed that the hue angle of the (orange) maximum colour *M* is located between the hue angle of the colours orange red *O* and yellow *Y*.

The calculation with the relative hue angle ratio $\alpha_{a,M}$ according to figure 11 requires more calculation time for large image matrices, for example of 1000 x 1000 points, compared to the method in figure 12 to 14 which uses the table data for the maximum colours *M* as function of the CIELAB hue angle $h_{ab,a}$.

Therefore the table method is preferred in the following. The necessary tables 3 and 4 for this calculation include data for the CIELAB hue angle difference of Δh_{ab} = 10 degree. The hue angle difference ΔH^*_a = 10 degree is for many applications to rough and therefore a table with the difference ΔH^*_a = 1 degree is recommended, see the data for 8 device systems (64 pages, 2,1 Mbyte) at the URL:

http://www.ps.bam.de/YE00/10L/L00E00NP.PDF

In many cases one may calculate colour profiles for the input and output. Also for the calculation of colour profiles with for example 25x25x25 *rgb* data and corresponding L^*, a^*_a, b^*_a data the tables are appropriate with the CIELAB hue angle difference of Δh_{ab} = 1 degree. In the following the faster calculation method with the table data is shown. The method of figure 12 uses table data and is an alternate method compared to figure 11.

Equations: colorimetric data transfer from LCH^*_a (CIELAB) to nce^* and olv^*_3			
Given: adapted CIELAB data of any colour L^*, $C^*_{ab,a}$, $h_{ab,a}$ = LCH^*_a = $LAB^*LCH^*_a$			
adapted CIELAB data L^*, $C^*_{ab,a}$, $h_{ab,a}$, a^*_a, b^*_a of eigth basic colours $X = OYLCVMNW$			
Aim: nce^* and rgb device data olv^*_3 of the given colour			
hue angle of of the given colour and of M	$h_{ab,a} = H^*_a$		(1)
CIELAB $LCH^*_{a,M}$ data of maximum colour M	L^*_M = function [$h_{ab,a}$]	(with table/equation)	(2)
	$C^*_{ab,a,M}$ = function [$h_{ab,a}$]	(with table/equation)	(3)
	$h_{ab,a,M} = h_{ab,a}$		(4)
relative lightness of the given colour	$l^* = [L^* - L^*_N] / [L^*_W - L^*_N]$		(5)
relative chroma of the given colour	$c^* = C^*_{ab,a} / C^*_{ab,a,M}$		(6)
relative triangle lightness of the given colour	$t^* = l^* - [L^*_M - L^*_N] / [L^*_W - L^*_N] c^* + 0{,}5\, c^*$		(7)
relative blackness of the given colour	$n^* = 1 - t^* - 0{,}5\, c^*$		(8)
relative whiteness of the given colour	$w^* = 1 - n^* - c^*$		(9)
elementary hue angle of the given colour	e^* = function [$h_{ab,a}$]	(with table or equation)	(10)
relative $olv^*_{3,M}$ data of maximum colour M	$o^*_{3,M}$ = function [$h_{ab,a}$]	(with table/equation)	(11)
	$l^*_{3,M}$ = function [$h_{ab,a}$]	(with table/equation)	(12)
	$v^*_{3,M}$ = function [$h_{ab,a}$]	(with table/equation)	(13)
relative olv^*_3 data of the given colour	$o^*_3 = w^* + c^*\, o^*_{3,M}$		(14)
	$l^*_3 = w^* + c^*\, l^*_{3,M}$		(15)
	$v^*_3 = w^* + c^*\, v^*_{3,M}$		(16)

ZE120−7

Figure 12 – Transformation of given data *$LAB^*LCH^*_a$* to *lab^*nce^** and *$lab^*olv^*_3$*

Figure 12 shows the transformations of the given data $LAB^*LCH^*_a$ to lab^*nce^* and $lab^*olv^*_3$. For this transformation the table data according to table 3 and 4 are used.

Equations: colorimetric data transfer from nce^* to olv^*_3 (*rgb* data) and LCH^*_a			
Given: nce^* data (similar NCS) of any colour nce^* = lab^*nce^* ($0 <= n^*, c^*, e^* <= 1$)			
adapted CIELAB data L^*, $C^*_{ab,a}$, $h_{ab,a}$, a^*_a, b^*_a of eigth basic colours $X = OYLCVMNW$			
Aim: rgb device data olv^*_3 and LCH^*_a of the given colour			
elementary hue number of a colour	e^*	($0 <= e^* <= 1$)	(1)
CIELAB hue angle of colour and maximum colour M	$h_{ab,a}$ = function [e^*]	(with table/equation)	(2)
relative whiteness of the given colour	$w^* = 1 - n^* - c^*$		(3)
relative triangle lightness of the given colour	$t^* = \ = 1 - n^* - 0{,}5\, c^*$		(4)
$olv^*_{3,M}$ data of maximum colour M	$o^*_{3,M}$ = function [$h_{ab,a}$]	(with table/equation)	(5)
	$l^*_{3,M}$ = function [$h_{ab,a}$]	(with table/equation)	(6)
	$v^*_{3,M}$ = function [$h_{ab,a}$]	(with table/equation)	(7)
relative olv^*_3 data of the given colour	$o^*_3 = w^* + c^*\, o^*_{3,M}$		(8)
	$l^*_3 = w^* + c^*\, l^*_{3,M}$		(9)
	$v^*_3 = w^* + c^*\, v^*_{3,M}$		(10)
adapted CIELAB $LCH^*_{a,M}$ data of maximum colour M	L^*_M = function [$h_{ab,a}$]	(with table/equation)	(11)
	$C^*_{ab,a,M}$ = function [$h_{ab,a}$]	(with table/equation)	(12)
	$h_{ab,a,M} = h_{ab,a}$		(13)
relative lightness of maximum colour M	$l^*_M = [L^*_M - L^*_N] / [L^*_W - L^*_N]$		(14)
relative lightness of the given colour	$l^* = t^* + l^*_M\, c^* + 0{,}5\, c^*$		(15)
adapted CIELAB LCH^*_a data of the given colour	$L^* = l^* [L^*_W - L^*_N] + L^*_N$		(16)
	$C^*_{ab,a} = c^*\, C^*_{ab,a,M}$		(17)
	$h_{ab,a} = h_{ab,a,M}$		(18)

ZE121−3

Figure 13 – Transformation of given data *lab^*nce^** to *$lab^*olv^*_3$* and *$LAB^*LCH^*_a$*

Figure 13 shows the transformations of the given data lab^*nce^* to $lab^*olv^*_3$ and $LAB^*LCH^*_a$.. For this transformation the table data according to table 3 and 4 are used.

Equations: colorimetric data transfer from $olv*_3$ to $nce*$ data and $LCH*_a$ data

Given: ***rgb*** **device data of any colour $olv*_3 = lab*olv*_3$**

adapted CIELAB data $L*$, $C*_{ab,a}$, $h_{ab,a}$, $a*_a$, $b*_a$ of eigth basic colours $X = OYLCVMNW$

Aim: $nce* = lab*nce*$ (similar to NCS data) and $LCH*_a$ data of the given colour $(0 <= e* <= 1)$

relative chroma of the given colour	$c* = max [olv*_3] - min [olv*_3]$	(1)
relative blackness of the given colour	$n* = 1 - max [olv*_3]$	(2)
relative triangle lightness of the given colour	$t* = 1 - n* - 0{,}5\ c*$	(3)
relative red-green chroma in 60 degree system s	$a*_{rs} = o*_3 \cos(30) + l*_3 \cos(150)$	(4)
relative yellow-blue chroma in 60 degree system s	$b*_{rs} = o*_3 \sin(30) + l*_3 \sin(150) + v*_3 \sin(270)$	(5)
hue angle in 60 degree system s	$h_{ab,s} = \arctan [b*_{rs} / a*_{rs}]$ $(0 <= h_{ab,s} <= 360)$	(6)
CIELAB hue angle in device system	$h_{ab,a}$ = function $[h_{ab,s}]$ (with table/equation)	(7)
elementary hue number of the given colour	$e*$ = function $[h_{ab,a}]$ (with table/equation)	(8)
adapted CIELAB $LCH*_a$ data of maximum colour M	$L*_M$ = function $[h_{ab,a}]$ (with table/equation)	(9)
	$C*_{ab,a,M}$ = function $[h_{ab,a}]$ (with table/equation)	(10)
	$h_{ab,a,M} = h_{ab,a}$	(11)
relative lightness of maximum colour M	$l*_M = [L*_M - L*_N] / [L*_W - L*_N]$	(12)
relative lightness of the given colour	$l* = t* + l*_M\ c* + 0{,}5\ c*$	(13)
adapted CIELAB $LCH*_a$ data of the given colour	$L* = l* [L*_W - L*_N] + L*_N$	(14)
	$C*_{ab,a} = c*\ C*_{ab,a,M}$	(15)
	$h_{ab,a} = h_{ab,a,M}$	(16)

ZE121−7

Figure 14 – Transformation of given data *lab*olv$_3$ to *lab*nce** and *LAB*LCH**$_a$**

Figure 14 shows the transformations of the given data $lab*olv*_3$ to $lab*nce*$ and $LAB*LCH*_a$. For this transformation the table data according to table 3 and 4 are used.

If tables by some reasons should not be used then there are equation examples under the following URL (1 page, 60 kByte)

http://www.ps.bam.de/ZE20/10L/L20E00NP.PDF

There are different equation examples:

1. For the transformation of the CIELAB hue angle ratio h_{ab} to the elementary hue number $e*$ and in the inverse direction
2. For the transformation of the standard hue angle $h_{ab,s}$ to the CIELAB hue angle $h_{ab,a}$

8. CIELAB colour system and application limits

The CIELAB system serves as basis for the description of properties of colour vision. A first test of each new colorimetric model of colour vision is therefore usually a comparison of the new model spacing compared to the spacing in the *Munsell* and/or the *OSA* colour order systems. CIELAB is based on the *Munsell colour system* and there is a paper of *Richter* (1980) with colour reproductions of the colour samples of the *Munsell* and the *OSA colour system* in different chromaticity diagrams. Some of the following pictures are taken from this paper and a book of *Richter* (1996).

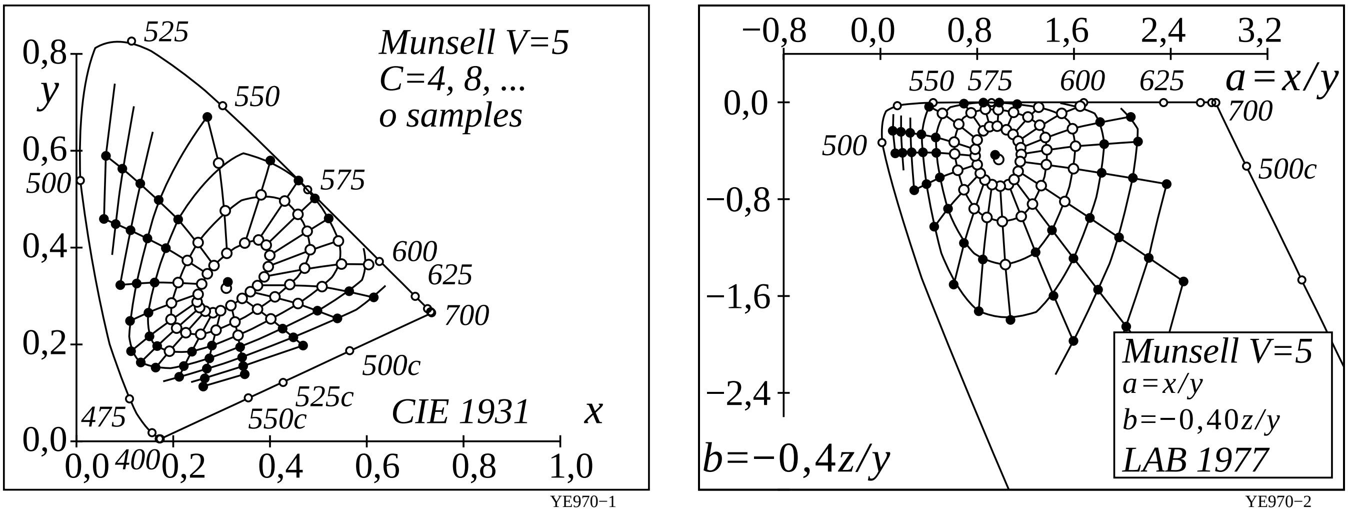


Figure 15 – Real and extrapolated samples of the *Munsell* colour order system (Value 5) in *(x, y)*, and *(a, b)*
Figure 15 shows real (o) and extrapolated samples (•) of the *Munsell* colour system for *Value* 5 in the CIE chromaticity diagram *(x, y)* and a special chromaticity diagram *(a, b*). The relation between the chromaticity coordinates *(a, b)* and *(x, y)* is linear and given in figure 15 *(right)*.

color valence metric (color data: linear relation to CIE 1931 data)		
linear color terms	**name and relationship to CIE tristimulues or chromaticity values**	**notes:**
luminous value	$Y = y\,(X + Y + Z)$	
chromatic value	*for linear chromatic value diagram (A, B)*	
red−green	$A = [X/Y - X_n/Y_n]\,Y = [a - a_n]\,Y$	*n=D65 (backgr.)*
	$= [x/y - x_n/y_n]\,Y$	
yellow−blue	$B = -0{,}4\,[Z/Y - Z_n/Y_n]\,Y = [b - b_n]\,Y$	
	$= -0{,}4\,[z/y - z_n/y_n]\,Y$	
radial	$C_{ab} = [A^2 + B^2]^{1/2}$	
chromaticity	*for (linear) chromaticity diagram (a, b)*	*compare to linear*
red−green	$a = X/Y = x/y$	*cone excitation*
yellow−blue	$b = -0{,}4\,[Z/Y] = -0{,}4\,[z/y]$	***P*/ (*P*+*D*)**
radial	$c_{ab} = [(a - a_n)^2 + (b - b_n)^2]^{1/2}$	***T*/ (*P*+*D*)**

YE971−7

Figure 16 – Coordinates of the lower colour metric and chromaticity coordinates *(a, b)*
Figure 16 shows coordinates of the lower colour metric and the chromaticity coordinates *(a, b)* which have a linear relation to the CIE chromaticity coordinates *(x, y).* The chromatic values *A* and *B* can be calculated by multiplying the luminance factor *Y* of the sample with the difference of the chromaticities *a* of the sample and a_n of the background.

