20 Years Colour Vision, Colorimetry and Colour Appearance on http://color.li.tu-berlin.de
(Prof. Dr. Klaus Richter, Berlin University of Technology, DfwG-Meeting, Cologne 2021)

Colorimetric models of colour vision describe relations between colour physiology, colour psychophysics and colour appearance. The colour appearance is based on two basic blocks:
1. Chromaticity functions: The chromaticities a' and b' are nonlinear (cube-root) transformations of the CIE chromaticities (German: Farbarten) x and y.
2. Lightness functions: The lightness L^* of CIELAB and CIELUV is a nonlinear function of $(Y/Y_n)^n$ with Y_n as standard tristimulus value of white and the exponent n=1/2,4 (see IEC 61966-2-1)..

However, physiology and psychophysics require a relation to the grey surround Y_u=18. For mate surface colours the white W is about 5 times lighter (Y_W=5Y_u=90) and the black N about 5 times darker (Y_N=Y_u/5=3,6). Our daily natural environment requires therefore a *logarithmic* and *antagonistic* lightness scaling and metric in relation to grey Y_u, which is justified.

In the information technology the metric of the chromaticity and lightness spacing changes with the sceeen contrast.white : black. For example between n=1 (linear) for a small contrast Y_W:Y_N=2:1 via n=1/2,4 (CIELAB- and sRGB-colour space according to IEC 61966-2-1) for the standard contrast Y_W:Y_N=25:1 up to about n=1/3,6 for the high contrast Y_W:Y_N larger 288:1.

Also the antagonistic colour-appearance attributes, for example the *blackness N** and the *brilliantness I** as well as the *whiteness W** and the *deepness T*, change with the scene contrast.*

For this paper in German see http://farbe.li.tu-berlin.de/DfwGG_21.PDF
For this paper in English see http://farbe.li.tu-berlin.de/DfwGE_21.PDF

line element of *Stiles*
(1946) with „color values" L_P, M_D, S_T

three separate color signal functions

$$F(L_P) = i \ln(1+9L_P)$$
$$F(M_D) = j \ln(1+9M_D)$$
$$F(S_T) = k \ln(1+9S_T)$$

***Taylor*-derivations:**

$$\Delta F(L_P, M_D, S_T) = \frac{dF}{dL_P}\Delta L_P + \frac{dF}{dM_D}\Delta M_D + \frac{dF}{dS_T}\Delta S_T$$

$$= \frac{9i}{1+9L_P}\Delta L_P + \frac{9j}{1+9M_D}\Delta M_D + \frac{9k}{1+9S_T}\Delta S_T$$

CEA51−1N

Figure 1: Line elements of *Stiles* (1946) for three receptors *L, M,* and *S*.

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Line-element examples for grey samples ($0,2 \leq x \leq 5$)

F(x) is called the line-element function of ***f(x)***.

The following relations are valid for $x = Y/Y_u = Y/18$:

$$\frac{d[F(x)]}{dx} = f(x) \qquad [1]$$

$$F(x) = \int \frac{f'(x)}{f(x)}\, dx \qquad [2]$$

Example for the normalized tristimulus value $x = Y/Y_u$:

