Data and Material for the Description of Colour Thresholds using the Report CIE R1-47

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For recent publications of the TUB group see: http://130.149.60.45/~farbmetrik/XY91FEN.html

Introduction

This paper considers the results of CIE TC1-55 "Uniform Colour Space for Industrial Colour Difference Evaluation" and CIE TC1-63 "Validity of the Range of CIE DE2000" that neither CIELAB nor CIE DE2000 work well for the description of colour thresholds.

The report CIE R1-47:2009 "Hue Angles of Elementary Colours" is used to define a linear *(A,B,Y)* opponent colour space which is applied for the description of colour thresholds. Especially the experimental results of *Holtsmark-Valberg* (1969) for equal threshold for complementary optimal colours and of *Evans* (1974) about colour thresholds are discussed.

Content

- **Definition of device and elementary colours and systems**
- **RG- and JB-chromatic values and optimal colours**

- ***BRGJ-I*-signals for spectral and surface colours**
- **Parable shape of sensitivities *(PDT=LMS)* of colour vision**
- **Equal colour threshold for complementary optimal colours**

- **Colour valence metric with *RG-* and *JB*-chromatic values *A* and *B***
- **Optimal colour diagrams *(A, B)* for CIE illuminant D65 and A**
- **Optimal colour data *(A, B)* for CIE standard Illuminants D65 and A**

- **Colour difference formula *ABY* for threshold of optimal colours**

Achromatic colours

five achromatic colours:

N black (french noir)
D dark grey
Z central grey
H light grey
W white

Elementary colours
"Neither-nor"-colours

four elementary colours:

R red
neither yellowish nor blueish
G green
neither yellowish nor blueish
B blue
neither greenish nor reddish
J yellow (french jaune)
neither greenish nor reddish

Device colours
Television (TV), Print (PR)
Photography (PH)

six device colours:

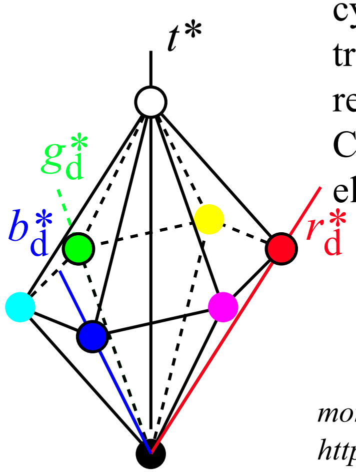
C cyanblue
M magentared
Y yellow
O orangered
L leafgreen
V violetblue

YE980–3

8 Device Colours in device (d) system

OYLCVM and NW

triangle system $\boldsymbol{rgb}^*_{\mathrm{d}}$
cylindric system tch^*
triangle lightness t^*
relative chroma c^*
CIELAB hue angle h_{ab}
elementary hue text u^*_{d}



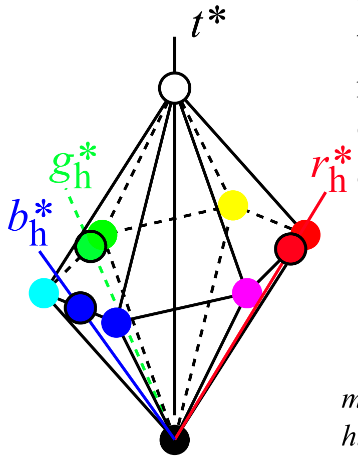
more coordinates and relations:
http://www.ps.bam.de/33872E

IE151-2N

8 Device Colours in elementary hue (h) system

OYLCVM and NW

triangle system $\boldsymbol{rgb}^*_{\mathrm{h}}$
cylindric system tch^*
triangle lightness t^*
relative chroma c^*
CIELAB hue angle h_{ab}
elementary hue text u^*_{h}



more coordinates and relations:
http://www.ps.bam.de/33872E

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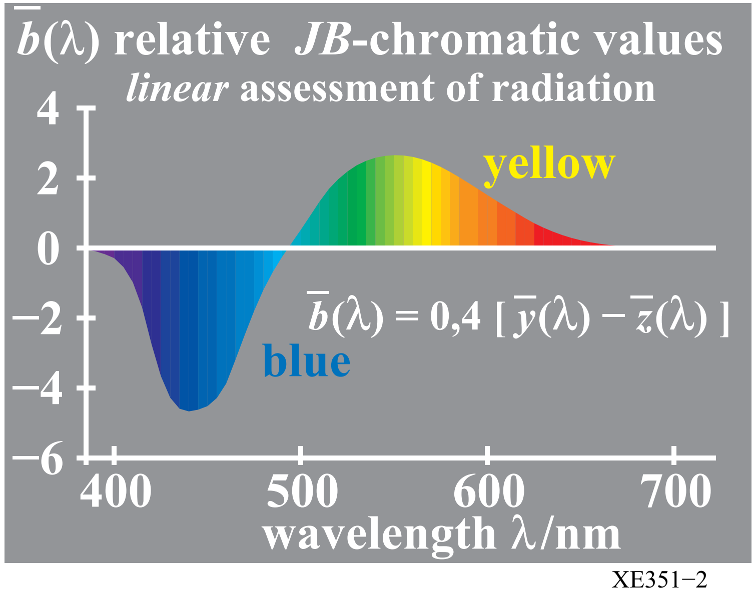
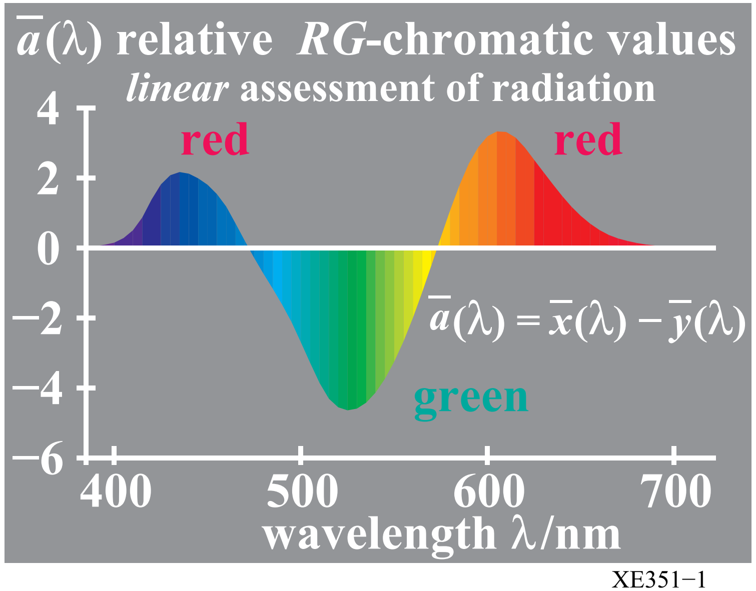
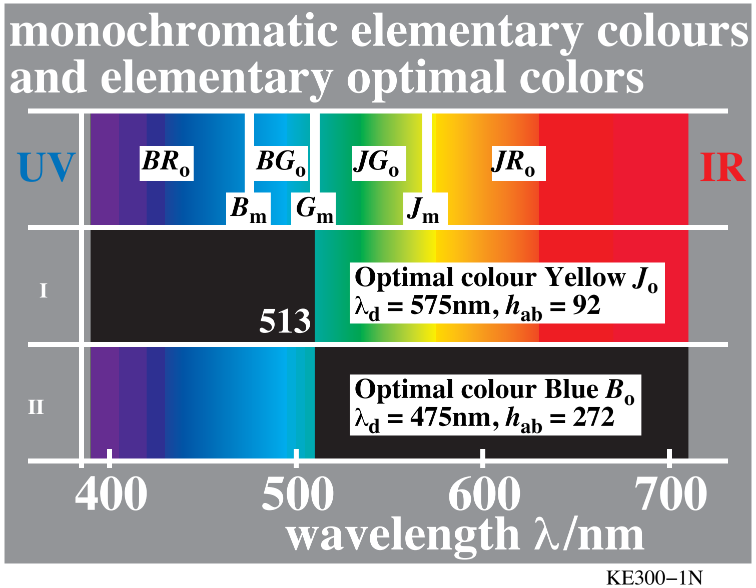
Figure 1: Definition of device and elementary colours and systems