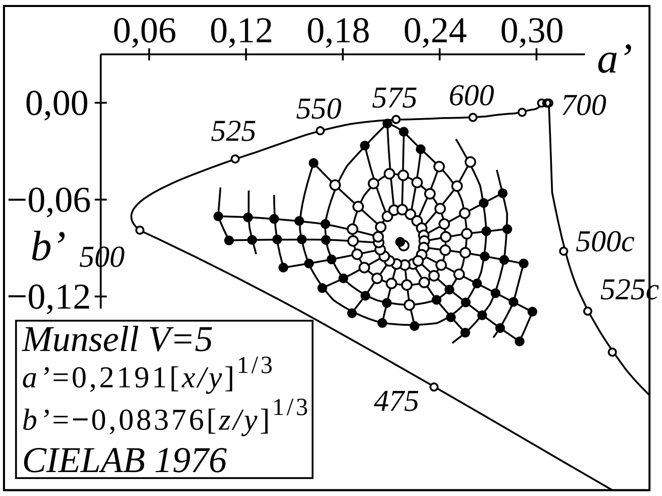
Additionally the chromaticity *a* and *b* can be compared with the saturation *P / (P+D)* and *T / (P+D)* of the three receptors *P, D* and *T* or *LMS* according to CIE 171-1:2005. For example in Figure 16 the ratio *Z / Y = z / y =* [*(1–x–y) / y*] is similar to the ratio *T / (P+D).*

Higher colormetric (color data: nonlinear relation to CIE 1931 data)		
non linear color terms	**name and relationship with tristimulues or chromaticity values**	**notes**
lightness	$L^* = 116\ (Y / 100)^{1/3} - 16 \quad (Y > 0{,}8)$	*CIELAB 1976*
	Approximation: $L^* = 100\ (Y / 100)^{1/2{,}4}$	
chroma	*non linear transform of chromatic values A and B*	
red−green	$a^* = 500\ [\ (X / X_n)^{1/3} - (Y / Y_n)^{1/3}\ ]$	*CIELAB 1976*
	$= 500\ (a' - a'_n)\ Y^{1/3}$	*n=D65 (backgr.)*
yellow−blue	$b^* = 200\ [\ (Y / Y_n)^{1/3} - (Z / Z_n)^{1/3}\ ]$	*CIELAB 1976*
	$= 500\ (b' - b'_n)\ Y^{1/3}$	
radial	$C^*_{ab} = [\ a^{*2} + b^{*2}\ ]^{1/2}$	
chromaticity	*nonlinear transform of chromaticities a=x/y and b=z/y*	
red−green	$a' = (1 / X_n)^{1/3}\ (x / y)^{1/3}$	*compare to log*
	$= 0{,}2191\ (x / y)^{1/3}$ for *D65*	*cone excitation*
yellow−blue	$b' = -0{,}4\ (1 / Z_n)^{1/3}\ (z / y)^{1/3}$	***log[P/ (P+D)]***
	$= -0{,}08376\ (z / y)^{1/3}$ for *D65*	***log[T/ (P+D)]***
radial	$c'_{ab} = [\ (a' - a'_n)^2 + (b' - b'_n)^2\ ]^{1/2}$	

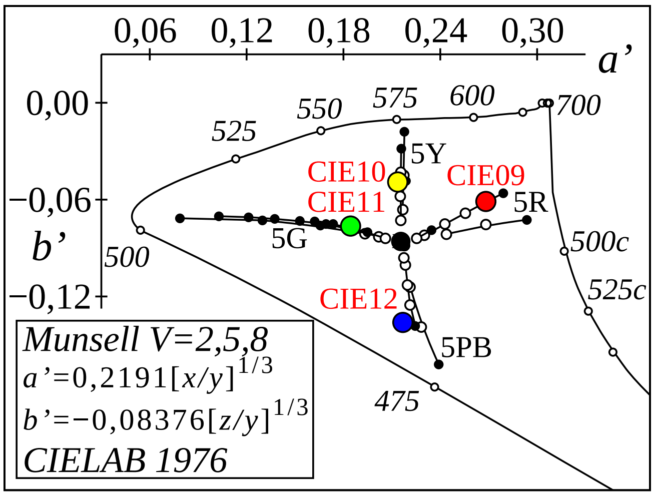
YE970−7

Figure 17 – Coordinates of the higher color metric with non linear chromaticity coordinates *(a’, b’)*

Figure 17 shows coordinates of the higher color metric with non linear chromaticity coordinates *(a’, b’)*. The CIELAB chroma data *a** and *b** can be calculated if the non linear chromaticity difference *a’* of the sample and a'_n of the background (n) is multiplied with the lightness *L** of the sample (in this case the approximation $Y^{1/3}$). This kind of calculation is similar compared to the calculation in the CIELUV colour space. Additionally the non linear chromaticity coordinates *a’* and *b’* are compared with the saturation log [*P/ (P+D)*] and log [*T / (P+D)*]. The cube root coordinates
$(Z / Y)^{1/3} = (z / y)^{1/3} = [\ (1–x–y) / y\ ]^{1/3}$ are similar to log [*T / (P+D)*].



YE970−3



YE970−4

Figure 18 – Real and extrapolated samples of the *Munsell* colour system *(Value 5)* in *(a’, b’)*

Figure 18 shows real (o) and extrapolated (•) samples of the *Munsell* colour system for *Value* V= 5 in the *non linear* chromaticity diagram *(a’, b’) (left)* and the elementary hues *5R, 5Y, 5G* and *5PB* of the *Munsell* colour order system for *Value* 2, 5, and 8 in the *non linear (cube root)* chromaticity diagram *(a’, b’) (right)*. Additionally the four CIE-test colours no. 9 to 12 of CIE 13.3 are shown. These four CIE-test colours serve as elementary colours in the field of image technology, see *Richter* (2007).

The CIELAB data of the four elementary colours have been calculated for the CIE standard illuminant D65 and the CIE illuminant D50, see table 7 in section 10. The CIE-test colours serve for example in DIN 33866-2, and in ISO/ IEC 15775 as reference colours for colour copiers and in ISO/IEC TR 24705 as reference colours for printer and monitor output and for scanner input.

In Figure 18 *(right)* the elementary colours yellow *J* and blue *B* are located approximately on a straight line through the white point (chromaticity of D65). The elementary colours red *R* and green *G* are not on a line through the white point. For D65 the elementary colours *RJGB* have the CIELAB hue angles h_{ab} = 26, 92, 162 and 272 degree in the CIELAB system.

DIN 33872-1 to -6 uses the *device independent* location of the CIELAB hue angles of the elementary colours, and increases so especially the user acceptance.

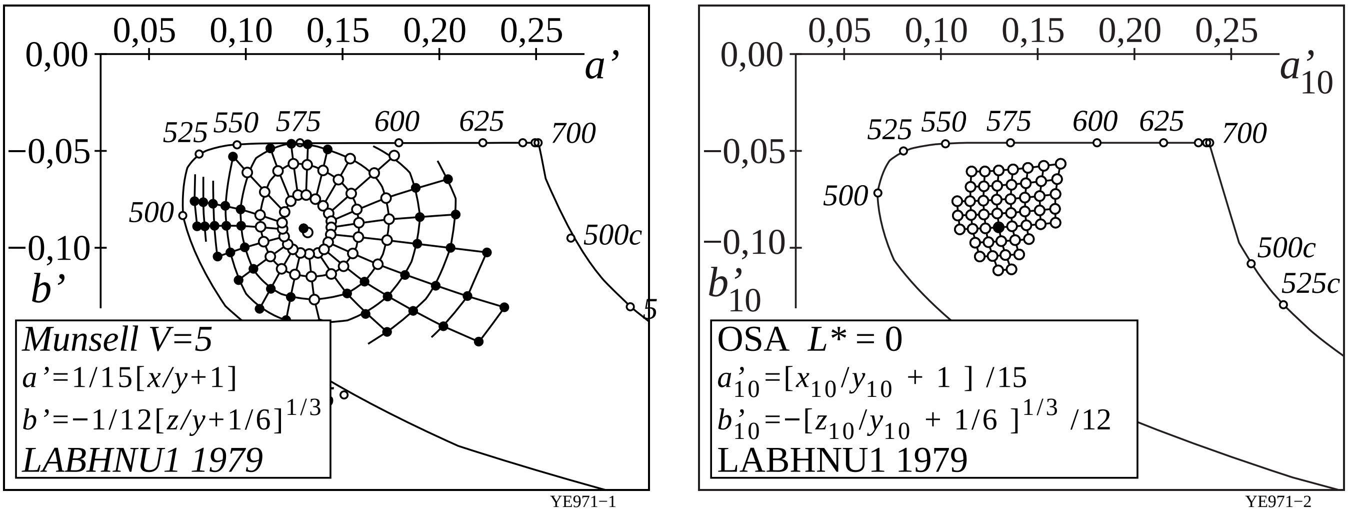


Figure 19 – Samples of the *Munsell* and *OSA* system in a modified chromaticity diagram *(a’, b’)*

Figure 19 shows samples *(Value 5)* of the *Munsell* colour system and samples (L^*_{OSA} = 0, corresponding to L^*_{CIE} = 50) of the *OSA* system in a modified chromaticity diagram *(a’, b’).* The relation between (*a’, b’*) and (*x, y*) and between (a'_{10}, b'_{10}) and (x_{10}, y_{10}) for the 10 degree observer is shown in the figure 19. According to the results of the output scaling, see *Richter* (1976 and 1996) it is possible to use a linear equation in red–green direction, see the definition of *a’* and a'_{10} in the figure. This *linear* equation is appropriate for the chromaticity range of surface colours which is represented by the real *Munsell (left)* and *OSA (right)* samples. This area of all real surface colours fills only about 30% of the optimal colour gamut. Optimal colours have rectangular reflection curves.

Application limits for colour spacing of CIELAB

Figure 19 shows in comparison to Figure 18 an improved colour spacing for the colour series White–Yellow for both the samples of the *Munsell* and the *OSA* system. This property is important for many applications with surface colours. There yellow standard surface colours of offset printing is located approximately on the spectral locus of the chromaticity diagrams (*x, y*) or (*a, b*) or (*a’, b’*). The yellow display colour which is mixed by the orange red *O* and leaf green device colours *L* is located much more inside the chromaticity diagrams. For example in CIELAB for a photo printer the chroma C^*_{ab} is equal to 115, and for the standard Monitor TLS18 the chroma C^*_{ab} is approximately equal to 60. Instead of the ratio 115/60 in CIELAB the ratio is 80/58 in the (corrected) space LABHNU1 1979. This ratio is much more appropriate compared to visual results. CIELAB therefore calculates a too large colour difference for saturated yellows in the white–yellow direction. Therefore the visual differences decrease for the series white–yellow towards saturated yellow if the series is equally spaced in CIELAB.

Additionally in red–green direction for saturated green colours a too high CIELAB difference is calculated in the white–green direction. This effect is less important in applications because there are usually no surface colours in this area.

In CIELAB the elementary hue blue *B* and the elementary hue red *R* show a hue angle shift as function of sample lightness. If we study for example the elementary blue colour *B* (h_{ab} = 272 degree) of the same chromaticity and for the three lightness data L^* = 20, 50 and 80 then the light sample appears more reddishcompared to the dark sample, compare figure 18 *(right)*.