$$\frac{d[a \ln(1+b\,x)]}{dx} = \frac{a\,b}{1+b\,x} \qquad [3]$$

$$a \ln(1+b\,x) = \int \frac{a\,b}{1+b\,x}\, dx \qquad [4]$$

CEA00−1N

Figure 2: Relation of the mathematical logarithmic line elements

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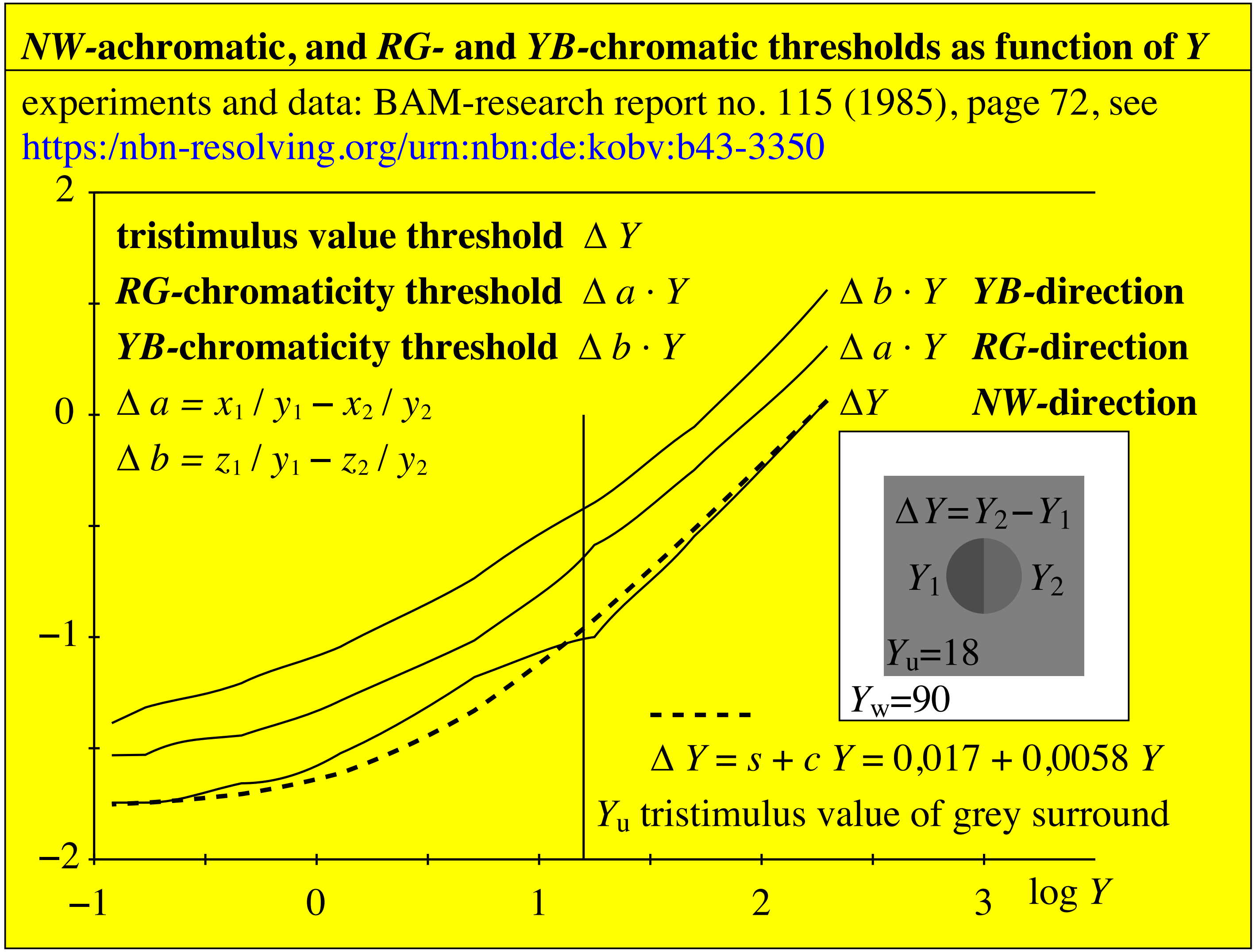

CEA01−3N

Figure 3: Experimental psychophysical grey discrimination.
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Line-element equations according to CIE 230:2019

Colour-discrimination function $f(x) = \Delta Y = \Delta x\, Y_u$ [0]

$\Delta Y=(A_1+A_2 Y)/A_0$ $A_0=1{,}5$, $A_1=0{,}0170$, $A_2=0{,}0058$

$$f_u(x) = \frac{\Delta Y}{\Delta Y_u} = \frac{1+b\,x}{1+b} \quad b=A_2 Y_u/A_1 \quad x=Y/Y_u \qquad [1]$$

$$F_u(x) = \int \frac{f'_u(x)}{f_u(x)}\, dx = \int \frac{b}{1+b\,x}\, dx \qquad [2]$$