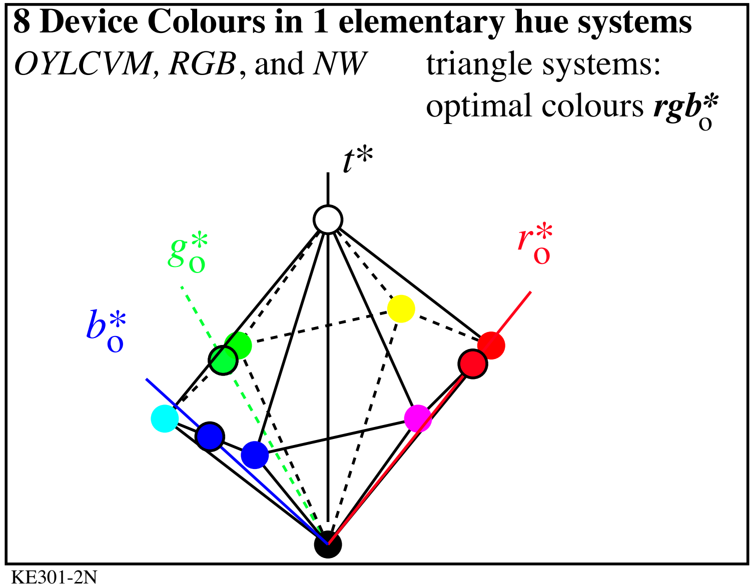


Figure 2: ***BRGJ-I*-signals for spectral colours. Background of white paper ($R(\lambda) = 0{,}9$) or display normalized to paper reflection factor**

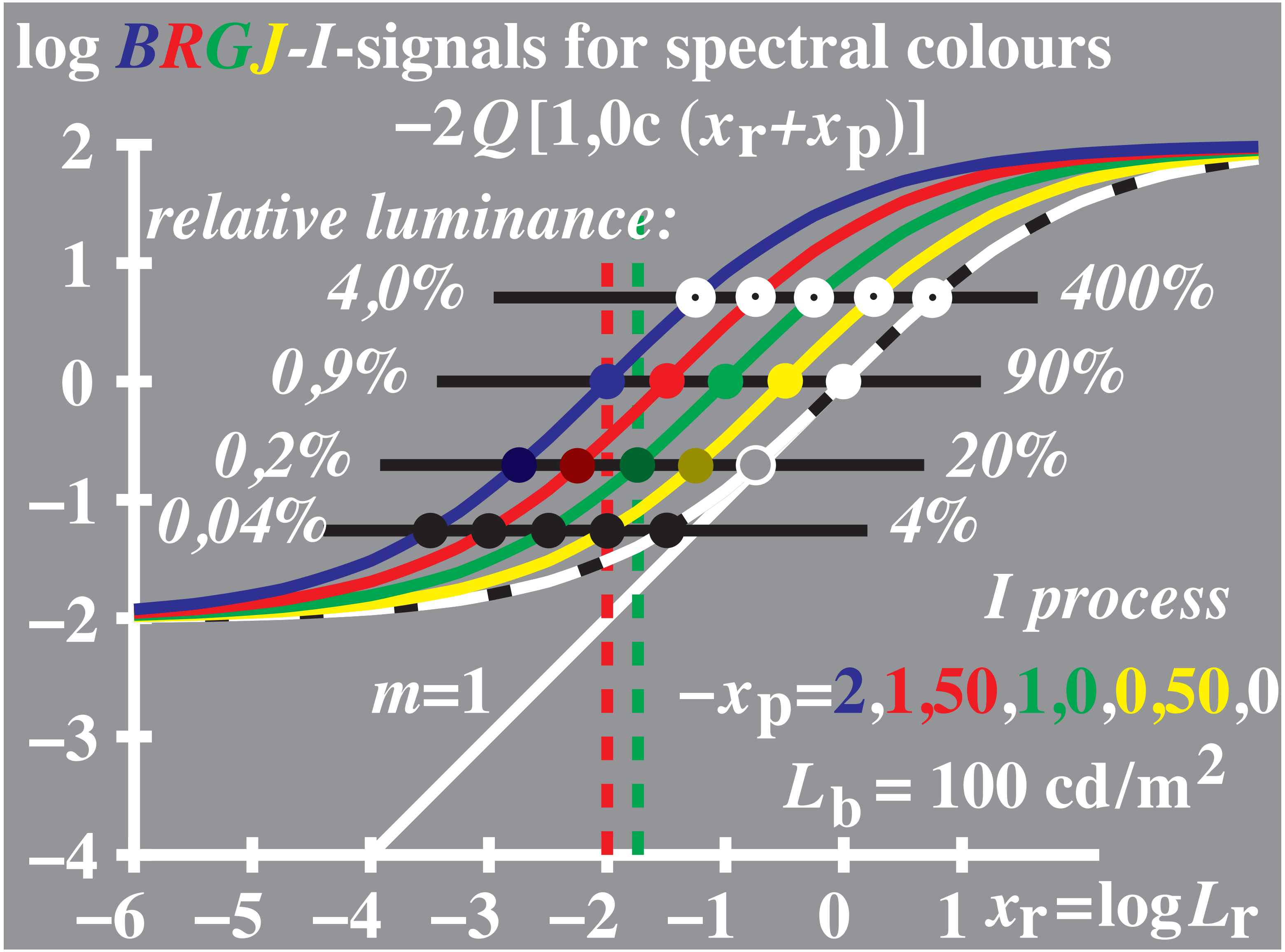
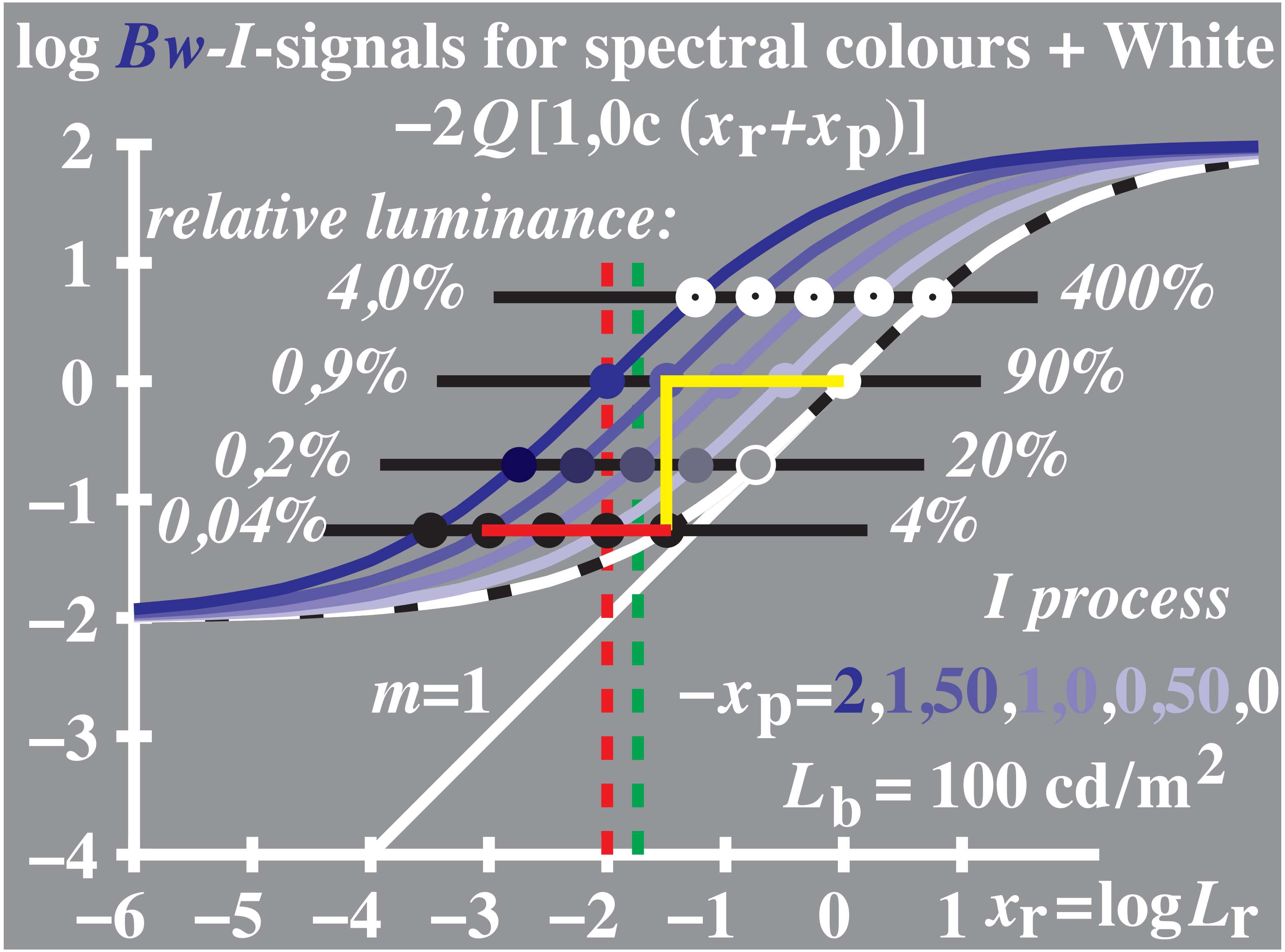