CIELAB has been developed for *separate* samples on a grey background. For *adjacent* samples the local *relative* adaptation luminance in dark-light direction is active which produces for small colour differences a change of the colour discrimination metric. For small colour differences and adjacent samples the colour difference formula CIEDE2000 has been developed, compare CIE 142:2001 which is applicable for colour differences between 0 and 5 CIELAB. In the field of colour image technology colour differences larger 5 CIELAB are usual between the original and the reproduction. Therefore the industrial colour difference calculation with the formula CIEDE2000 of CIE 142 is not applicable. The application range of CIEDE2000 is under study in the technical committee CIE TC1-63 “Validity of CIEDE2000“ with *K. Richter* as chairman.

Application range of CIELAB for different illuminants

CIE 15:2005 defines two CIE standard illuminants D65 (daylight) and A (approximately tungsten light) and other illuminants, for example D50, D55, and D75. For the field of standardisation CIE 15 recommends one of the two standard illuminants D65 or A. In the office area the standard illuminant D65 is produced by daylight in the office. CIE TC1-66 defines at present an “indoor daylight“ which assumes D65 outside the office and which considers the absorption of the technical window glass. The window glass absorbs especially in the UV region. There are only small changes of the spectral power distribution of D65 in the visible range.

According to DIN 18599-4 in the EU for every new office building an energy passport is required. Both the use of daylight in the office with enough large windows and not to large room deepness increase the saving of energy.

According to IEC 61966-2-1 the standard monitor has the default chromaticity of white D65.If the standard office luminance is produced then a new monitor adjustment to the chromaticity D50 requires in general an electrical energy consumption which is 20% to 30% higher compared to D65. Therefore the use of the chromaticity D50 in the office is not appropriate to save energy. Therefore in the office area for the comparison of hardcopy and softcopy by many reasons the CIE standard illuminant D65 is recommended.

In DIN 33866-1 and ISO/IEC 15775 the CIE standard illuminant D65 is used in all steps: for the definition of the analog test charts, for the production, for the visual assessment, and for the colorimetric specification.

D65 and D50 in standards

ISO TC 130 (Graphic Technology) has chosen the CIE illuminant D50 as standard by different reasons. By history at the printing work places at first slides, which have been optimized for CIE standard illuminant A in the projection, must be compared with the prints under daylight D65. For this comparison the illuminant D50 was a compromise as this illuminant is intermediate between the illuminants A and D65.

At the printing work places the use of daylight by windows is often not possible because of the large room deepness which is necessory for the large printing machines and often too variable. Therefore artificial light sources, at least in a colour comparison booth, are necessary with defined properties. The light sources in any colour comparison booth usually produce the CIE standard illuminants D65 and A, and the standard illuminant D50 of ISO TC 130.

The illuminant D50 produces only about half of the radiation near 400 nm compared to D65. For example this different radiation changes the colour of white office or photo paper which both include optical brighteners. This colour change by fluorescence is often 10 CIELAB for D65 and about 5 CIELAB for D50. However, most of the *xy*-standard devices used in the graphic industry can not measure this colour change by fluorescence, compare CIE 163. However, these changes influence the visual assessment to a high degree, for example by the comparison of the softcopy (monitor without fluorescence) and the hardcopy (paper with fluorescence). Because of these application problems and for the often necessary comparison of the measuring results the measurement **without** fluorescence is under consideration in new draft standards of ISO TC 130.

The Technical Report CIE 163 describes the possibilities of an improved colour measurement in the standard case of fluorescent white office papers with measurement devices which include quartz optic instead of artificial optic. An improved measurement method for images is described in a Ph.D. thesis of *S. Jaeger* (2006). A BAM-CIELAB camera measures the spectral power distribution at each image point (including the visual spectral power distribution produced by fluorescence) for a D65 illumination and calculates the CIELAB data for each image point with a standard deviation of 2 CIELAB. For an appropriate measurement of fluorescence a D65 taking illuminant is necessary.

Problems in application of CIELAB for D50

The colour space CIELAB has been developed for the CIE standard illuminant D65, and is based on experimental results with the former CIE standard illuminant C which is replaced by the CIE standard illuminant D65 with approximately the same colour temperature. CIE 15 limits the application for illuminants **similar** to D65. A tolerance is not defined.

The application of CIELAB for D50 is therefore allowed according to CIE 15. CIELAB for D50 and for D65 use both an included “*von Kries* chromatic adaptation formula“. In CIE 160 a different chromatic adaptation formula is recommended for tests. According to this technical report the “*von Kries* chromatic adaptation formula“ of CIELAB is less appropriate. Therefore indirectly the use of CIELAB for D50 is *not* recommended but the use of CIELAB for D65 and the chromatic adaptation formula according to CIE 160 if D50 must be used by some reasons.

Relative colour image reproduction for D65 and D50

DIN 33872 uses the CIE standard illuminant D65 and is therefore in agreement with the recommendations of CIE 15. The studies of the colour spacing which use as alternate the CIE standard illuminant D65 and the CIE illuminant D50 in section13 show that the *relative* equidistant spacing under D65 is at the same time a *relative* equidistant spacing under D50. This is valid if the visual tolerance of 3 CIELAB is allowed which is defined in ISO/IEC 15775 for colour copiers. Therefore the visual properties under D65 are equal to the properties under D50. There is no need for an additional assessment in the office area for the illuminant D50, compare the calculations for D65 and D50 (24 pages, 700 kByte)

http://www.ps.bam.de/De19/10L/L19e00NP.PDF

9. Definition and basis for colours of equal blackness *N**

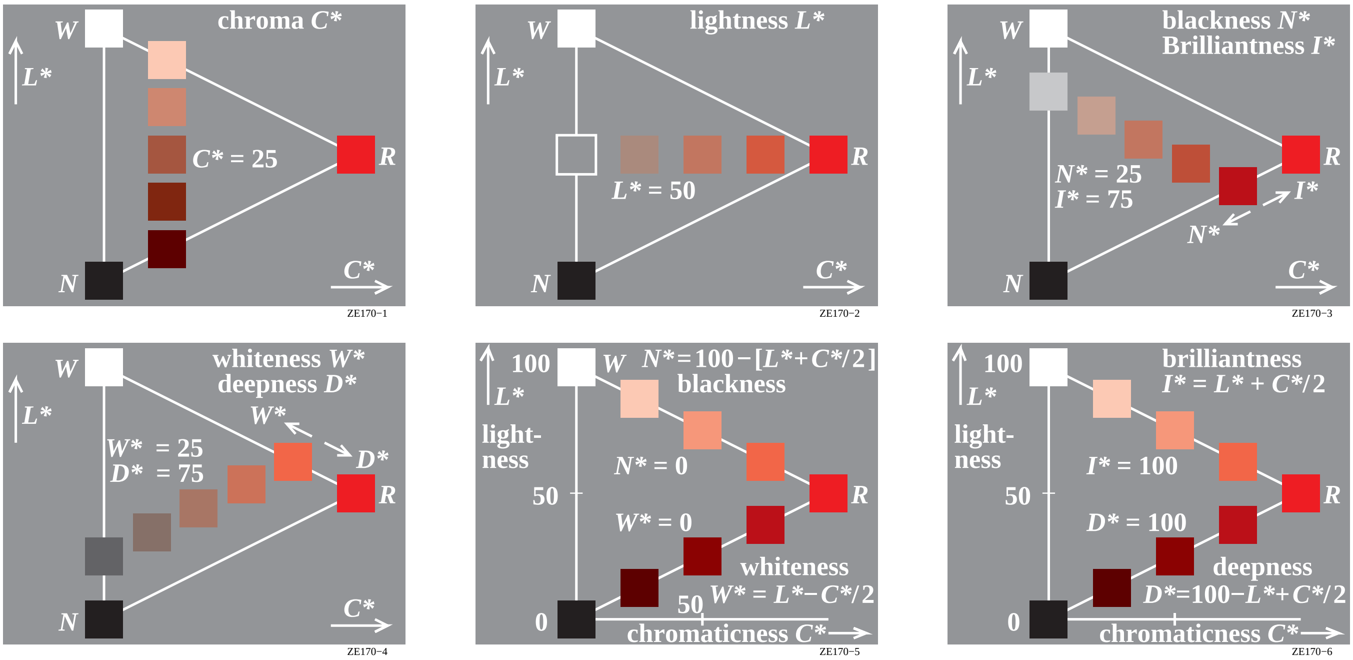


Figure 20: Colour attributes *N, *I** *W** *D** and relation to chroma *C** and lightness *L****

Figure 20 shows the relations between the colour attributes blackness N^*, brilliantness I^*, whiteness W^*, colour deepness D^*, chroma C^* and lightness L^*. The colour attributes blackness N^* and brilliantness I^* as well as whiteness W^* and colour deepness D^* are complementary to each other, which means it is valid:

$$N^* = 100 - I^* \qquad (1)$$

$$W^* = 100 - D^* \qquad (2)$$

Further there is the triangle equation of *Ostwald* (1930)

$$N^* + W^* + C^* = 100 \qquad (3)$$

Additionally there are linear relations between the four coordinates N^*, W^*, L^* and C^*

$$N^* = 100 - [\, L^* + 0{,}5\, C^* \,] \qquad (4)$$

$$W^* = L^* - 0{,}5\, C^* \qquad (5)$$

Nearly all colorimetric colour systems use three colour attributes for the specification of colours. Nearly all use the hue as the first and most important colour attribute and distinguish in the choice of the two others. In the CIELAB colour system which is based on the *Munsell* colour order system the two colour attributes chroma C^* and lightness L^* are preferred. In the *Swedish Natural colour system NCS* the blackness N^* instead of this lightness L^* has been chosen as the more important colour attribute.

Figure 20 shows in all parts *separate* colour samples on a grey background. In many cases *adjacent* colours on a grey background are shown. In both cases the colour difference is viewed and evaluated as the important colour attribute for reproduction.

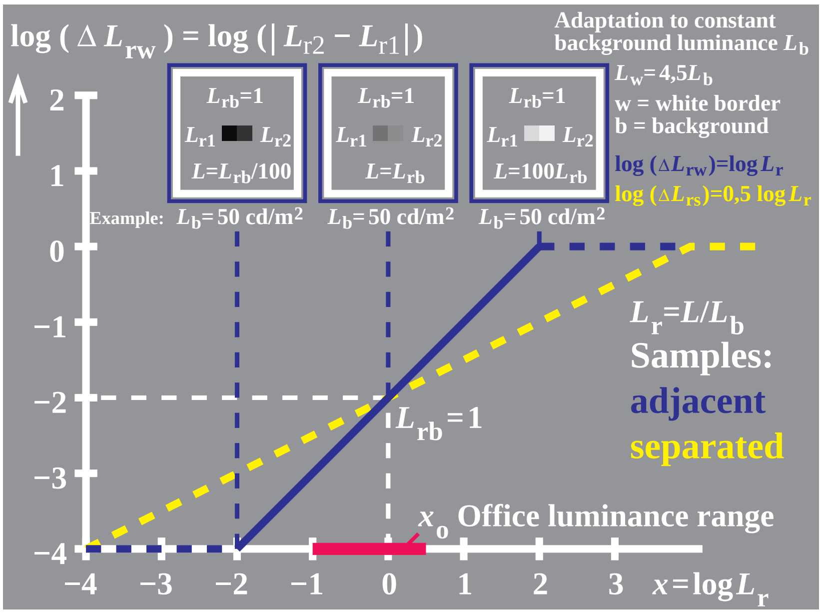
For both viewing situations *separated* and *adjacent* colours the colorimetry has developed different colour metrics for the description of the colour differences. In the case of *separate* colour samples the CIELAB colour difference formula of 1976, see CIE 15, and in the case of *adjacent* colour sample the colour difference formula CIEDE2000, see CIE 142 is recommended.

The reason for the different metrics for the description of the different experimental results are to a high degree unknown. CIEDE2000 is recommended for small colour differences of *adjacent* colours in the range between 0 <= ΔE^*_{ab} <=5. For the *Munsell* colour order system the colour differences of the samples of is near $\Delta E^*_{ab} = 10$ for both the lightness and the chroma direction in any hue plane. Therefore in general CIELAB seems appropriate for large colour differences and CIEDE2000 for small colour differences. Additionally CIELAB seems more appropriate for *separate* colours and CIEDE2000 more for *adjacent* colours in a mean grey background.

Colorimetric supplement for DIN 33872-1 to -6

The following colorimetric model of this paper describes achromatic thresholds for the viewing situations *separate* and *adjacent achromatic* colours. Later in this paper the model describes properties of *chromatic* colours.