Example for $L^*(x)$ & ΔY with $x=Y/Y_u$, $x_u=1$, $b=6{,}141$:

$$L^*_u(x) = \frac{L^*(x)}{L^*(x_u)} = \frac{\ln(1+b\,x)}{\ln(1+b)} \qquad [3]$$

$$f_u(x) = \frac{\Delta Y}{\Delta Y_u} = \frac{1+b\,x}{1+b} \qquad [4]$$

CEA00−5N

Figure 4: Logarithmic line elements of the tristimulus value Y

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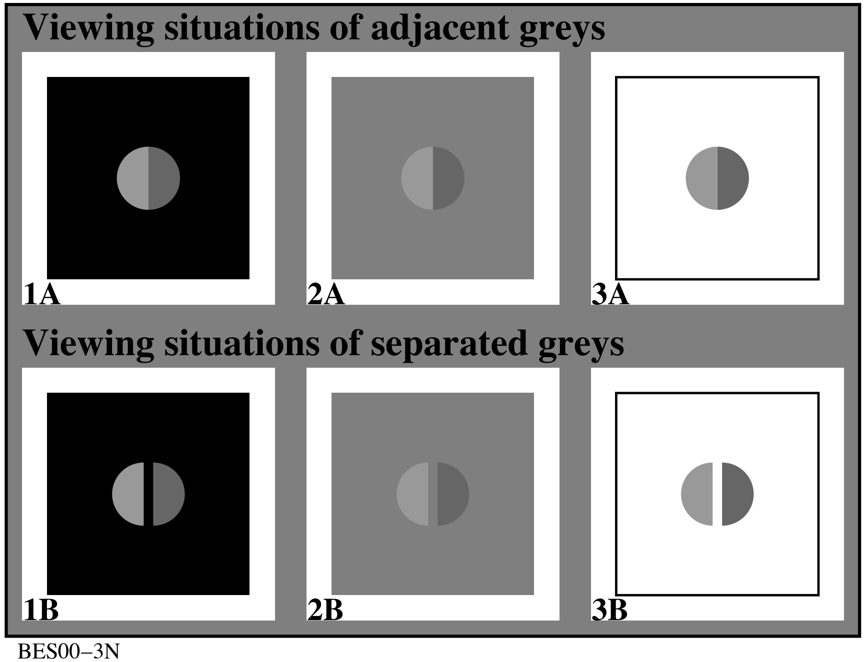