Figure 3: ***BRGJ-I*-signals for spectral colours. Background of white paper (*R(λ)* = 0,9) or display normalized to paper reflection factor**



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***BRGJ-I*-signals for spectral and unsaturated blue colours.**
Background of white paper (*R(λ)* = 0,9) or display; Vision window *J*

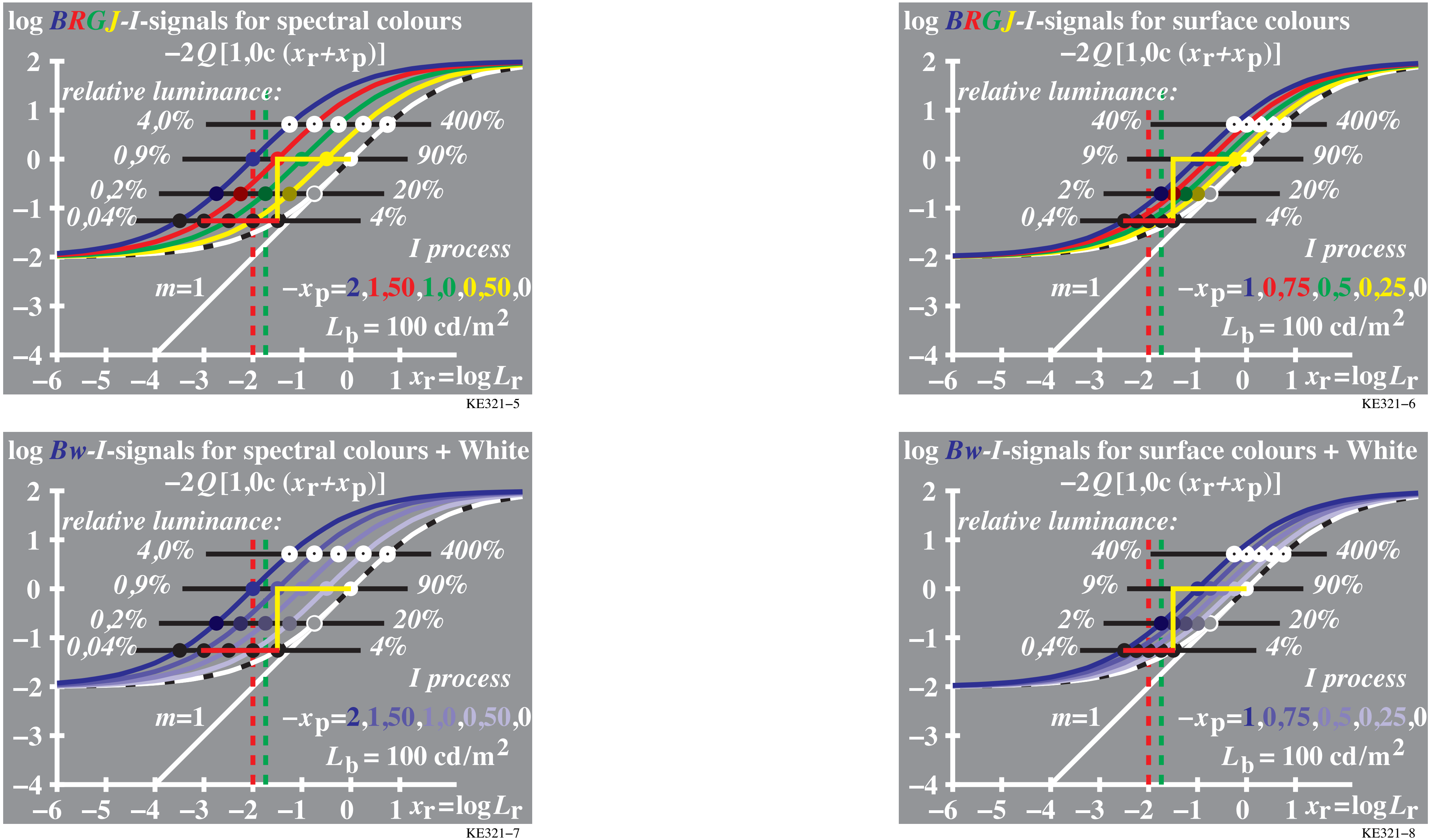
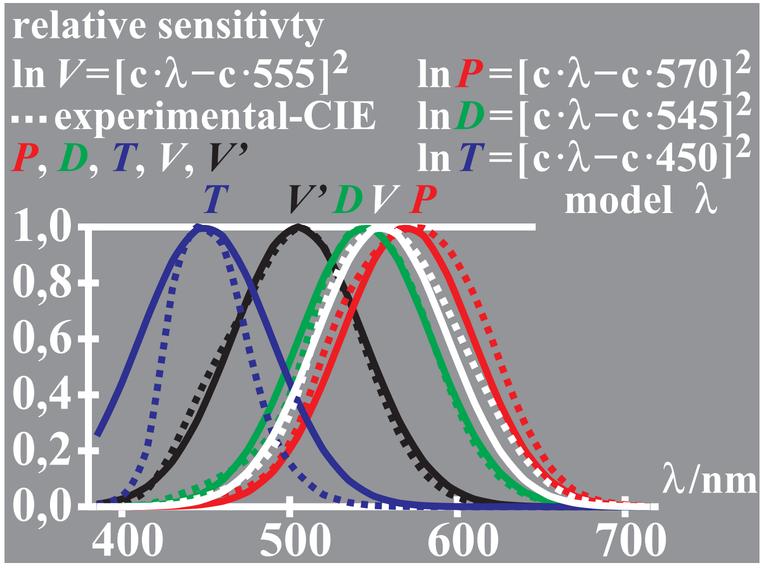
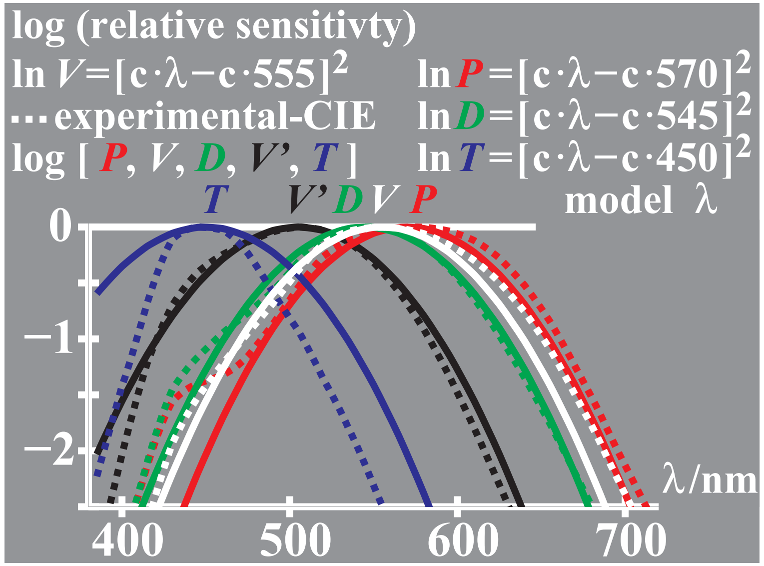