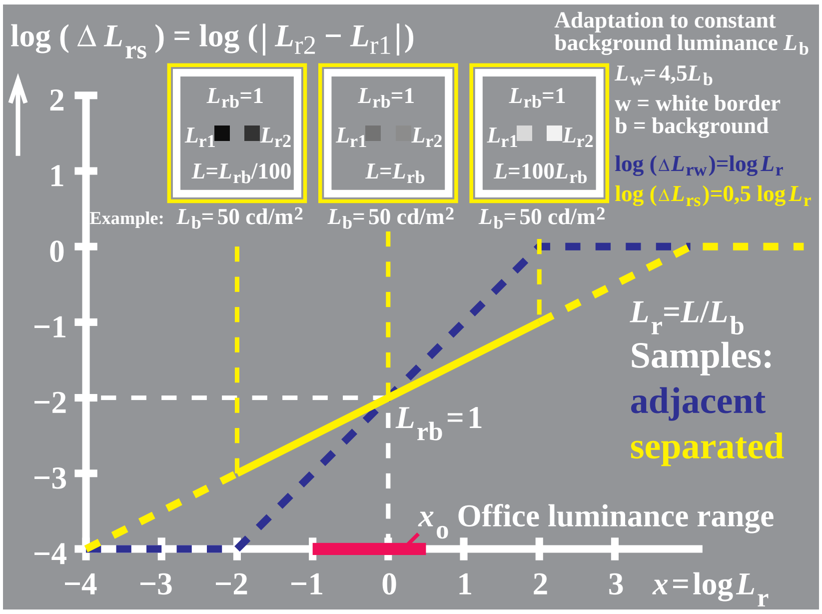
At first we will repeat the law of *Weber-Fechner* (*Fechner* 1860) which describes the colour threshold for achromatic *adjacent* colours. Then we will repeat the law of *Stevens* (1961) which describes the colour thresholds and scaling for achromatic *separate* colours. Then both laws are combined in a new model for achromatic colours. Later in this paper we will extend the model for chromatic *adjacent* and *separate* colours.



ZE290−5

Figure 21 – *Weber-Fechner* threshold ΔL_{rw} and relative luminance L_r for *adjacent* achromatic colours

Figure 21 shows the *Weber-Fechner* threshold ΔL_{rw} as function of the relative luminance L_r for *adjacent* achromatic colours. The viewing situation and the results are shown with ***blue*** colours. At the same time the additional results for the viewing situation *separate* colours are shown by yellow dashed lines.



ZE290−6

Figure 22 – *Stevens* threshold ΔL_{rs} and relative luminance L_r of separate achromatic colours

Figure 22 shows the *Stevens* threshold ΔL_{rs} as function of relative luminance L_r for *separate* achromatic colours. The viewing situation and the results are given in ***yellow*** colours. At the same time the results for the viewing situation *adjacent* colours are shown by blue dashed lines.

The combination of the laws of *Weber-Fechner* and *Stevens* has been described in a model by Richter (2006c). A simple explanation is given in the following:

The colour vision is a *border* vision and the *local* relative adaptation luminance L_{ra} is determined by the mean luminance at the border. The *relative* **adaptation** (index a) luminance L_{ra} at the border of a just noticeable *relative* luminance difference (no. 1 and 2) in the grey relative background luminance $L_{rb} = 1$ is for:
1. adjacent samples and with L_{r1} approximately equal to $L_{r2} = L_r$:

$$\log L_{ra} = 0{,}5\ (\log L_{r1} + \log L_{r2}) = \log L_r \qquad (6)$$

2. *separate* samples and because $L_{rb} = 1$:

$$\log L_{ra} = 0{,}5\ (\log L_r + \log L_{rb}) = 0{,}5 \log L_r \qquad (7)$$

Figure 21 shows a linear relation with the slope 1 for the *Weber* luminance difference ΔL_{rw}

$$\log \Delta L_{rw} = \log L_r \qquad (8)$$

Figure 22 shows a linear relation with the slope 0,5 for the *Stevens* luminance difference ΔL_{rs}

$$\log \Delta L_{rs} = 0{,}5 \log L_r \qquad (9)$$

With the equations (6) and (7) there is a combination of the laws of *Weber-Fechner* and *Stevens* which leads to a linear model relation for both viewing situations *adjacent* and *separate:*

$$\log \Delta L_r = \log L_{ra} \qquad (10)$$

or with linear terms

$$\Delta L_r = \mathrm{const}\ L_{ra} \qquad (11)$$

In the following the results are applied also for chromatic colours. A description for the colour attribute equal blackness N^* will be given using the relative luminance L_r and the purity p.

Experimental results of *Evans* (1974) and physiological models of colour vision seem to indicate that the colour attribute blackness N^* is of equal or higher importance compared to the lightness L^*. For naive observers it is more difficult to order colour samples on a grey or a white background according to the colour attribute lightness L^* in comparison to the colour attribute blackness N^*. This is one reason that in the *NCS* colour system the colour attribute blackness N^* is preferred compared to the lightness L^*.

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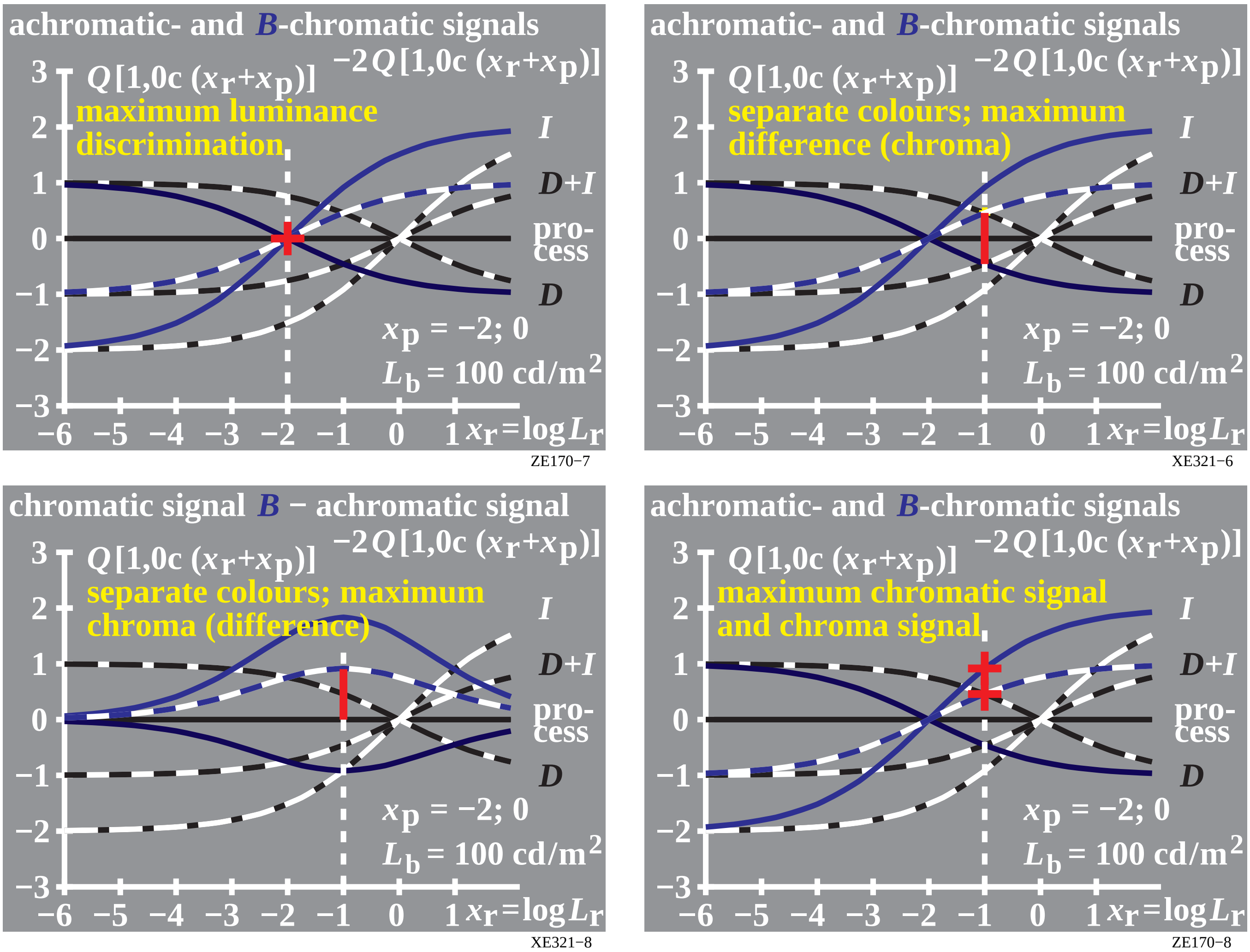

Figure 23 – Achromatic and chromatic signals for achromatic and blue colours

Figure 23 shows achromatic and chromatic signals as function of relative luminance L_r for both achromatic colours and a blue *spectral* colour with the purity p = 100 (or x_p = – log p = –2).

According to *Valberg* (2005) there are three physiological processes: *I* (Increment), *D* (decrement) and sum *(D+I).* For the achromatic colour the signals are shown in black and white. For a blue *spectral* colour the signals are shown in blue. The signals for the blue *spectral* colour are shifted by two logarithmic units to the left.

Therefore for achromatic colours the maximum luminance discrimination (maximum difference of the signals of the *I*-process) is at the grey background luminance L_{rb} = 1. For blue spectral colours the maximum luminance discrimination is shifted by two log units to the left *(top left).*

Figure 23 shows further a mark at the largest difference between the blue and achromatic signal *(top right)* for the *D+I*-process which is calculated for all relative luminance *(bottom left)* and may be interpreted as chroma signal. Two marks show the relative luminance of the maximum chromatic and chroma signals for the *I*- and *D+I*-process *(bottom right)*. The model calculates the difference of the two signals for the blue and achromatic colours at any relative luminance. For the *I*- and the *D+I*-process the maximum chromatic and chroma signals are shifted only one (and not two) log units to the left *(bottom right)*, compare *Richter* (2006c).

Among the surface colours there are only very dark blue colours of higher purity p. The more chromatic blue surface colours may have a purity ten times less compared to the spectral colour. Because of 4% surface reflection for all mate surface colours there are in a grey background with the reflection of 20% only relative luminances of 20% compared to grey (4% of 20%). Therefore the properties for the 1% relative background field luminance shown in Figure 23 can not be tested with surface colours. Only optical devices with lights and colour lasers are appropriate for this test.

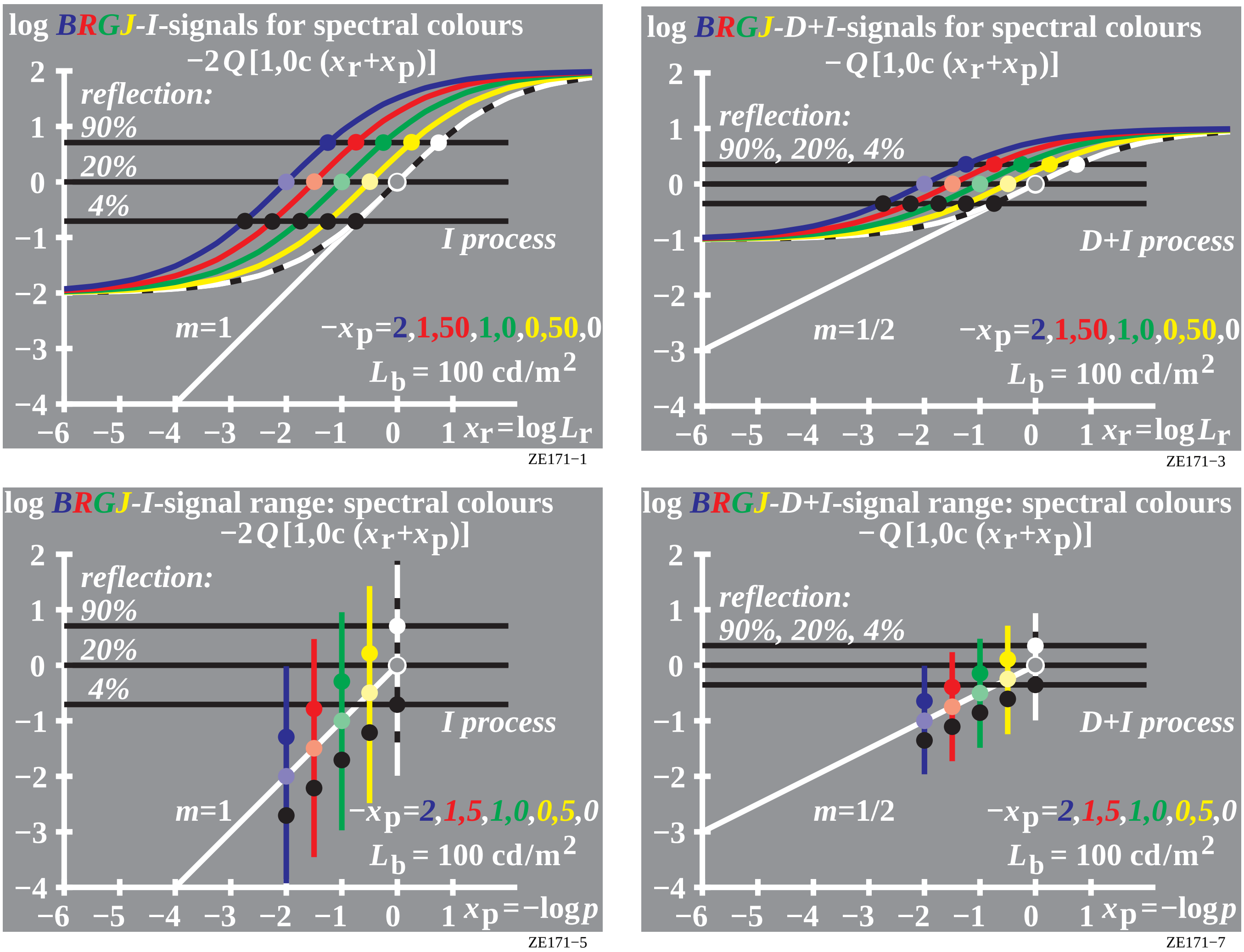

Figure 24 – Physiological signals and range for changes of relative luminance and purity

Figure 24 shows physiological signals with the slope m = 1 (*Weber-Fechner* law) and with the slope m = 0.5 (*Stevens* law), see *top left and right.* A model which combines the two laws of *Weber-Fechner* (*Fechner* 1860) and *Stevens* (1961) for the achromatic colour series black–grey–white was developed by *Richter* (2006c). This model leads to the equations (10) or (11). This model for achromatic colours will be extended now for the chromatic colour series between Black and the colours Yellow, Green, Red and Blue. The purity p of these series increases from p = 1 to p =100. On a log scale then with the log purity x_p = – log p the range of x_p is between –2 and 0, see figure 24.