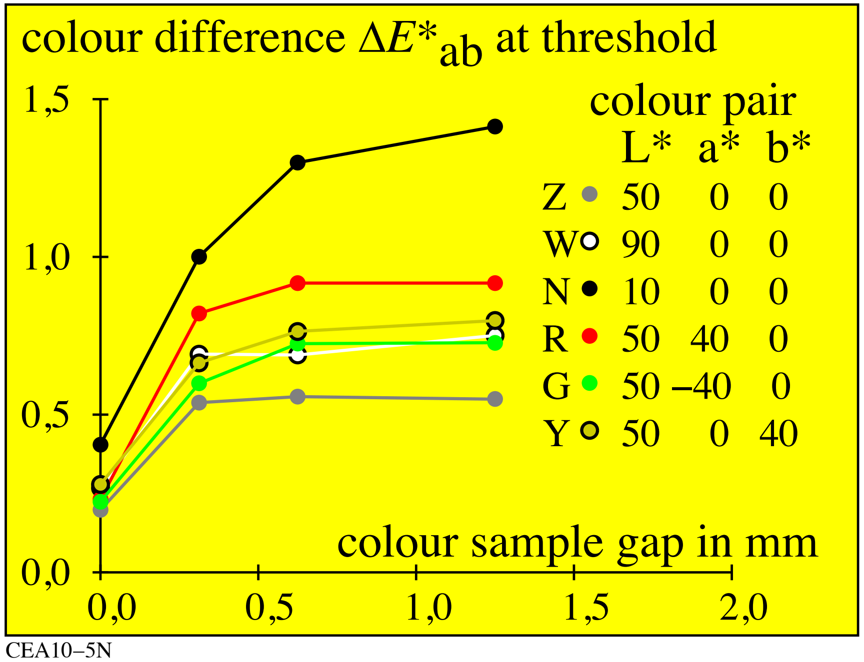

Figure 5: Discrimination of adjacent and separate greys.
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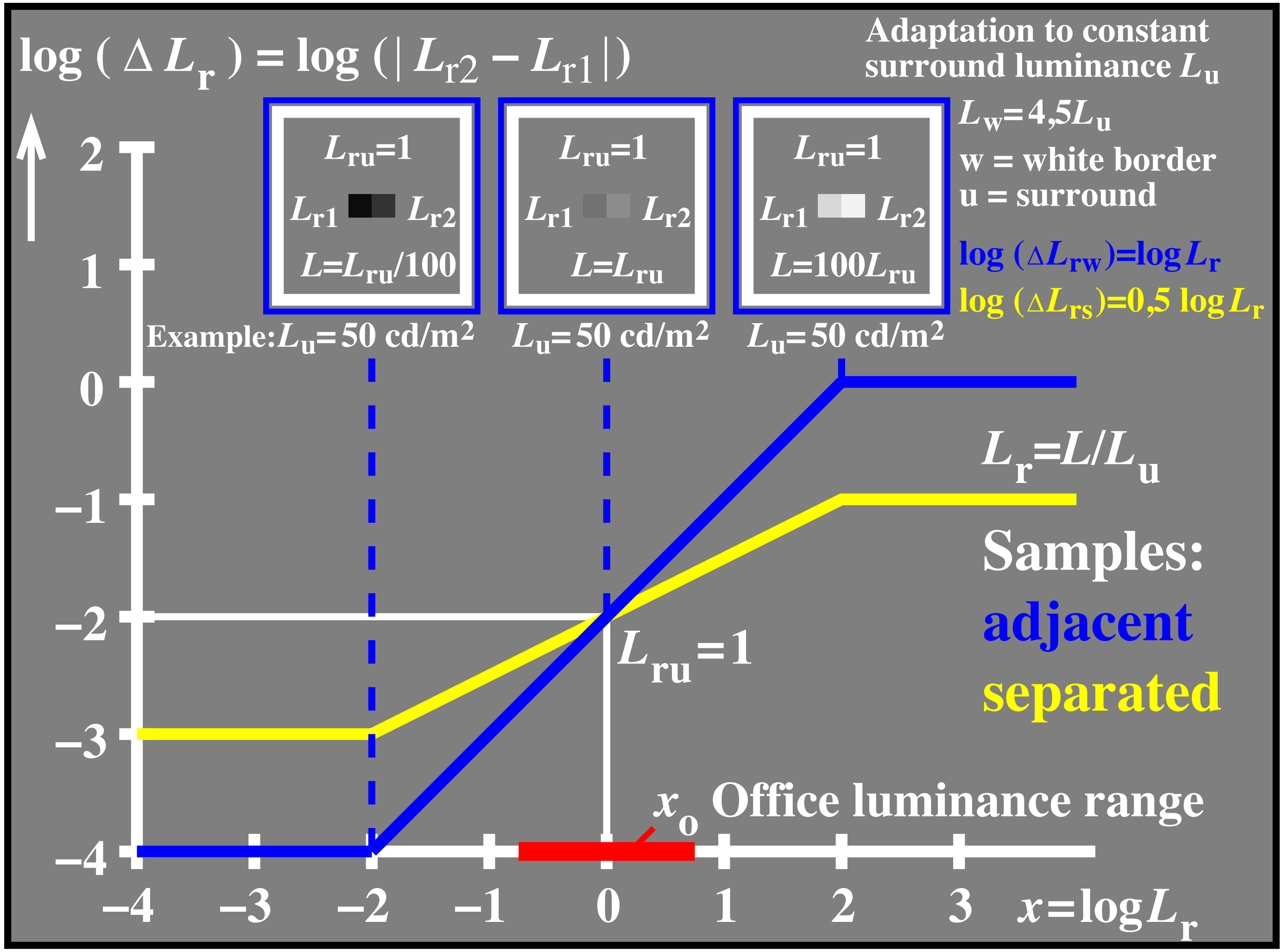

CEA30–3N

Figure 6: Two lightness functions for adjacent and separate greys.