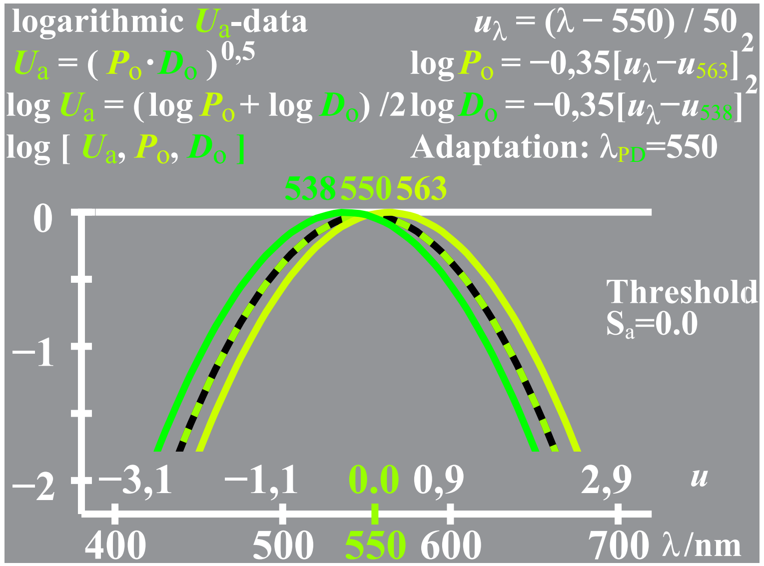
Figure 4: *BRGJ-I*-signals for spectral and surface colours. The horizontal line of the *yellow* vision window defines zero blackness



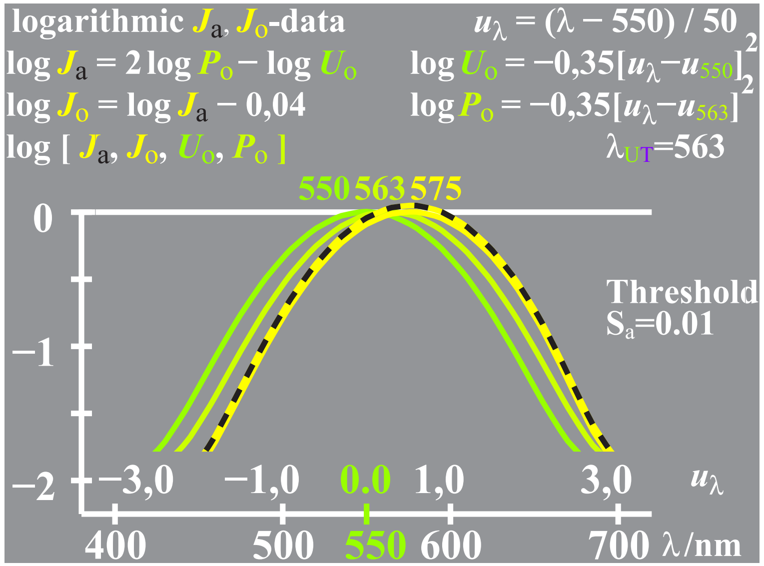
JE530−1



JE530−2



JE531−2



Uon2Po−>JaJo

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Sensitivities *(PDT=LMS)* of colour vision. Parables on a log scale are used to calculate sensitivity *U(λ)= V(λ)* or yellow *J(λ)*