Figure 24 shows the physiological signals I (=Increment) and D+I (sum process) with the slopes m = 1 and m = 0,5 (*top left and right*). The border vision and the *local relative* adaptation luminance L_{ra} are the basis for the slopes m = 1 and m = 0,5 near the background luminance. This is known from Figures 21 and 22. Additionally the signals are described here for very low and very large *relative* luminance, compare *Richter* (1996). The relative luminance range is several log units larger compared to the relative luminance range used in offices, compare Figures 21 and 22.

It is for example well known from the colorimetry of the colour space CIECAM02, see CIE 159, that the physiological signals are reduced with decreasing background luminance. Similar for increasing background luminance the signals increase. The signal range between the maximum and minimum value increases with approximately an exponent of the value 1/6 as function of background luminance, compare *Richter* (1996).

It is less known in colorimetry that the physiological signals of spectral colours are shifted to the *left* with increasing purity p. Purity p increases from white to the spectral colours between the value 1 and a maximum value depending on the dominant wavelength. The colorimetric experimental results of *Evans* (1974) lead for achromatic colours to a purity value equal to the value 1, and for chromatic spectral colours the values are approximately 2 for yellow, 10 for green, 20 for red and 100 for blue.

The purity shift by two logarithmic units for a blue spectral colour is shown in all parts of Figure 24. In the lower two figures the purity is used in x_p = – log p instead of the luminance in x_r = log L_r.

The local relative adaptation luminance L_{ra} has the value 1/100 for the blue *adjacent* spectral colours. For the blue spectral *separate* colours in a grey background the local relative adaptation luminance L_{ra} has only the value 1/10 compared to the adaptation luminance of the background.

The local adaptation at the border leads to the half adaptation luminance and to the slope *m*=0,5 in Figure 25 (*bottom right*) in comparison to *m*=1 in Figure 24 (*bottom left*).

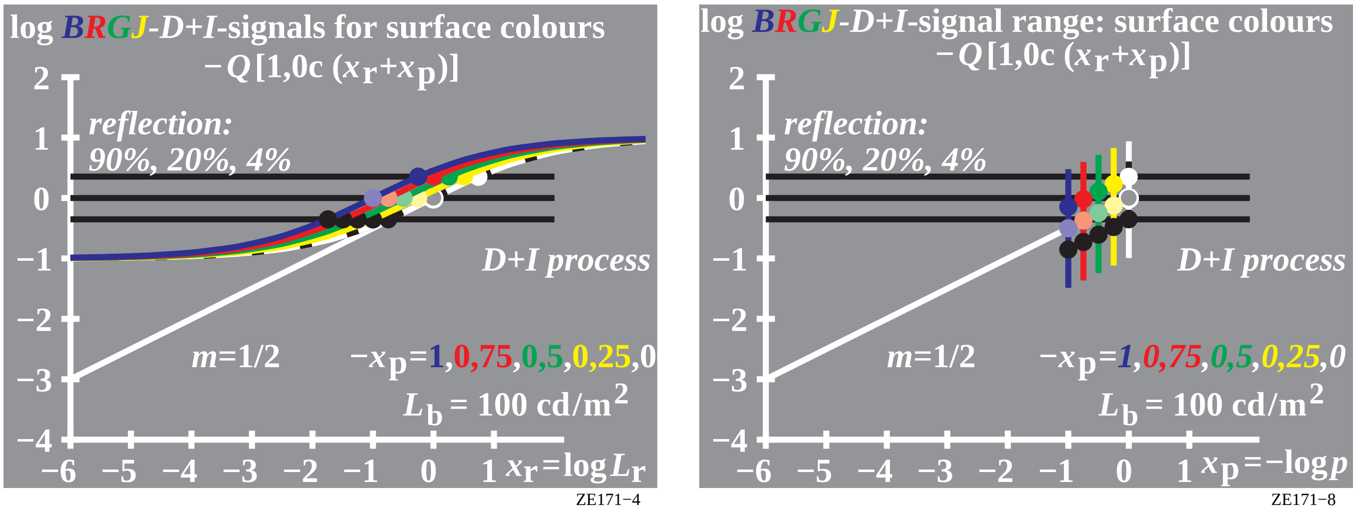

Figure 25 – Physiological signals as function of relative luminance and purity for surface colours

Figure 25 shows a model of the physiological signals as function of relative luminance (L_r) and purity *(p)* for *surface colours*.. The purity *p* for blue spectral colours has approximately the value 100. For x_p this leads with x_p = – log *p* to the value – 2, compare Figure 24. For blue surface colours the purity reasches the maximum value 10. For x_p this leads with x_p = – log *p* to the value – 1, compare Figure 25. The changes for the relative luminance (L_r) and the purity *(p*) are shown in figure 25 for these surface colours.

It is appropriate to interprete the colour series with decreasing "lightness" signals as function of the purity *p* in Figure 25 (*bottom right)* as colours of equal blackness *N**. A interpretation as "colours of equal lightness *L**" is forbidden, because the relative luminance L_r decreases for these series. For the description of such a series it is valid according to Figure 25 *(bottom right).*

$$\log L_{ra} = 0 \text{ for } \log p = 0 \quad (12)$$

und therefore

$$\log L_{ra} = –0{,}5 \log p = 0{,}5\, x_p \quad (13)$$

and with

$$\log L_{ra} = 0{,}5\,(\log L_r + \log L_u) = 0{,}5 \log L_r \quad (14)$$

it follows

$$\log L_r = – \log p \quad (15)$$

or

$$L_r = 1 / p \quad (16)$$

Therefore the relative luminances L_r, which we interprete in this model as colours of equal blackness *N** = constant, are inverse proportional to the purity *p*. A colorimetric confirmation of this model output is missing up to now. The experimental results of *Evans* (1974) and of the *NCS* colour system may contribute to the confirmation of this model result.

We will study again the experimental results for colours of the blackness cero (G0-colours) of *Evans* (1974) in the *Munsell* and CIELAB colour system. For achromatic colours of the blackness *N** = 0 the lightness value is *L** = 100. With increasing chroma C*ab the lightness L* decreases for equal blackness according to the following formula, compare Figure 20:

$$N^* = 100 – (L^* + 0{,}5\, C^*_{ab,a}) \quad (17)$$

The lightness is for the blackness *N** = 0 given by

$$L^*_{N^*=0} = 100 – 0{,}5\, C^*_{ab,a} \quad (18)$$

The lightness is for the blackness *N** = 50 given by

$$L^*_{N^*=50} = 50 – 0{,}5\, C^*_{ab,a} \quad (19)$$

It was the aim of this model to present the unique property of the colours of equal blackness *N** in comparison to the colours of equal lightness *L**. In the *NCS* colour system the colour attribute blackness *N** has been choosen instead of the lightness *L** of the CIELAB colour system or the lightness (Value) of the *Munsell* colour system (*Value* = 10 corresponds to *L** = 100).

One can mention the following reason for the development of the visual system in the direction of blackness *N**: All chromatic colours decrease in their lightness *L** approximately *linear* with increasing chroma $C^*_{ab,a}$, compare equation (18) or (19). Therefore for the colour vision it is appropriate to create a special colour attribute for these colour series. This leads to the colour attribute blackness *N**, which is described in CIELAB by the simple equation (4) in agreement with *Evans* (1974) and *Richter* (1980). For the relationship between the three colour attributes *N**, *L** and *C** the equation (4) is valid.The relationship between these three colour attributes is shown in Figure 20.

10. Definition and basis for elementary colours and elementary hue *E**

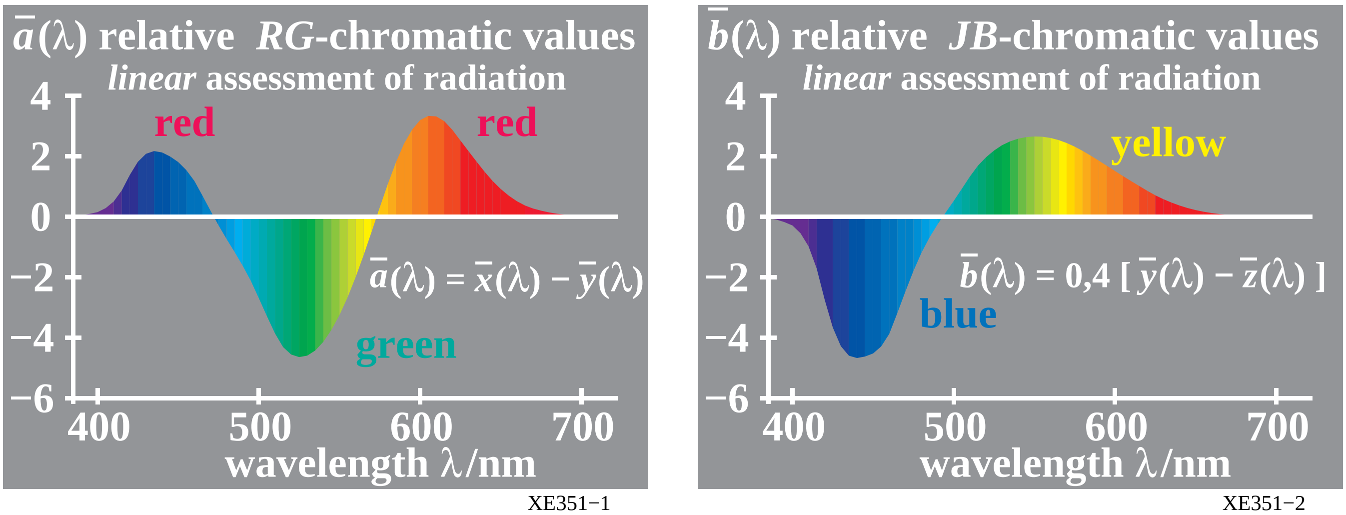

Figure 26 – Elementary hues *BGJ* in the spectrum according to the model of *Hurvich* and *Jameson*

Figure 26 shows the two spectral chromatic values blue–yellow and red–green which are described by two simple transformations of the CIE tristimulus values according to *Hurvich* (1981). The transformation equations are included in Figure 26. The values of the two chromatic functions are cero at approximately the wavelength 475nm, 503nm and 575nm. Colours of these wavelength describe the three spectral elementary colours Blue *B*, Green *G* and Yellow *J*.

The cero points correspond to the visual criteria for the elementary hues *Blue* and *Yellow* as neither reddish nor greenish *(left)* or to the elementary hue *Green* as neither bluish nor yellowish *(right)*. For the elementary colour *Red* the cero value appears if an appropriate mixture is used of the two spectral colours 700nm and 400nm.This mixture is on the purple line in the CIE chromaticity diagram and can be described by the complementary wavelength λ_c = 494nm compared to CIE chromaticity of D65.

In the following we will look at experimental data of surface colours instead of spectral colours. *Miescher* (1948) has developed three symmetric hue circles of surface colours with 24, 96 and 400 steps. The four elementary colours have been defined with 28 observers under natural daylight (north sky). At that time CIE illuminant C which is now replaced by CIE standard illuminant D65 has served for the colorimetric calculations. The CIE measurement data for CIE illuminant C are similar to the CIE measurement data for CIE standard illuminant D65 if the samples are not fluorescent. All samples of the *Miescher* elementary hue circle are free of fluorescence.