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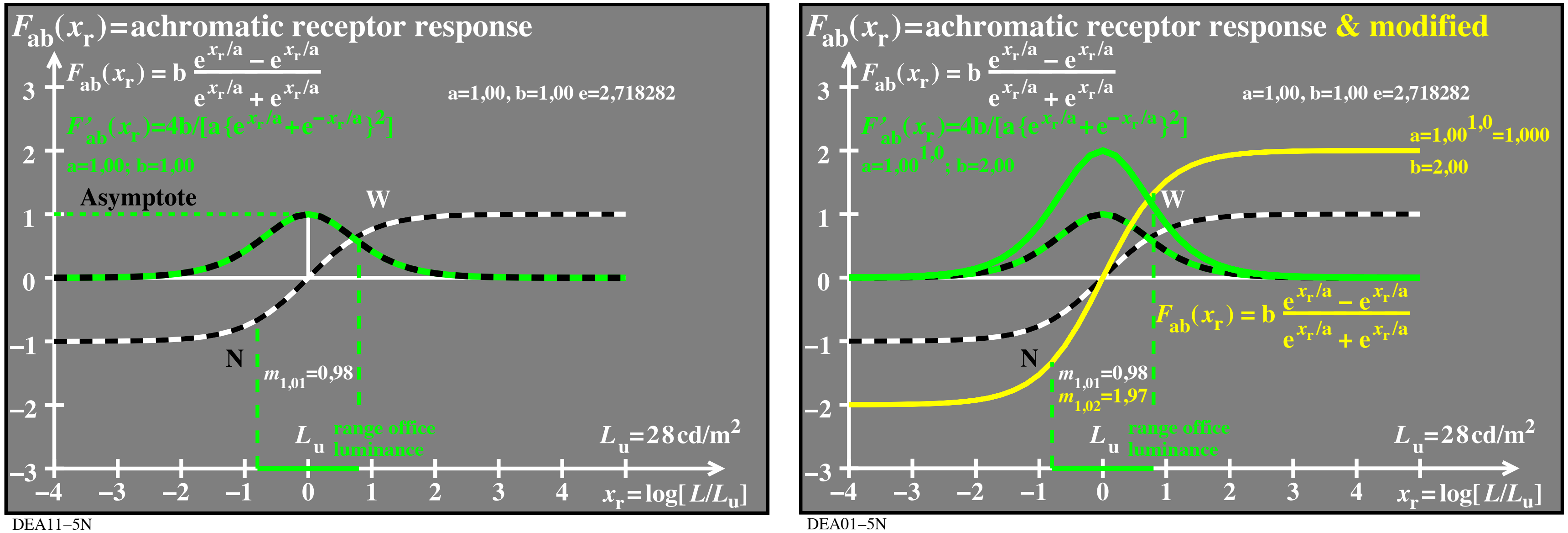

Figure 7: Logarithmic line elements of grey samples and for discrimination.

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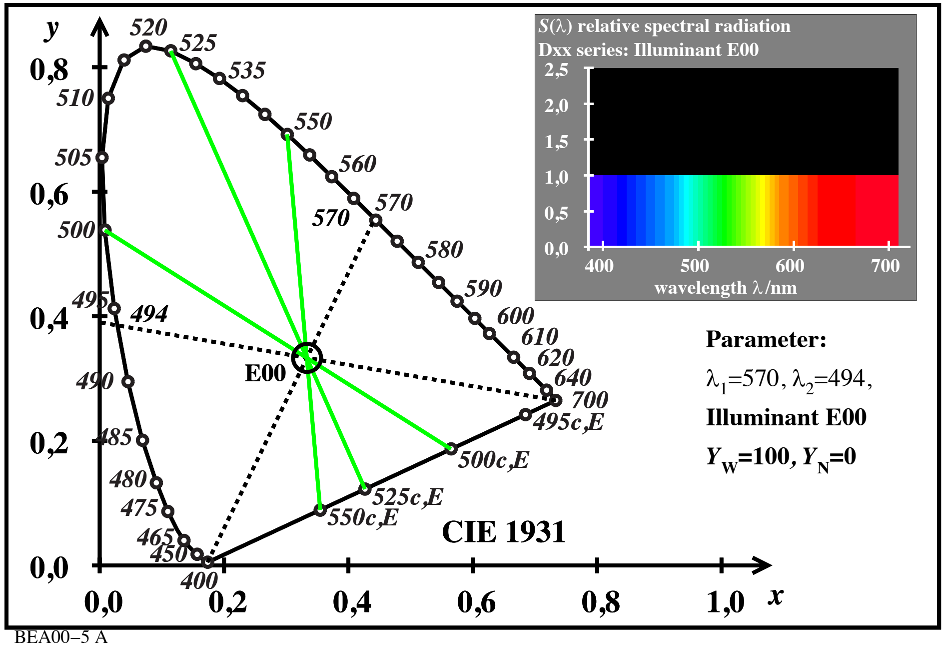