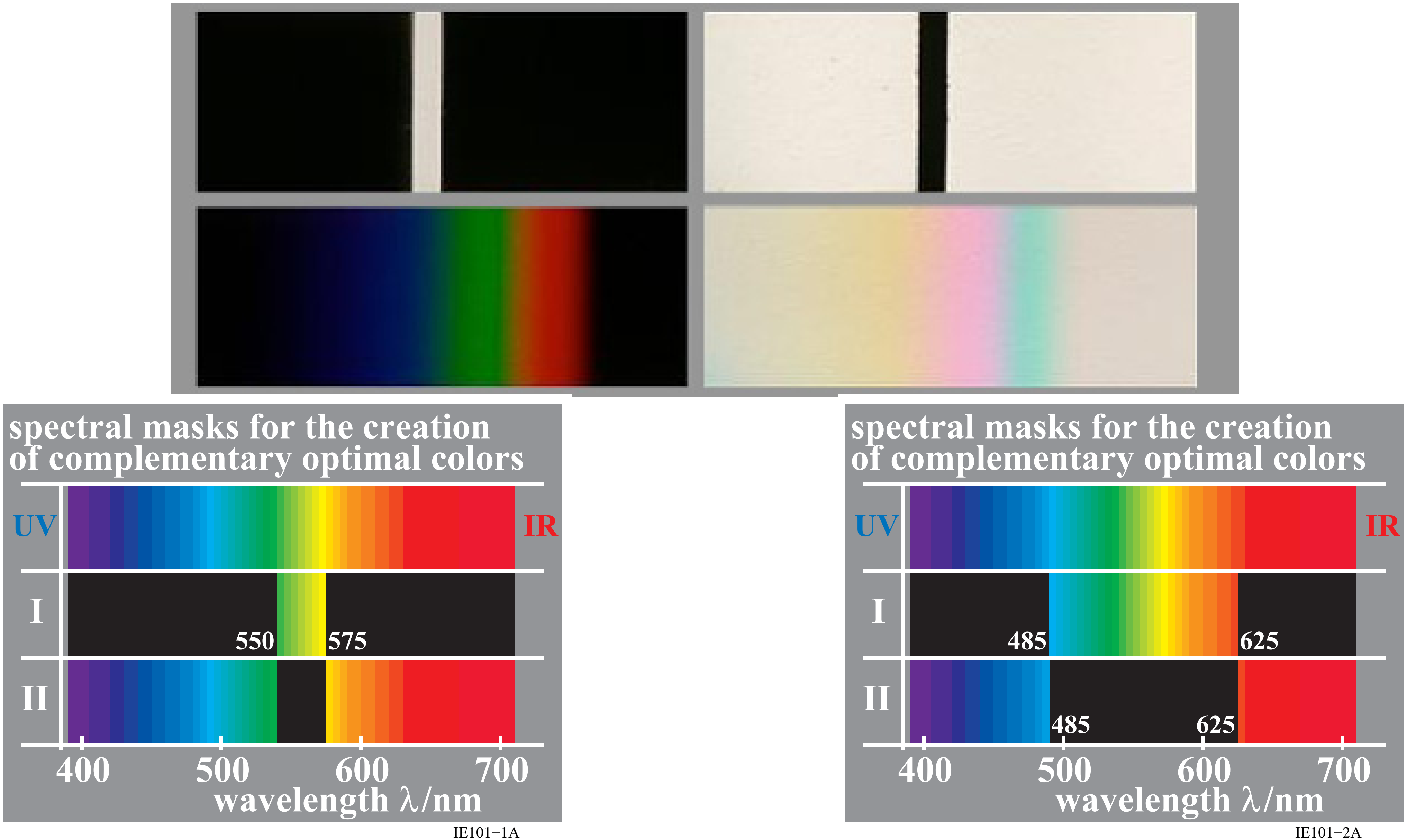

Figure 5: *OLV* and complementary colours *CMY* produced by prism, and complementary masks for a 2-beam spectral colour integrator