Table 5 – *Miescher* elementary hues and corresponding *Munsell* notations

Four elementary colours and four intermediate colours		CIE tristimulues values and chromaticity for illuminant C and 2 degree observer				
Hue circle	**Miescher/Munsell hue**	X_C	Y_C	Z_C	x_C	y_C
Elementary Red R	08/6.0R−V5	32,53	18,11	5,32	0,5813	0,3236
red yellow R50J	05/3.7YR−V5	60,31	45,44	5,55	0,5419	0,4083
Elementary Yellow J	02/8.5Y−V5	70,52	77,82	10,18	0,4449	0,4909
yellow green J50G	23/9.5GY−V5	25,23	45,15	14,00	0,2990	0,5351
Green G	20/5.9G−V5	8,51	20,24	16,28	0,1890	0,4495
green blue G50B	17/8.5BG−V5	8,83	14,56	31,55	0,1607	0,2650
Blue B	14/5.3PB−V5	11,92	9,35	48,79	0,1701	0,1335
blue red B50R	11/7.4P−V5	16,15	8,47	30,90	0,2909	0,1526

XE350−1

Table 5 shows the CIE data of the *Miescher* elementary colours and the corresponding *Munsell* notations for the lightness *L** = 50 (*Munsell Value V* = 5). In a first approximation the elementary hues Red *R*, Yellow *J*, Green *G* and Blue *B* correspond to the *Munsell* notations *5R, 5Y, 5G* and *5PB*. These hues have been selected for the CIE-test colours no. 9 to 12 (Red, Yellow, Green and Blue) in CIE 13.3 (“colour rendering”).

Table 6 – *Miescher* elementary hues, *Munsell* notations, and dominant wavelength

.

Elementary and intermediate colours					Munsell Notation (Value 5) and dominant wavelength	
Hue	**Observer K.R.**	**G.W.**	**A.V.**	**K.M.**	**mean Munsell value and dominant wavelength**	**correction for Bezold-Brücke effect**
Red R	6.5R 700	5.8R 494c	6.0R 494c	5.8R 494c	6.0R 494c	494c 700 495c
R50J	3.75YR 592	4.2YR 591	3.5YR 593	3.7YR 592	3.7YR 592±1	590±2
Yellow J	7.5Y 575	8.5Y 574	10.0Y 572	10.0Y 572	8.5Y 574±2	572±2
J50G	10GY 542	8.75GY 550	9.0GY 548	0.5G 536	9.5GY 544±8	542±10
Green G	6.0G 502.5	5.0G 504	6.0G 502.5	6.7G 501.5	5.9G 503±2	503±2
G50B	7.5BG 488.5	8.75BG 487.5	8.0BG 488	10.0BG 486.5	8.5BG 488±2	489±2
Blue B	5.6PB 472	5.0PB 474.5	5.1PB 474	5.0PB 474.5	5.3PB 474±2	472±2
B50R	7.5P 558c	7.5P 558c	7.0P 560c	7.5P 558c	7.4P 559c±1	559c±1

XE350−7

Table 6 shows experimental *Munsell* notations and the dominant wavelength for elementary and intermediate colours of the *Miescher* elementary hue circle according to *Richter* (1969). In Table 6 the wavelengths near 475nm, 503nm and 575nm represent the elementary hues Blue *B*, Green *G* and Yellow *J.*

Table 7 – Colorimetric data of the CIE-test colours no. 9 to 12 as reference for the four elementary hues

Elementary colour and CIE illuminant		CIELAB data, CIE tristimulus values and CIE chromaticity for the CIE standard illuminant D65 and D50 and the 2 degree observer									
CIE-test colour	**Ill.**	L^*	a^*	b^*	C^*_{ab}	h_{ab}	X	Y	Z	x	y
09, Red R	D65	40,04	58,98	28,32	65,43	25,7	20,64	11,27	4,34	0,5693	0,3110
10, Yellow J		81,30	−2,99	71,82	71,89	92,4	54,89	59,01	12,02	0,4359	0,4686
11, Green G		52,27	−42,40	13,64	44,54	162,2	12,15	20,38	15,34	0,2538	0,4258
12, Blue B		30,52	1,21	−46,35	46,37	271,5	6,24	6,45	27,59	0,1550	0,1601
09, Red R	D50	41,88	62,00	31,82	69,69	27,2	23,31	12,42	3,24	0,5982	0,3188
10, Yellow J		81,97	1,81	71,59	71,61	88,5	58,84	60,24	9,50	0,4576	0,4685
11, Green G		51,62	−41,12	11,52	42,70	164,4	12,10	19,81	11,95	0,2759	0,4515
12, Blue B		29,20	−5,28	−49,34	49,62	263,9	5,25	5,92	21,25	0,1621	0,1825

XE350−3

Table 7 shows the colorimetric data of the CIE-test colours no. 9 to 12 which show a good approximation of the four elementary hues for CIE standard illuminant D65 and probably also for the CIE illuminant D50. The CIE-test colours correspond approximately to the colours with the *Munsell* notations *5R, 5Y, 5G* and *5PB* and serve as reference for the four elementary hues Red *R*, Yellow *J,* Green *G* and Blue *B*.

The spectral data of the CIE-test colours are specified in CIE 13.3. There are real samples of the *BAM* and other sources which approximate the spectral reflections of the CIE-test colours. Additionally there are metameric samples for D65 available, for example one set of metameric CIE-test colours has been produced with standard offset colours, in the test charts according to ISO/IEC 15775 for the test of colour copiers.

11. Equal hue triangles and 16 step colour scales

In image technology the CIELAB hue angles $h_{ab,a}$ = 26, 92, 162, and 272 are used for *RJGB*, to produce the elementary hues for these angles. For a real printer 10 pages with the six elementary colours *X* = *OYLCVM*, and the four elementary hues *X* = *RJGB* have been produced. For a printer output example the following file (350 kByte, 10 pages) may be used.

http://www.ps.bam.de/VE39/10L/L39E00NP.PDF

The CIELAB hue angles $h_{ab,a}$ are shown in the output for both types of hue angles: the six basic device hues *X* = *OYLCV (page 1 to 6)*, and the four CIE elementary hues *X* = *RJGB (page 7 to 10)* which are approximately produced by the printer.

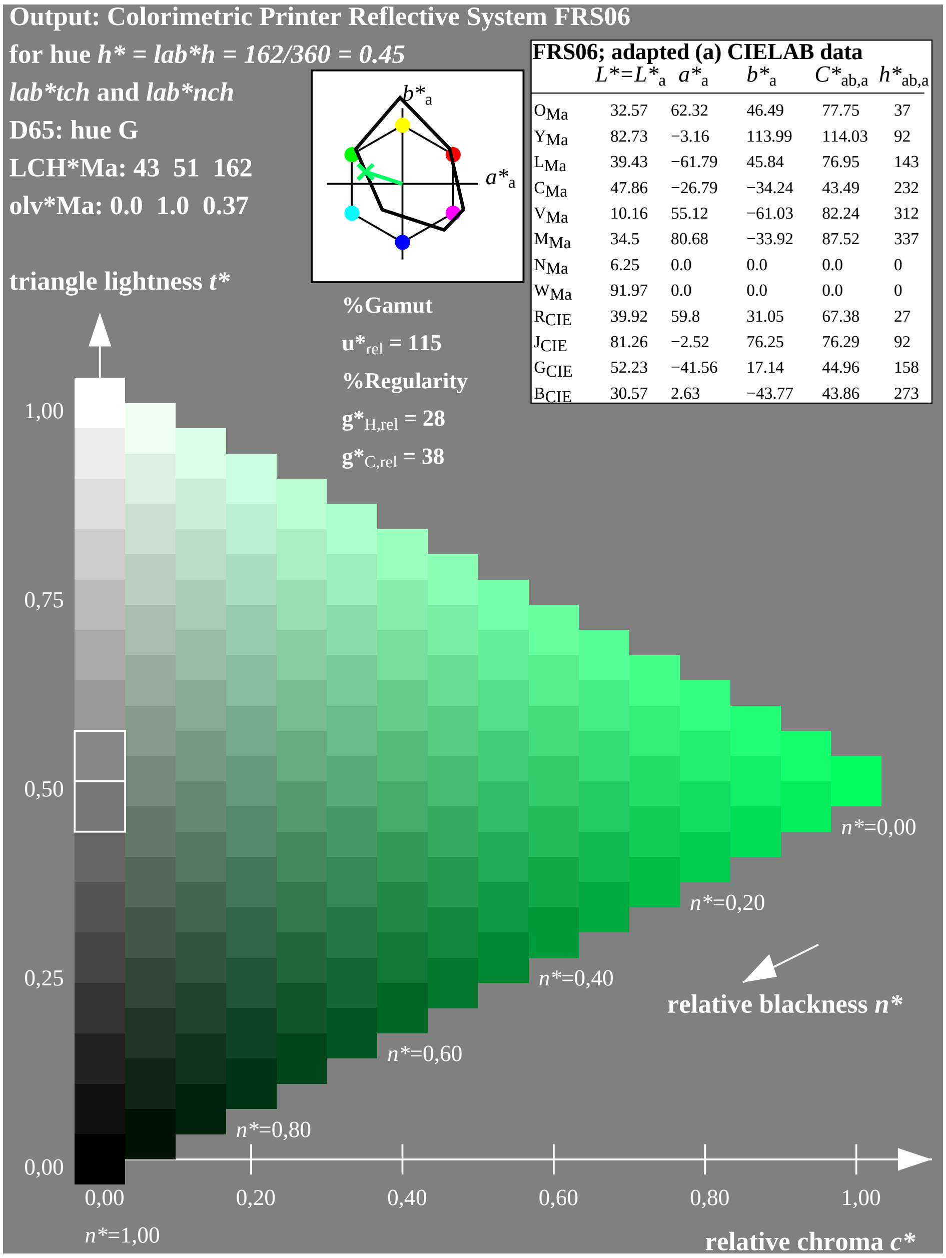

Figure 27 – 16 step colour scales in a hue triangle with the elementary hue green *G*

Figure 27 shows the 16 step colour series in a hue triangle with the elementary hue green of the CIELAB hue angle $h_{ab,a}$ = 162 degree. The three 16 step series black–white, white–green, and green–black shall be equally spaced in the output. This is for many devices often not the case. ISO/IEC TR 19797 describes a linearisation method which reaches this aim in the output.

A hue triangle includes the colour attributes *relative* whiteness ***w****, *relative* chroma ***c****, and *relative* blackness ***n**** which are all three in the range between 0 and 1. According to *Ostwald* (1930) there is the colorimetric relation

relative whiteness + *relative* blackness + *relative* chroma = 1

or

$$w^* + n^* + c^* = 1$$

In a hue triangle two of the three colour attributes are sufficient to describe the location in the triangle.The *Swedish Natural Colour System* NCS (1982) has choosen in a hue triangle as primary attribute the elementary hue text ***u**** and additionally the *relative* blackness ***n**** and the *relative* chroma ***c****. One can compare these colour attributes with the specifications of the *Munsell* colour system: hue, chroma, and lightness (*Value*). The definitions of the triangle lightness ***t**** and the *relative* blackness ***n**** in figure 27 is different compared to the definition of the CIE lightness L^*, and CIE chroma C^*_{ab}, see Richter (2006a).

The 16 step colour series in a hue triangle requires at first the CIELAB data of the six chromatic device colours *OYLCVM*, and the two achromatic device colours *NM*. The output of the elementary colours require the hue angles 26, 92, 162, and 272 degree of the four elementary colours *RJGB* in CIELAB. Figure 27 includes these data as table *(top right)*. For the production of elementary green *G* the three *rgb* input data 0, 1, and 0 are interpreted as elementary *rgb** input data. For the output of the elementary colour green *G* the device related data olv^*_{Ma} = 0, 1, and 0.37 are calculated, see data in figure 27.

