

Weber-Fechner law in CIE 230:2019 for threshold colour differences of surface colours

The Weber-Fechner law describes the lightness L^* , as logarithmic function of L_r . The Stevens law describes the lightness $L^*_{REL,LAB}$ as potential function of $L_r=3/5$. IEC 61966-2-1 uses a similar potential function $L^*_{TIC} = m L_r^{1/2.4}$.

The Weber-Fechner law is equivalent to the linear equation: $\Delta L_r = c L_r$ [1]
 Integration leads to the logarithmic equation: $L^* = k \log(L_r)$ [2]
 Derivation for $\Delta L_r = 1$ leads to the linear equation: $L_r \Delta L_r = k_1$ ($k_0=46, k_1=63$) [3]
 For colours in offices the standard contrast range is 25:1=90:3.6.

Table 1: CIE tristimulus value Y, luminance L_u , and lightness L^*

Colour (matte)	Tristimulus value Y	office luminance L_u [cd/m ²]	relative luminance $L_r = L_u/L_u$	CIE lightness L^*	relative lightness L^*_r
White W (paper)	90	142	5	94	44
Grey Z (paper)	18	28.2	1	50	40
Black N (paper)	3.6	5.6	0.2	18	-32

For the lightness range between $L^*_r = -40$ and 40 the constant is: $k=40 \log(5) = 57$

Weber-Fechner law in CIE 230:2019 for threshold colour differences of surface colours and two ranges $0.2 \leq L_r \leq 1$ and $1 \leq L_r \leq 5$

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The Weber-Fechner law is equivalent to the linear equation: $\Delta L_r = c L_r$ ($c=0.1$) [1]
 Integration leads to the logarithmic equation: $L^* = k_1 \log(L_r)$ ($k_1=63$) [2]
 Derivation leads for $L^*_r = 1$ to the linear equation: $L_r \Delta L_r = k_1$ ($k_0=46, k_1=63$) [3]
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For the two lightness ranges it is $k_0 = -32 \log(0.2) = 46$ and $k_1 = 44 \log(5) = 63$.

line element of Stiles (1946) with „color values“ L_P, M_D, S_T

three separate color signal functions

$$F(L_P) = i \ln(1 + 9 L_P)$$

$$F(M_D) = j \ln(1 + 9 M_D)$$

$$F(S_T) = k \ln(1 + 9 S_T)$$

Taylor-derivations:

$$\Delta F(L_P, M_D, S_T) = \frac{dF}{dL_P} \Delta L_P + \frac{dF}{dM_D} \Delta M_D + \frac{dF}{dS_T} \Delta S_T$$

$$= \frac{9i}{1+9L_P} \Delta L_P + \frac{9j}{1+9M_D} \Delta M_D + \frac{9k}{1+9S_T} \Delta S_T$$

line element of Vos&Walraven (1972) with „color values“ L_P, M_D, S_T

three separate color signal functions

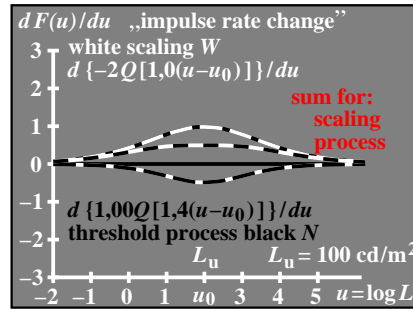
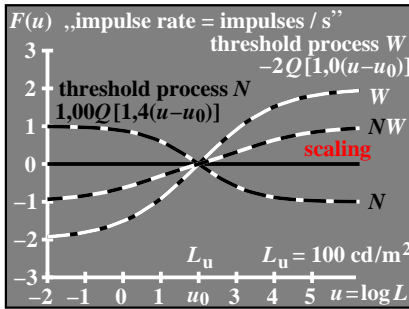
$$F(L_P) = -2i \sqrt{L_P}$$

$$F(M_D) = -2j \sqrt{M_D}$$

$$F(S_T) = -2k \sqrt{S_T}$$

Taylor-derivations:

$$\Delta F(L_P, M_D, S_T) = \frac{dF}{dL_P} \Delta L_P + \frac{dF}{dM_D} \Delta M_D + \frac{dF}{dS_T} \Delta S_T$$

$$\Delta F(L_P, M_D, S_T) = \frac{i}{\sqrt{L_P}} \Delta L_P + \frac{j}{\sqrt{M_D}} \Delta M_D + \frac{k}{\sqrt{S_T}} \Delta S_T$$


functions $q[k(u-u_0)]$

„achromatic signal“-description

with $u = \log L$ ($L = \text{luminance}$)
 $u_0 = \log L_u$ ($L_u = \text{surround luminance}$)

$$q[k(u-u_0)] = 1 + 1/[1 + \sqrt{2} e^{k(u-u_0)}]$$

function values:

$$q[k(u-u_0) \rightarrow +\infty] = 1$$

$$q[k(u-u_0) = 0] = \sqrt{2}$$

$$q[k(u-u_0) \rightarrow -\infty] = 2$$

„achromatic signal“-description

functions $Q_{1m}[k(u-u_0)]$

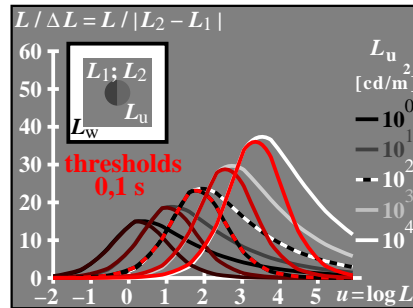
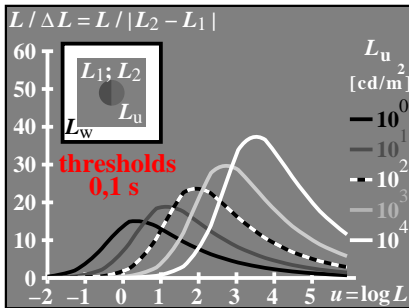
with $u = \log L$ ($L = \text{luminance}$)
 $u_0 = \log L_u$ ($L_u = \text{surround luminance}$)

$$Q_{1m}[k(u-u_0)] = \frac{l}{\ln \sqrt{2}} \ln q[k(u-u_0)] - m$$

function values with $l = m = 1$:

$$Q[k(u-u_0) \rightarrow +\infty] = -1$$

$$Q[k(u-u_0) = 0] = 0$$

$$Q[k(u-u_0) \rightarrow -\infty] = 1$$


„achromatic signal“ discrimination

as function of relative light density
 $h = \ln H = k(u-u_0)$, $\ln = \text{natural log.}$

$$Q' = \frac{d}{dH} [\ln\{1 + 1/(1 + \sqrt{2} H)\}] / \ln \sqrt{2}$$

$$= -\sqrt{2} / [\ln \sqrt{2} (1 + \sqrt{2} H)(2 + \sqrt{2} H)]$$

function values:

$$Q'[k(u-u_0) \rightarrow +\infty] = 0$$

$$Q'[k(u-u_0) = 0] = -0,5$$

$$Q'[k(u-u_0) \rightarrow -\infty] = 0$$

luminance discrimination

possibility $L/\Delta L$ as function of H

with: $L = 10^u$ $H = e^h = 10^{\log e k(u-u_0)}$

$$dL/dL = \ln 10 L$$

$$dH/dH = k H$$

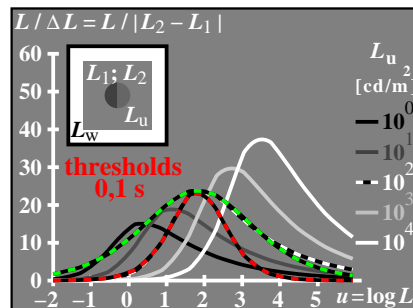
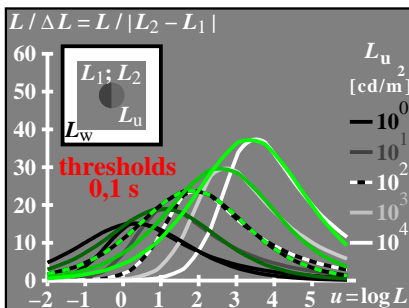
it follows: $L/\Delta L = [kH]/(dH \ln 10)$

$$\frac{L}{\Delta L} = \text{const } H / [(1 + \sqrt{2} H)(2 + \sqrt{2} H)]$$

function values:

$$Q'[k(u-u_0) \rightarrow +\infty] = 0$$

$$Q'[k(u-u_0) = 0] = \text{maximum}$$

$$Q'[k(u-u_0) \rightarrow -\infty] = 0$$


double line element of Richter (1987) for the lighting technology with the luminance $L = f(L_P, M_D, S_T)$

$$F(L) = \int_{-\infty}^L (L/\Delta L) dL \text{ (relative } L, M, S?)$$

$$F(L) = iQ(H) = \begin{cases} iQ(\bar{H}) & (u < u_0) \\ iQ(\bar{H}) & (u \geq u_0) \end{cases}$$

with: $k=1,4$ $\bar{k}=1$ $i=1$ $\bar{i}=-2$
 $u = \log L$ $u_0 = \log L_u$
 $H = e^{k(u-u_0)}$ $\bar{H} = e^{\bar{k}(u-u_0)}$

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$$F(L) = \int_{-\infty}^L (L/\Delta L) dL \text{ (relative } L, M, S?)$$

$$F(L) = iQ(H) \quad H = e^{k(u-u_0)}$$

$$Q(H) = [\ln\{1 + 1/(1 + \sqrt{2} H)\}] / \ln \sqrt{2} - 1$$

Taylor-derivations:

$$\Delta F(L) = \frac{dF}{dL} \Delta L = i \frac{dQ}{dH} \Delta H$$

$$= -i \sqrt{2} \Delta H / [\ln \sqrt{2} (1 + \sqrt{2} H)(2 + \sqrt{2} H)]$$

see similar files: http://farbe.li.tu-berlin.de/CEA5/CEA5.HTM
 technical information: http://farbe.li.tu-berlin.de or http://130.149.60.45/~farbmetrik

TUB registration: 20201101-CEA5/CEA5L0NP.PDF /.PS
 application for evaluation and measurement of display or print output
 TUB material: code=rh4ta