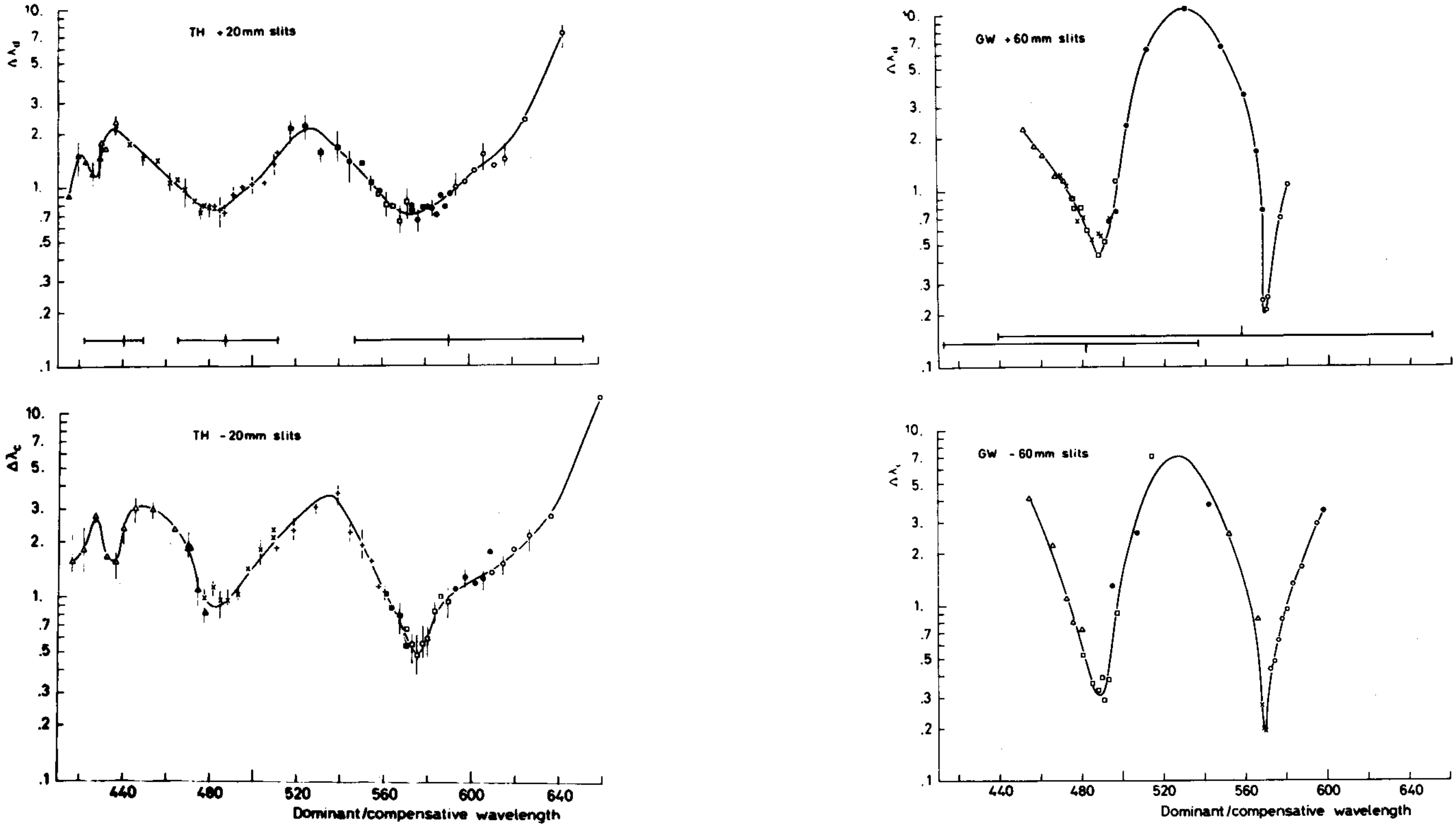

Figure 6: Equal colour threshold for complementary masks (small and broad slit) of a 2-beam spectral colour integrator

color valence metric (color data: linear relation to CIE 1931 data)		
linear color terms	**name and relationship to CIE tristimulues or chromaticity values**	**notes:**
luminous value	$Y = y\,(X + Y + Z)$	
chromatic value	*for linear chromatic value diagram (A, B)*	
red−green	$A = [\,X/Y - X_n/Y_n\,]\,Y = [\,a - a_n\,]\,Y$	*n=D65 (backgr.)*
	$= [\,x/y - x_n/y_n\,]\,Y$	
yellow−blue	$B = -0{,}4\,[\,Z/Y - Z_n/Y_n\,]\,Y = [\,b - b_n\,]\,Y$	
	$= -0{,}4\,[\,z/y - z_n/y_n\,]\,Y$	
radial	$C_{ab} = [\,A^2 + B^2\,]^{1/2}$	
chromaticity	*for (linear) chromaticity diagram (a, b)*	*compare to linear*
red−green	$a = X/Y = x/y$	*cone excitation*
yellow−blue	$b = -0{,}4\,[Z/Y] = -0{,}4\,[z/y]$	***P*/(*P*+*D*)=*L*/(*L*+*M*)**
radial	$c_{ab} = [\,(a - a_n)^2 + (b - b_n)^2\,]^{1/2}$	***T*/(*P*+*D*)=*S*/(*L*+*M*)**

JE441−7

Figure 7: Linear colour valence metric with *RG-* and *JB*-chromatic values *A* and *B* and a chromaticity diagram *(a, b)* in luminance units

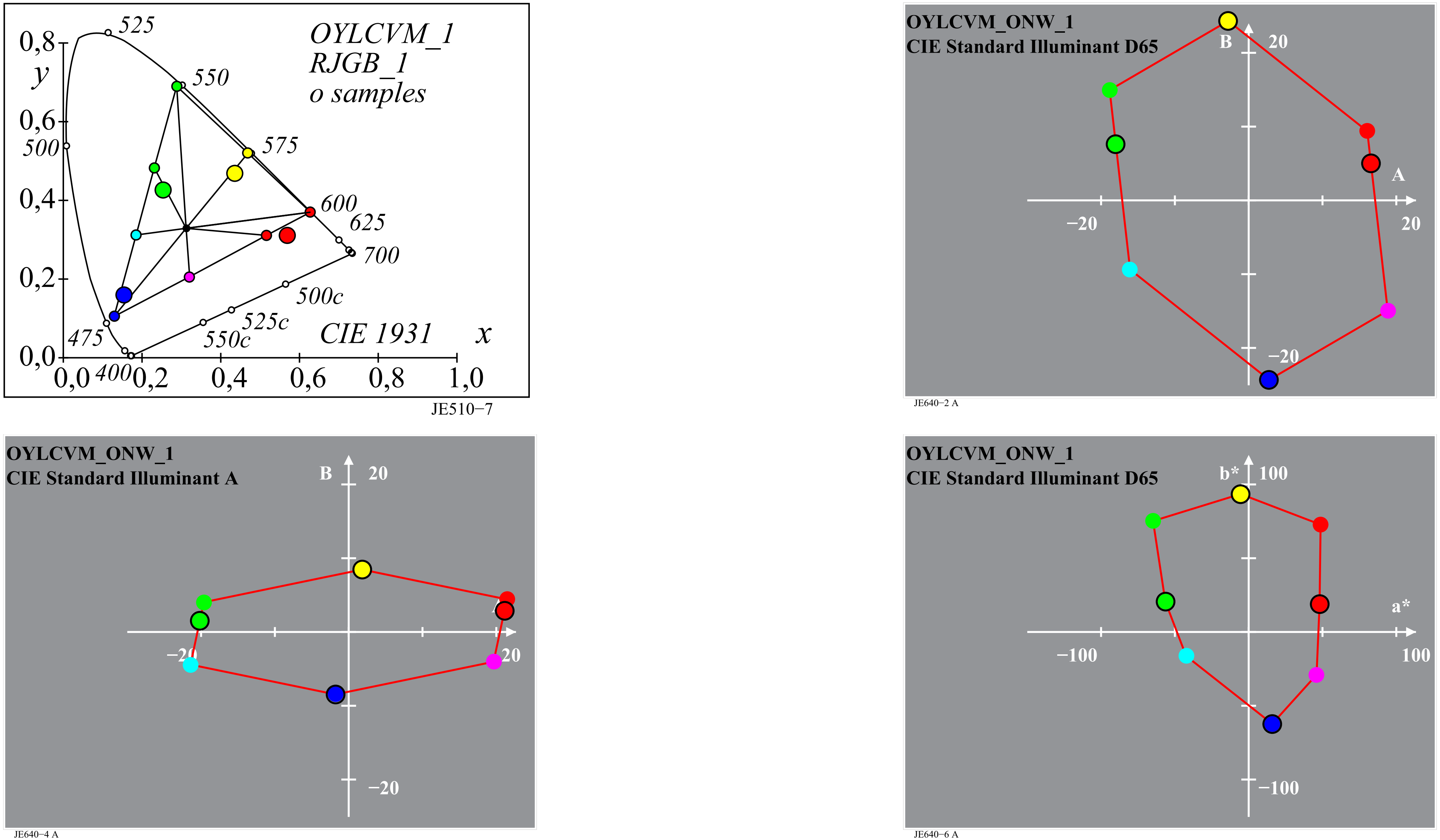

Figure 8: Optimal colours in diagrams *(x, y)*, *(A, B)* for illuminant D65 and A, *(a*, b*)*. Equal values *(A, B)* for complementary colours