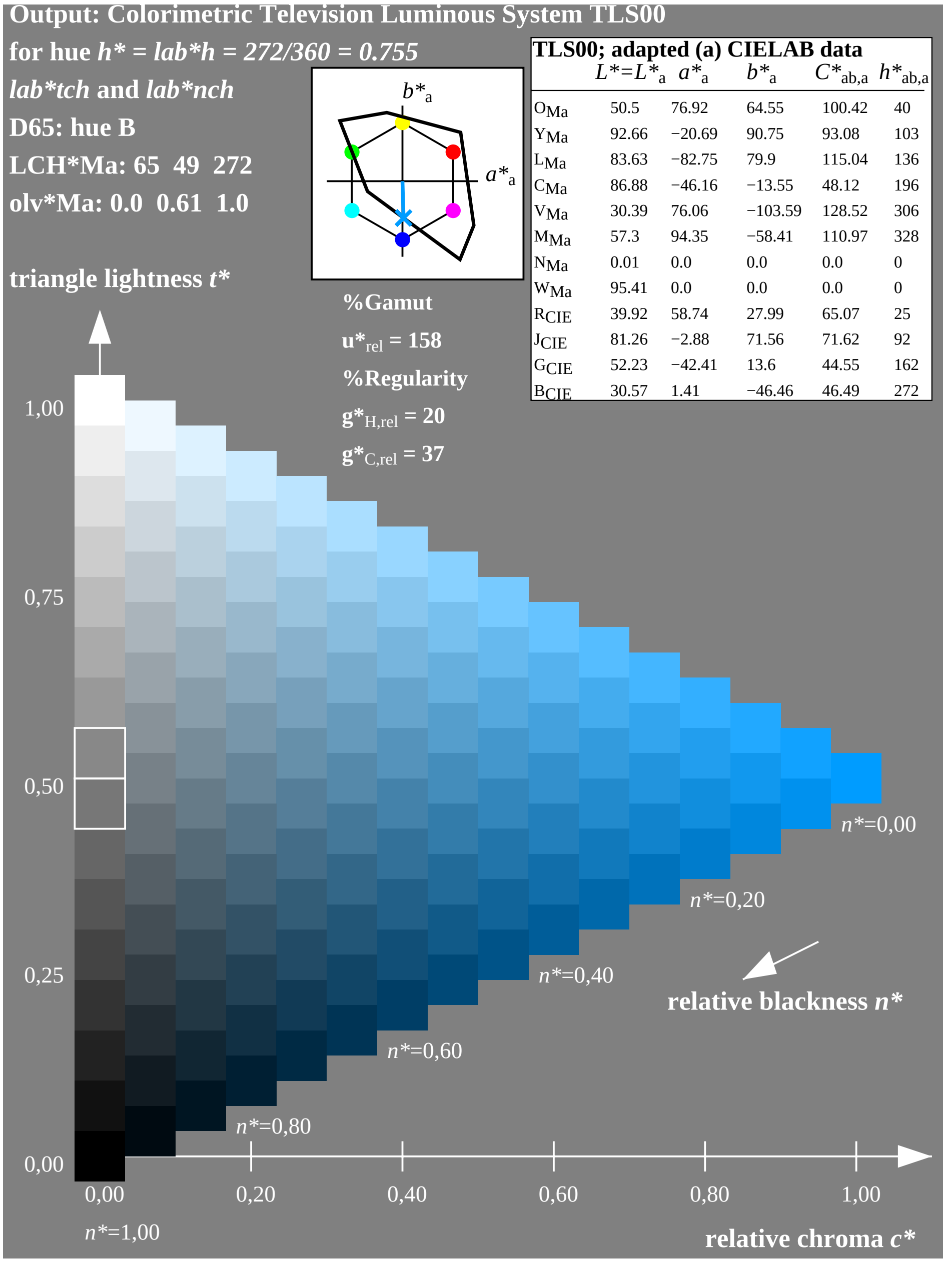

Figure 28 – 16 step equally spaced colour series for the elementary hue blue *B* on a standard monitor TLS00

Figure 28 shows the hue triangle with 16 step equally spaced colour series of the elementary hue blue *B* for the standard monitor TLS00. For the standard monitor TLS00 the ***rgb**** coordinates (0, 0, 1) are transferred to the ***olv****-coordinates (0, 0,61, 1) with an appropriate *PostScript* program code in the PS and PDF file .

Figure 28 includes the standard colour hexagon with the regular 60 degree hue hexagon. Additionally the standard monitor colours are shown in comparison to this regular hue hexagon. The device colour violet blue *V* has the hue angle 306 degree. This is a large hue difference compared to 272 degree of elementary blue *B*.

For the production of a equally spaced output of the elementary hue blue *B* on the monitor, for example the linearisation method according to ISO/IEC TR 19797 is used. An improved method has been developed by *Witt* (2006). This method works with a colour table, for example of 9x9x9 *rgb* input colours, and the produced *L*a*b** output device colours. With this method 16 step equally spaced colour series can be produced in any hue triangle, for example for the hue angles of the six device colours *OYLCVM*, and the hue angles of the four elementary colours *RJGB*, and additionally for any other hue angle.

In the application of this paper the measured device table *rgb – L*a*b** is included in the *PostScript* code of the test file which linearizes the output of this file or any *PS* or *PDF* file, compare ISO/IEC TR 19797. With this *PostScript* code the *rgb* input data of any *PS* or *PDF* file may be interpreted as either device colour data *(rgb –> olv*)* or as elementary colour data *(rgb –> rgb*)*. In a first step both interpretations and output methods are realized and described for *PostScript* vector code, see Richter (2006b).

12. User coordinates and colour workflow

The standard DIN 33872 includes the output of digital test files on many output devices, for examples on monitors and printers for different viewing situations. In any hue plane the output colours can be described in a good approximation by a hue triangle in the CIELAB colour space with the coordinates *adapted* chroma $C^*_{ab,a}$, and lightness L^*. The following figures show complementary hue planes and usual transfers between two devices in real applications.

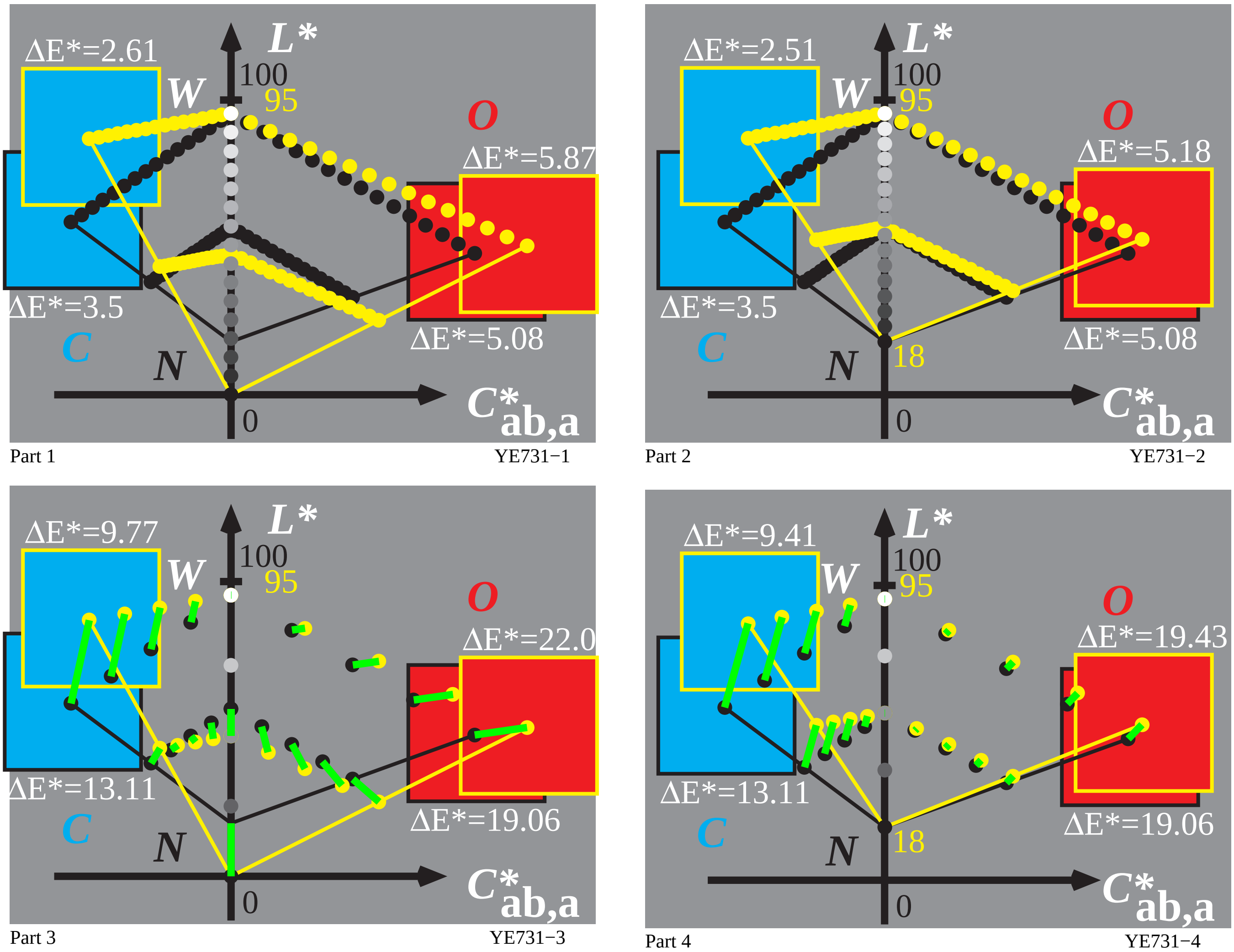

Figure 29 – 16 step colour series of hue planes *O–C* of the two systems TLS00–ORS18 and TLS18–ORS18

Figure 29 shows 16 step *(part 1 and 2)* and 5 step *(part 3 and 4)* colour series for the complementary hue planes *O – C* of the systems TLS00 – ORS18 *(left)* and TLS18 – ORS18 *(right)*. The data ΔE^* describe the CIELAB-colour differences of 16 and 5 steps between neighboring colours.

With the transformation method of DIN 33872 the 16 step colour series are transferred by an *affine* transformation. This transformation is well defined between any two hue triangles. This kind of reproduction remains the most important colour attribute hue constant.However, for example for the cyan hue a darker colour series is produced on the printer *(black balls)* compared to the monitor *(yellow balls)*, compare figure 29 part 2 and part 4.

In figure 29 *(part 3 and 4)* the transformations are shown by green lines for the 5 step series. In a first view the transfers seem to be complex. But the transfer structure and the colorimetric realisation is simple and well defined.

In ISO 9241-306 the *affine* transfer is also applied to different reflections of both the office lighting on the monitor surface and the data projector display. If for example in an extreme case the luminance of the data projector on the display is equal to the luminance of the office lighting on the display area, then the luminance contrast between white and black is 2 : 1. Then the lightness L^* of the gray scale is limited to the lightness range L^* =70 to 95, compare for example the lightness range L^* = 0 to 95 or L^* = 18 to 95 in figure 29 *(left and right)*. A small grey range of 25 CIELAB steps allows still the discrimination of 16 visual grey steps, if the relative luminances are spaced appropriately. In this case this is reached by an approximately *linear* relative luminance of the grey steps, see the example file of ISO 9241-306 (16 pages, 1,7 Mbyte)

http://www.ps.bam.de/ME15/10L/L15E00FP.PDF

In this case it shall be valid: gamma=1 and not gamma=2.2 which is defined in the sRGB colour space of IEC 61966-2-1 for a high luminance contrast range. This high luminance contrast range. is larger 255:1, which can only be realized with a monitor in a dark room with no room light reflections on the monitor surface. At real work places the luminance contrast range may be between 20:1 and 40:1 for both monitors and printers. Therefore the colour space sRGB of IEC 61966-2-1 seems not appropriate for this office case.

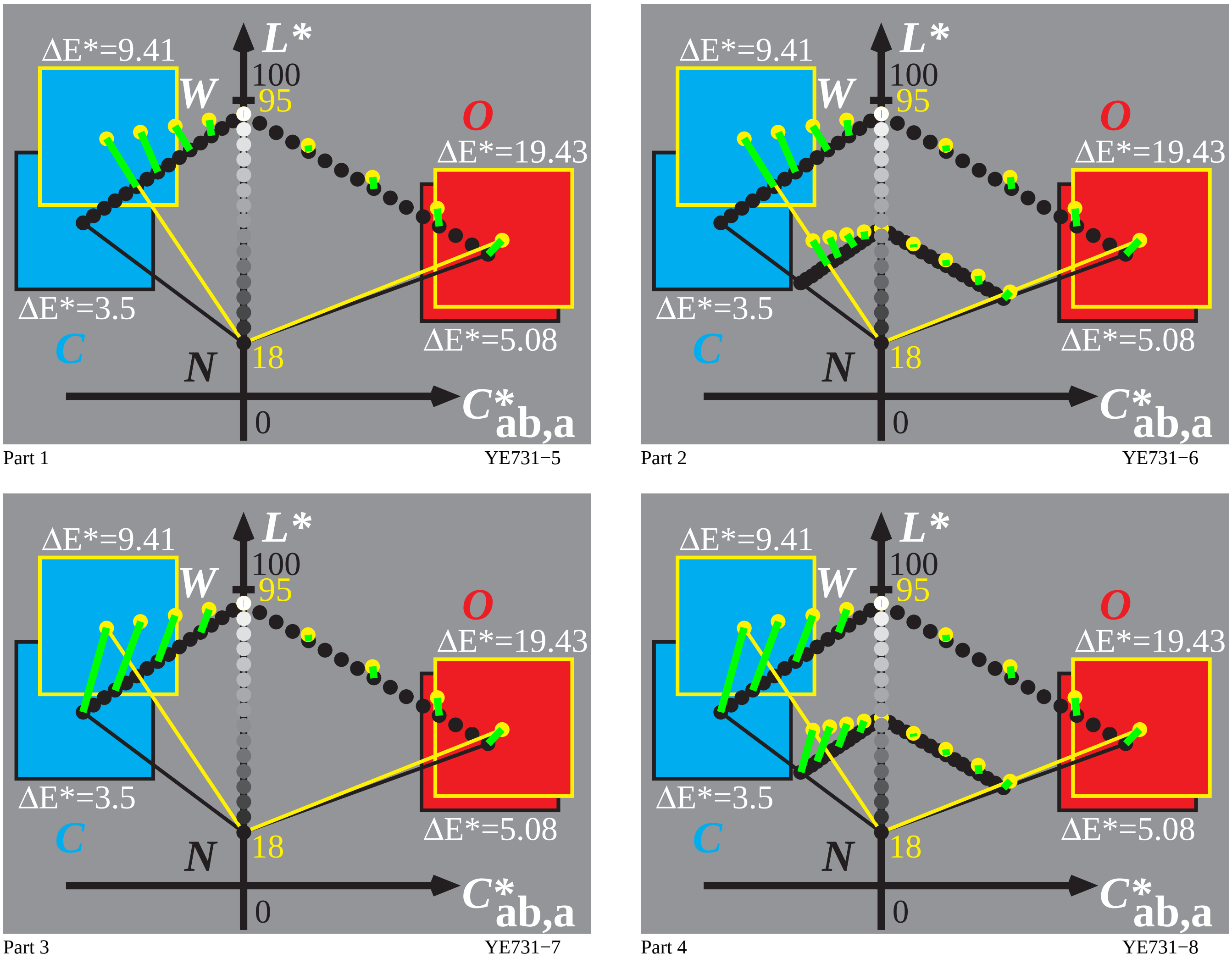

Figure 30 – Complementary hue triangles *O* – *C* for TLS00 and ORS18, and colour management

Figure 30 shows complementary hue triangles for the standard television luminous system TLS18 (yellow balls), and the standard offset reflective system ORS18a *(black balls).* The transformations differ considerably compared to the *affine* transformation in figure 29. Only the *affine* transformation remains the hue, the colour discrimination and the equal *relative* spacing on the different device systems.

