

**Line-element examples for grey samples (0.2 ≤ x ≤ 5)**

$F(x)$  is called the line-element function of  $f(x)$ .

The following relations are valid for  $x=Y/Y_0=1/18$ :

$$\frac{dF(x)}{dx} = f(x) \quad (1)$$

$$F(x) = \int \frac{f(x)}{f(x)} dx \quad (2)$$

Example for the normalized tristimulus value  $x=Y/Y_0$ :

$$\frac{d(a \ln(1+b \cdot x))}{dx} = \frac{ab}{1+b \cdot x} \quad (3)$$

$$a \ln(1+b \cdot x) = \int \frac{ab}{1+b \cdot x} dx \quad (4)$$

CEAO-1X

**Line-element examples for grey samples (0.2 ≤ x ≤ 5)**

$f_{00}(x)$  is called the line-element function of  $f_{00}(x)$ .

Both functions are normalized to the surround value:

$$\frac{d[F_{00}(x)]}{dx} = f_{00}(x) \quad (1)$$

$$F_{00}(x) = \int \frac{f_{00}(x)}{f_{00}(x)} dx = \int \frac{b}{1+b \cdot x} dx \quad (2)$$

Example for  $L^*(x)$  and  $\Delta Y$  with  $x=Y/Y_0$ ,  $x_0=1$ ,  $b=6,141$ :

$$L^*(x) = \frac{L^*(x)}{L^*(x_0)} = \frac{\ln(1+b \cdot x)}{\ln(1+b)} \quad (3)$$

$$f_{00}(x) = \frac{\Delta Y}{\Delta Y_0} = \frac{1+b \cdot x}{1+b} \quad (4)$$

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**Line-element equations according to CIE 230:2019**

Colour-discrimination function  $f(x) = \Delta Y = \Delta x \cdot Y_0$  [0]  
 $\Delta Y = (A_1 + A_2 \cdot Y^2) / A_0$ ,  $A_0=1.5$ ,  $A_1=0.0170$ ,  $A_2=0.0058$

$$f_{00}(x) = \frac{\Delta Y}{\Delta Y_0} = \frac{1+b \cdot x}{1+b} \quad b=A_2 \cdot Y_0^2 / A_1 \quad x=Y/Y_0 \quad (1)$$

$$F_{00}(x) = \int \frac{f_{00}(x)}{f_{00}(x)} dx = \int \frac{b}{1+b \cdot x} dx \quad (2)$$

Example for  $L^*(x)$  and  $\Delta Y$  with  $x=Y/Y_0$ ,  $x_0=1$ ,  $b=6,141$ :

$$L^*(x) = \frac{L^*(x)}{L^*(x_0)} = \frac{\ln(1+b \cdot x)}{\ln(1+b)} \quad (3)$$

$$f_{00}(x) = \frac{\Delta Y}{\Delta Y_0} = \frac{1+b \cdot x}{1+b} \quad (4)$$

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**Line-element equations for thresholds and scaling**

Colour-discrimination function  $f(x) = \Delta Y = \Delta x \cdot Y_0$  [0]  
 $\Delta Y = 1/(1+x)(2+x) = 1/(1+x) - 1/(2+x)$ ,  $x=\sqrt{2} \cdot e^{0.10 \cdot (0-x)}$

$$f_{00}(x) = \frac{\Delta Y}{\Delta Y_0} = \frac{1-x}{2} - \frac{2-x}{3} \quad x=Y/Y_0 \quad (1)$$

$$F_{00}(x) = \int \frac{f_{00}(x)}{f_{00}(x)} dx = \int \frac{1}{1+x} dx - \int \frac{1}{2+x} dx \quad (2)$$

Example for  $L^*(x)$  and  $\Delta Y$  with  $x=Y/Y_0$ ,  $x_0=1$ :

$$L^*(x) = \frac{L^*(x)}{L^*(x_0)} = \frac{\ln(1+x)}{\ln(2)} - \frac{\ln(1.5)}{\ln(2)} \quad (3)$$

$$f_{00}(x) = \frac{\Delta Y}{\Delta Y_0} = \frac{1-x}{2} - \frac{1.5}{1.5} \quad (4)$$

CEAO-1X

see K. Richter (1985), Computer Graphic and Colorometry, p. 113-127  
<http://color.li.tu-berlin.de/BU/AMFB.PDF>

**Line-element examples for grey samples (0.2 ≤ x ≤ 5)**

$F_{00}(x)$  is called the line-element function of  $f_{00}(x)$ .