Optimal colours for CIE Standard Illuminant D65

X	*Y*	*Z*	*x*	*y*	*A*	*B*	C_r	*OYLCVM_ONW_1*		
54.8	32.3	0.0	0.628	0.37	24.1	14.0	27.9	%O=JR	00	575_770
76.8	85.2	1.6	0.469	0.52	-4.1	36.4	36.7	%Y=J=JG+JR	01	515_770
22.0	52.9	1.6	0.288	0.69	-28.2	22.4	36.0	%L=JG	02	515_575
27.5	57.3	33.7	0.231	0.483	-27.0	11.4	29.3	%G	03	0,70*L+0,30*C
40.2	67.6	108.8	0.185	0.312	-24.1	-14.0	27.9	%C=L+V	04	380_575
18.1	14.7	107.2	0.129	0.105	4.1	-36.4	36.7	%V=B=BR+BG	05	380_515
72.9	47.0	107.2	0.321	0.206	28.2	-22.4	36.0	%M=V+O	06	380_515+575_770
58.0	34.9	19.3	0.516	0.311	24.8	7.4	25.9	%R	07	0,18*M+0,82*O
54.8	32.3	0.0	0.628	0.37	24.1	14.0	27.9	%O=JR	08	575_770
0.0	0.1	0.1	0.311	0.327	0.0	0.0	0.0	%N0(β=0,001)	09	380_770
95.0	100.0	108.8	0.312	0.329	0.0	0.0	0.0	%W1(β=1,000)	10	380_770

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Optimal colours for CIE Standard Illuminant A

X	*Y*	*Z*	*x*	*y*	*A*	*B*	C_r	*OYLCVM_ONW_1*		
83.5	46.7	0.0	0.64	0.358	32.1	6.6	32.8	%O=JR	00	575_770
104.3	92.5	1.2	0.526	0.467	2.7	12.6	12.9	%Y=J=JG+JR	01	515_770
20.8	45.8	1.1	0.307	0.675	-29.4	6.0	30.0	%L=JG	02	515_575
22.5	48.0	11.4	0.274	0.585	-30.2	2.2	30.3	%G	03	0,70*L+0,30*C
26.3	53.2	35.5	0.228	0.462	-32.1	-6.6	32.8	%C=L+V	04	380_575
5.4	7.4	34.3	0.115	0.157	-2.7	-12.6	12.9	%V=B=BR+BG	05	380_515
88.9	54.1	34.4	0.501	0.305	29.4	-6.0	30.0	%M=V+O	06	380_515+575_770
84.4	48.0	6.2	0.608	0.346	31.6	4.3	31.9	%R	07	0,18*M+0,82*O
83.5	46.7	0.0	0.64	0.358	32.1	6.6	32.8	%O=JR	08	575_770
0.1	0.0	0.0	0.445	0.405	0.0	0.0	0.0	%N0(β=0,001)	09	380_770
109.8	99.9	35.5	0.447	0.407	0.0	0.0	0.0	%W1(β=1,000)	10	380_770

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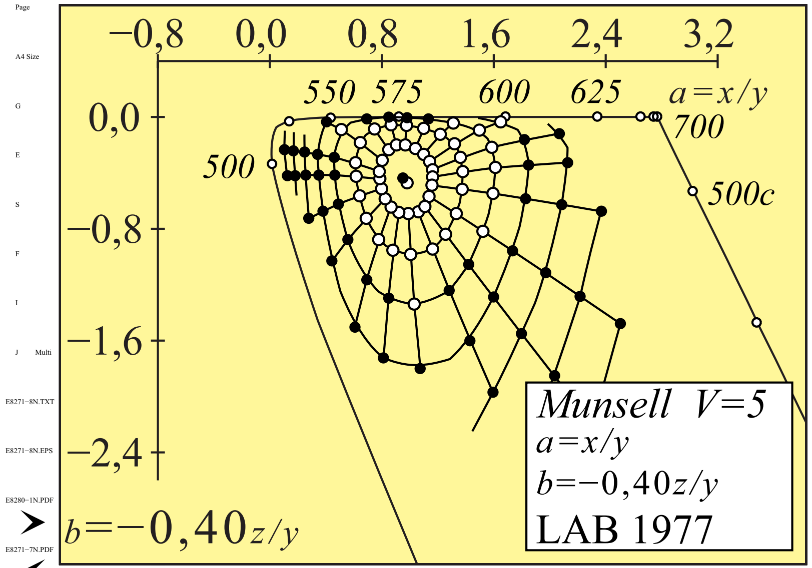
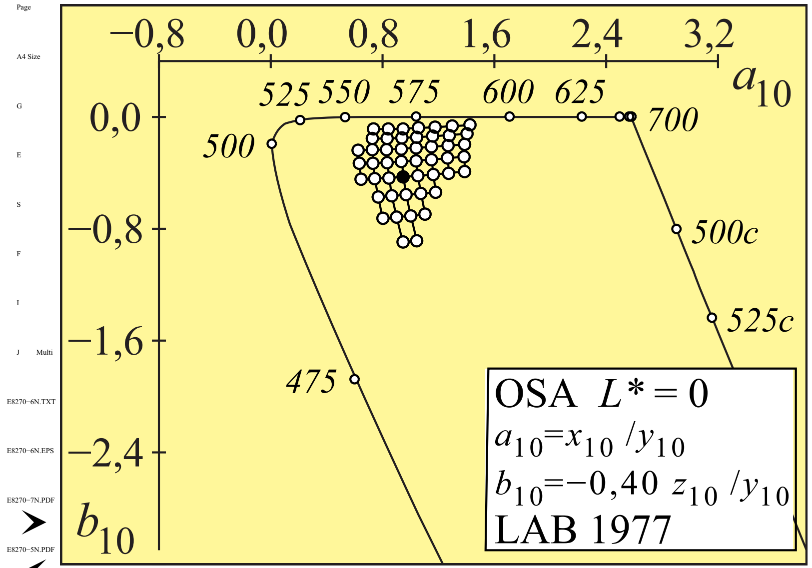
Figure 9: Optimal colour data for CIE standard Illuminants D65 and A, compare e. g. anti-symmetric data *(A, B)* of colours *Y* and *V*



B8120_2, E8271−8, B4_17_1, N=4_17_1



B8220_2, E8270−6, B4_14_1, N=4_14_1

color threshold formula LABJNDS 1985 (JND=just noticeable difference)

$$\Delta E^*_{JND} = Y_0\,[\,(\Delta Y)^2 + (a_0\,\Delta a''\cdot Y)^2 + (b_0\,\Delta b''\cdot Y)^2]^{1/2} \,/\, (s + d\,Y^e)$$