Figure 30 part 1 and 3 includes a cutting ("clipping") of monitor colours and transfer on the surface of the printer colour space. Figure 30 part 2 and 4 includes a colour shift also within the colour space similar compared to the *affine* transformation in figure 29. The *affine* transformation in figures 29 and 30 define the affine colour management method.

According to ISO 15076-1 there is a "standard" method for "colour management" which includes the above standard case monitor – printer. In application the *relative* output according to ISO 15076-1 produces for this standard case very different outputs on a printer. According to ISO 15076-1 the many different transfers of figure 30 are possible and additionally many different gamma transformations of the steps are allowed. The different color management modules (*CMMs*) and computer operating systems use many allowed and different transfers according to ISO 15076-1. By clipping of the output differences of 30 CIELAB may appear on the same device. This difference is large compared to the output tolerance of 3 CIELAB for colour copiers according to ISO/IEC 15775.

With the different *CMMs,* for example of the companies *Apple, Adobe, Windows, Heidelberg and others,* different results are produced according to figure 30. Only by the application of the *affine* transformation of figure 29 the results are expected on the same device within the visual and colorimetric tolerance of 3 CIELAB.

13. Colorimetric output specification for the illuminants D50 and D65

Colour measurement is the basis for the colorimetric output specification, for example defined for the 5 and 16 step colour series in DIN 33866-1 Annex G, or in ISO/IEC 15775 Annex G. The test files according to DIN 33872-6 include additionally 9 step series, for example for the hue planes *O–C, Y–V*, and *L–M* which include central grey Z.

The colorimetric output specification is based on the output of the test file of figure 8 in DIN 33872-1 with 24 colour series in the rows A to X each of 17 colour steps. For the corresponding *rgb* test file see (3 pages, 120 kByte)

http://www.ps.bam.de/De11/10L/L11e00NP.PDF

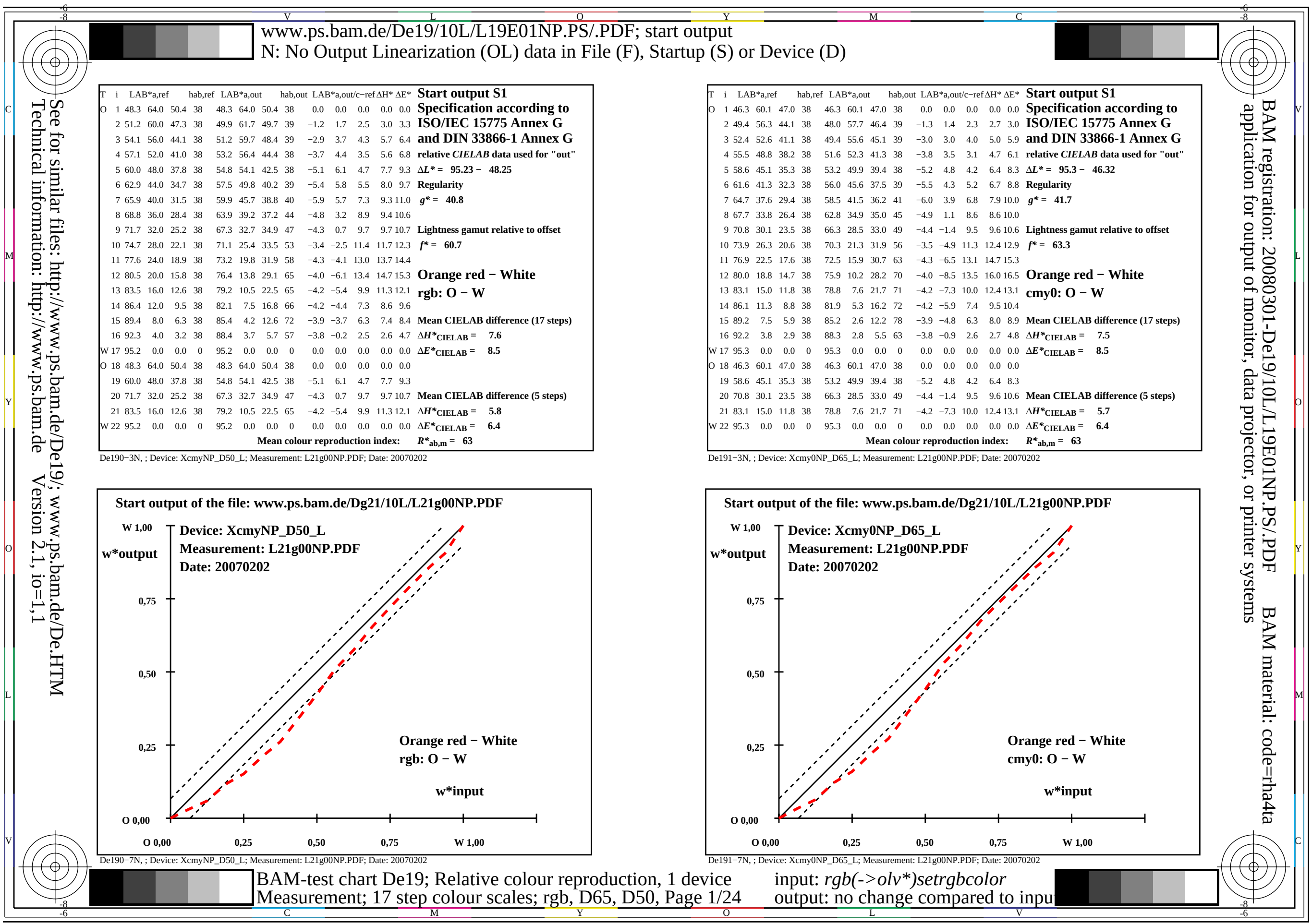

Figure 31 – Colorimetric output specification of the colour series orange red – white for D50 and D65

Figure 31 shows the colorimetric output specification for *rgb* input data. The example shows the 17 step colour series orange red – white of the row A for the device system X. The standard specification is given for the CIE illuminant D50 (*left*) and the CIE standard illuminant D65 *(right*).

The basis for any colorimetric specification requires the output of an *rgb* test file, see for example (3 pages, 120 kByte)

http://www.ps.bam.de/De11/10L/L11e00NP.PDF

or/and the output of a *cmy* test file, see for example (3 pages, 120 kByte)

http://www.ps.bam.de/De21/10L/L21e00NP.PDF

The measured *L*a*b** CIELAB data may be included in three special *PS* files which can be downloaded from the following three internet addresses. If the measured *L*a*b** CIELAB data are included in a *PS* file.Then a *PDF* file is produced from the *PS* file with a *PS* interpreter. All necessary calculations are included in the *PS* program code.

For the colorimetric specification with *rgb* colour data of two device systems F and X for D65 the measured data tables *rgb – L*a*b** are included at the beginning of the following *PS* file, see (*PS* code, 220 kByte)

http://www.ps.bam.de/De1710L/L17e00NP.PS

For the colorimetric specification with *rgb* colour data of two device systems F and X for D65 the following *PDF* file is produced from the *PS* file with the *PS* interpreter *Adobe Acrobat Distiller 3.0*, see (24 pages, 700 kByte)

http://www.ps.bam.de/De17/10L/L17e00NP.PDF

Similar, for the colorimetric specification with *rgb* and *cmy0* colour data of the device system X for D65 the following *PS* file is used and transferred to the *PDF* file, see (220 kByte of *PS* code and 24 pages, 700 kByte of *PD*F file)

http://www.ps.bam.de/De18/10L/L18e00NP.PS

http://www.ps.bam.de/De18/10L/L18e00NP.PDF

Similar, for the colorimetric specification with *cmy0* colour data of the device system X for D50 and D65 the following *PS* file is used and transferred to the *PDF* file, see (220 kByte of *PS* code and 24 pages, 700 kByte of *PD*F file)

http://www.ps.bam.de/De19/10L/L19e00NP.PS

http://www.ps.bam.de/De19/10L/L19e00NP.PDF

14. References

14.1 Standard and similar documents

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CIE15:2004: Colorimetry, 3rd edition

CIE 17.4:1987: International lighting vocabulary, 4th edition (Joint publication IEC/CIE)

CIE 142:2001, Improvement to industrial colour difference evaluation

CIE160:2004, A review of chromatic adaptation transforms

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DIN 5033-3:1992, Farbmessung – Farbmaßzahlen

DIN 5033-4:1992, Farbmessung – Spektralverfahren

DIN 5033-7:1983, Farbmessung – Meßbedingungen für Körperfarben

DIN 5035-2:1990, Beleuchtung mit künstlichem Licht – Richtwerte für Arbeitsstätten in Innenräumen und im Freien

DIN 6164:1980, DIN-Farbenkarte – System der DIN-Farbenkarte für den 2^{o}-Normalbeobachter

DIN 6160:1996, Anomaloskope zur Diagnose von Rot-Grün-Farbenfehlsichtigkeit nach *Nagel*

DIN 6169-1:1976, Farbwiedergabe – Allgemeine Begriffe

DIN 6174:1979, Farbmetrische Bestimmung von Farbabständen bei Körperfarben nach der CIELAB-Formel oder

DIN 6174, Norm-Entwurf, 2006, Farbmetrische Bestimmung von Farbmaßzahlen und Farbabständen im angenähert gleichförmigen CIELAB-Farbenraum

DIN EN IEC 61966-2-1: 2005, Multimedia-Systeme und Geräte – Farbmessung und Farbmanagement – part 2.1: Farbmanagement, RGB-Vorgabe-Farbraum – sRGB

DIN EN IEC 61966-7-1: 2006, Multimedia-Systeme und Geräte – Farbmessung und Farbmanagement – part 7.1: RGB-Farbdrucker

RAL DESIGN Atlas - Color charts with 1688 colour samples 1,7 cm x 1,8 cm, silky glossy

NOTE: The RAL DESIGN Atlas includes colour samples for 36 CIELAB hue angle differences of 10 degree, and for CIELAB chroma and lightness differences ΔC^*_{ab} =10 and ΔL^* = 10 for the CIE standard illuminant D65 and the CIE 10-degree-observer.

ISO 2846–1:1997, Graphic technology – Colour and transparency of ink sets for four-colour-printing – Part 1: Sheetfed and heat-set web offset lithographic printing.

ISO 15076-1:2005, Image technology colour management – Architecture, profile format, and data structure – Part 1: Based on ICC.1:2004-10.

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ISO/IEC TR 19797:2004, Information technology – Office machines – Device output of 16 colour scales, output linearization method (LM) and specification of the reproduction properties. For information and test charts according to ISO/IEC TR 19797 see

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http://opus.kobv.de/tuberlin/volltexte/2006/1405/

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NOTE: The elementary hues are often called unique hues. They are in hue identical with the elementary colours of DIN 33872. The elementary colours of the Natural Colour System NCS also have the elementary hues.

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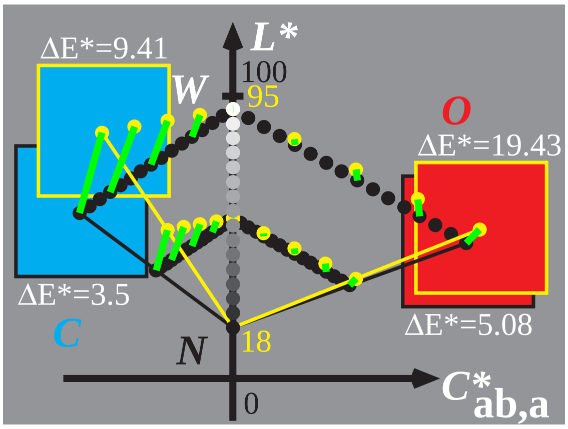

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