Both functions are normalized to the surround value:

$$\frac{d[F_{00}(x)]}{dx} = f_{00}(x) \quad (1)$$

$$F_{00}(x) = \int \frac{f_{00}(x)}{f_{00}(x)} dx \quad (2)$$

Example for the normalized functions with  $x_0=1$ :

$$F_{00}(x) = \frac{F(x)}{F(x_0)} = \frac{\ln(1+b \cdot x)}{\ln(1+b)} \quad (3)$$

$$f_{00}(x) = \frac{f(x)}{f(x_0)} = \frac{1+b \cdot x}{1+b} \quad (4)$$

CEAO-2X

**Line-element equations according to CIE 230:2019**

Colour-threshold (1) function  $f_0(x) = \Delta Y_1 = \Delta x \cdot Y_0$  [0]

$\Delta Y_1 = (A_1 + A_2 \cdot Y_0^2) / A_0$ ,  $A_0=1.5$ ,  $A_1=0.0170$ ,  $A_2=0.0058$

$$f_{00}(x) = \frac{\Delta Y_1}{\Delta Y_1^0} = \frac{1+b \cdot x}{1+b} \quad b=A_2 \cdot Y_0^2 / A_1 \quad x=Y/Y_0 \quad (1)$$

$$F_{10}(x) = \int \frac{f_{00}(x)}{f_{00}(x)} dx = \int \frac{b}{1+b \cdot x} dx \quad (2)$$

Example for  $L^*(x)$  and  $\Delta Y_1$  with  $x=Y/Y_0$ ,  $x_0=1$ ,  $b=6,141$ :

$$L^*(x) = \frac{L^*(x)}{L^*(x_0)} = \frac{\ln(1+b \cdot x)}{\ln(1+b)} \quad (3)$$

$$f_{00}(x) = \frac{\Delta Y_1}{\Delta Y_1^0} = \frac{1+b \cdot x}{1+b} \quad (4)$$

CEAO-2X

**Line-element equations for thresholds and scaling**

Colour-discrimination function  $f(x) = \Delta Y = \Delta x \cdot Y_0$  [0]  
 $\Delta Y = 1/(1+x)(2+x) = 1/(1+x) - 1/(2+x)$ ,  $x=\sqrt{2} \cdot e^{0.10 \cdot (0-x)}$

$$f_{00}(x) = \frac{\Delta Y}{\Delta Y_0} = \frac{1+b \cdot x}{1+b} - \frac{1+0.5b \cdot x}{1+0.5b} \quad b=1, \quad x=Y/Y_0 \quad (1)$$

$$F_{00}(x) = \int \frac{f_{00}(x)}{f_{00}(x)} dx = \int \frac{b}{1+b \cdot x} dx - \int \frac{0.5b}{1+0.5b \cdot x} dx \quad (2)$$

Example for  $L^*(x)$  and  $\Delta Y$  with  $x=Y/Y_0$ ,  $x_0=1$ ,  $b=1$ :

$$L^*(x) = \frac{L^*(x)}{L^*(x_0)} = \frac{\ln(1+b \cdot x)}{\ln(1+b)} - \frac{\ln(1+0.5b \cdot x)}{\ln(1+0.5b)} \quad (3)$$

$$f_{00}(x) = \frac{\Delta Y}{\Delta Y_0} = \frac{1+b \cdot x}{1+b} - \frac{1+0.5b \cdot x}{1+0.5b} \quad (4)$$

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see K. Richter (1985), Computer Graphic and Colorometry, p. 113-127  
<http://color.li.tu-berlin.de/BU/AMFB.PDF>

**Line-element equations for thresholds and scaling**

Colour-discrimination function  $f(y) = \Delta Y = \Delta y \cdot Y_0$  [0]  
 $\Delta Y = 1/(1+y)(2+y) = 1/(1+y) - 1/(2+y)$ ,  $y=(1+\sqrt{2} \cdot e^{0.10 \cdot (0-y)})$

$$f_{00}(y) = \frac{\Delta Y}{\Delta Y_0} = \frac{y}{2} - \frac{1+y}{3} \quad y=1+Y/Y_0, \quad dy=dx \quad (1)$$

$$F_{00}(y) = \int \frac{f_{00}(y)}{f_{00}(y)} dy = \int \frac{1}{y} dy - \int \frac{1}{1+y} dy \quad (2)$$

Example for  $L^*(y)$  and  $\Delta Y$  with  $y=1+Y/Y_0$ ,  $y_0=2$ :

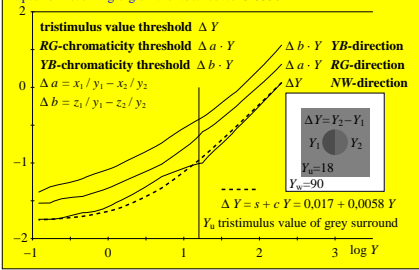
$$L^*(y) = \frac{L^*(y)}{L^*(y_0)} = \frac{\ln(y)}{\ln(2)} - \frac{\ln(1+y)}{\ln(3)} \quad (3)$$

$$f_{00}(y) = \frac{\Delta Y}{\Delta Y_0} = \frac{1-y}{2} - \frac{1.5}{1.5} \quad (4)$$

CEAO-2X

see K. Richter (1985), Computer Graphic and Colorometry, p. 113-127  
<http://color.li.tu-berlin.de/BU/AMFB.PDF>

**NW-achromatic, and RG- and YB-chromatic thresholds as function of Y experiments and data: BAM-research report no. 115 (1985), page 72, see <https://nbn-resolving.org/urn:nbn:de:kobv:b43-3350>**



CEAO-3X