$a = x/y \quad a_n = x_n/y_n \quad b = -0{,}4\,z/y \quad b_n = -0{,}4\,z_n/y_n$

$a'' = a_n + (a - a_n)/(1 + 0{,}5\,|a - a_n|)$

$b'' = b_n + (b - b_n)/(1 + 0{,}5\,|b - b_n|)$

n = D65 or A (surround)

$Y = (Y_1 + Y_2)/2 \quad \Delta Y = Y_1 - Y_2 \quad \Delta a'' = a''_1 - a''_2 \quad \Delta b'' = b''_1 - b''_2$

$s = 0{,}0170 \quad d = 0{,}0058 \quad e = 1{,}0$

$a_0 = 1{,}0 \quad b_0 = 1{,}8 \quad Y_0 = 1{,}5$ *surround D65*

$a_0 = 1{,}0 \quad b_0 = 1{,}7 \quad Y_0 = 1{,}0$ *surround A*

B7351_7, E8560−7, BT4_01, N=4_1 2x2

Figure 10: ***Munsell and OSA colours in chromaticity diagram (a,b) and colour threshold (JND) formula based on coordinates a and b.***

Threshold colour difference formula *ABY-JND* for optimal colours

This formula is one which is in agreement with the *Holtsmark–Valberg* (1969) threshold results: *the threshold is equal for complementary optimal colours.*

$$\Delta \boldsymbol{E}^{*}_{\mathrm{ABY}} = Y_0 \, \{ [a_0 \ \ \Delta \boldsymbol{A}_{01}]^2 + [b_0 \ \ \Delta \boldsymbol{B}_{01}]^2 + [(\Delta \boldsymbol{Y}_{01}) \, / \, \boldsymbol{Y}_{01}]^2 \}^{1/2}$$

For complementary (c) colours the absolute values A_{01} and A_{01c} are equal.

Proof: With the normalization $X_{01} = X / X_n$, $Y_{01} = Y / Y_n$, $Z_{01} = Z / Z_n$

then for the complementary optimal colours it is always valid:

$$X_{01c} = 1 - X_{01}, \qquad Y_{01c} = 1 - Y_{01}, \qquad Z_{01c} = 1 - Z_{01}$$

Therefore

$$A_{01c} = X_{01c} - Y_{01c} = 1 - X_{01} - (1 - Y_{01}) = Y_{01} - X_{01} = - A_{01}$$

and similar for *B*.

According to the *Weber-Fechner* law $\Delta Y_{01} \, / \, Y_{01}$ is constant in the surface colour range, which includes the complementary optimal colours.

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Annex A: Equations and possible metric to describe achromatic and chromatic thresholds

In this annex the linear equations in Fig. 11 are used for the description of the *Holtsmark- Valberg* experiments. The red-green chromatic value for the basic colour is

$$A = (a - a_n) \; Y = (x / y - x_n / y_n) \; Y \qquad \text{(B;1)}$$

Then it is valid with the normalization of image technology for the range 0 to 1 (Index 01)

$$A_{01} = (a_{01} - a_{01n}) \; Y_{01} = (x_{01} / y_{01} - 1) \; Y_{01} = (X_{01} / Y_{01} - 1) \; Y_{01} = X_{01} - Y_{01}$$

For the complementary colours it is always valid

$$X_{01c} = 1 - X_{01}, \qquad Y_{01c} = 1 - Y_{01,} \qquad Z_{01c} = 1 - Z_{01}$$

Therefore

$$A_{01c} = X_{01c} - Y_{01c} = 1 - X_{01} - (1 - Y_{01}) = Y_{01} - X_{01} = - A_{01} \qquad \text{(B;2)}$$

If we use the three-dimensional difference in the linear space, then we have for the basic colours at threshold (th)

$$\text{delta } E^*_{ABY,th} = \{ [\text{ delta } A_{01}]^2 + [\text{ delta } B_{01}]^2 + [\text{ delta } Y_{01}]^2\}^{1/2} \qquad \text{(B;3)}$$

and for the complementary colours at threshold

$$\text{delta } E^*_{ABY,th,c} = \{ [\text{ delta } A_{01c}]^2 + [\text{ delta } B_{01c}]^2 + [\text{ delta } Y_{01c}]^2\}^{1/2} \qquad \text{(B;4)}$$

The absolute hue discrimination is for the complementary optimal colours the same because of equation (B;2)

$$A_{01c} = A_{01} \quad \text{and} \quad B_{01c} = B_{01} \qquad \text{(B;5)}$$

The last term delta Y_{01} is for the complementary colours different. If one colour is dark then the complementary is light. By the *Weber-Fechner* law it is valid for the achromatic discrimination along the luminance axis

$$\text{delta } Y_{01} = c_Y \; Y_{01}(\qquad \text{(B;6)}$$

Therefore the above equations are only a solution for the special case that the luminance threshold is below the hue threshold. This is not always true in the *Holtsmark-Valberg* experiments because they report to see in some regions only a lightness difference.

In this case we must look for a possibility to modify the threshold model. We can look at the **contrast sensitivity**

$$Y_{01C} / (\text{delta } Y_{01C}) = Y_{01} / (\text{delta } Y_{01}) \quad \text{(B;7)}$$

which is according to the *Weber-Fechner* law the **same for complementary colours**.

So instead of the equation (B;3) the following metric is in **complete agreement with the *Holtsmark–Valberg* threshold results for complementary optimal colours**

$$\text{delta } E^*_{\text{ABY,th}} = \{ [\text{delta } A_{01}]^2 + [\text{delta } B_{01}]^2 + [(\text{delta } Y_{01}) / Y_{01}]^2\}^{1/2} \quad \text{(B;8)}$$

In the colour space ABY and at threshold this formula will calculate the same value for complementary optimal colours

$$\text{delta } E^*_{\text{ABY,th}} = \text{delta } E^*_{\text{ABY,th,c}} \quad \text{(B;9)}$$

Equation (B;8) may be the first equation which describes the surprising results of *Holtsmark-Valberg* for thresholds.

Remark 1: During the AIC-symposium in Soesterberg in 1971 there have been very controversial discussions about the *Hotlsmark-Valberg* results.

We must be careful about the interpretation of equation (B;8). This equation does not tell us at the moment how to scale A_{01}. In other words if

$$\text{delta } A_{01} = \text{delta } A_{01c}$$

then it is also valid

$$(\text{delta } A_{01}) / A_{01} = (\text{delta } A_{01c}) / A_{01c}$$

The following speculative **equation for complementary optimal colours**

$$\text{delta } E^*_{\text{ABY,th}}=\{[(\text{delta } A_{01}) /A_{01}]^2 + [(\text{delta } B_{01}) /B_{01}]^2 + [(\text{delta } Y_{01}) /Y_{01}]^2\}^{1/2} \quad \text{(B;10)}$$

is also in full agreement with the *Holtsmark-Valberg* results.