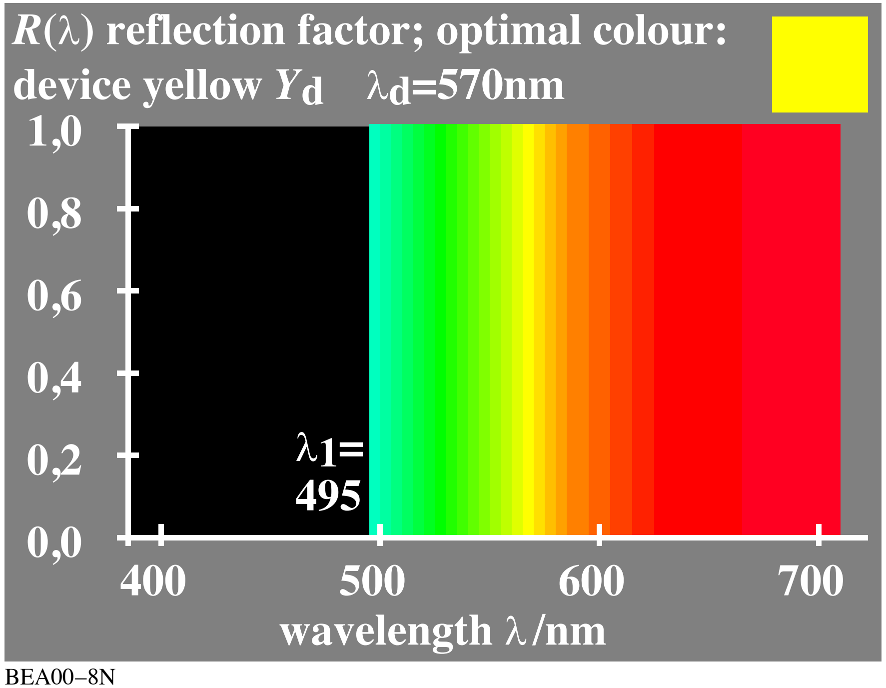

Figure 8: Wavelength limits of *Ostwald* optimal colours for CIE illuminant E.

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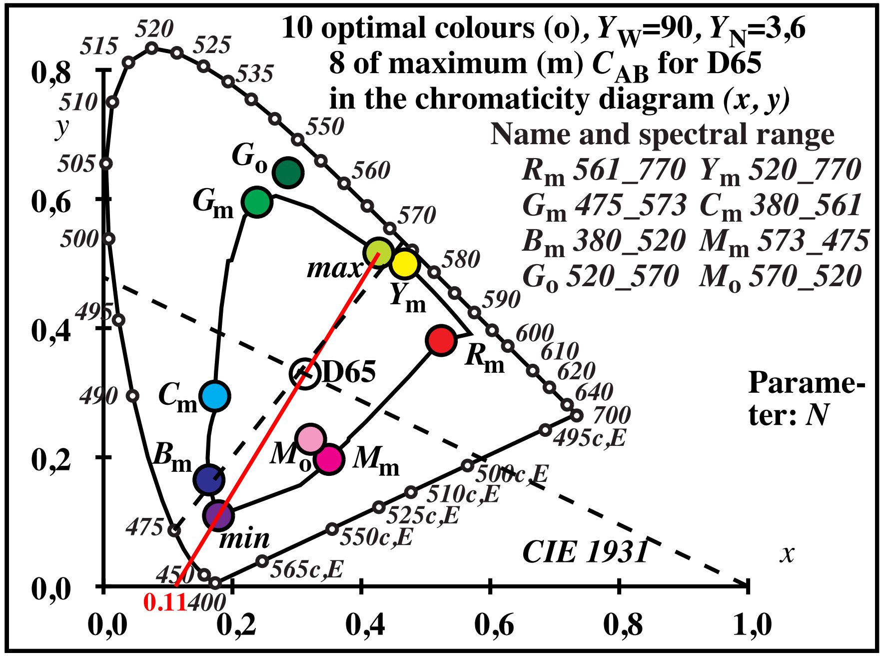
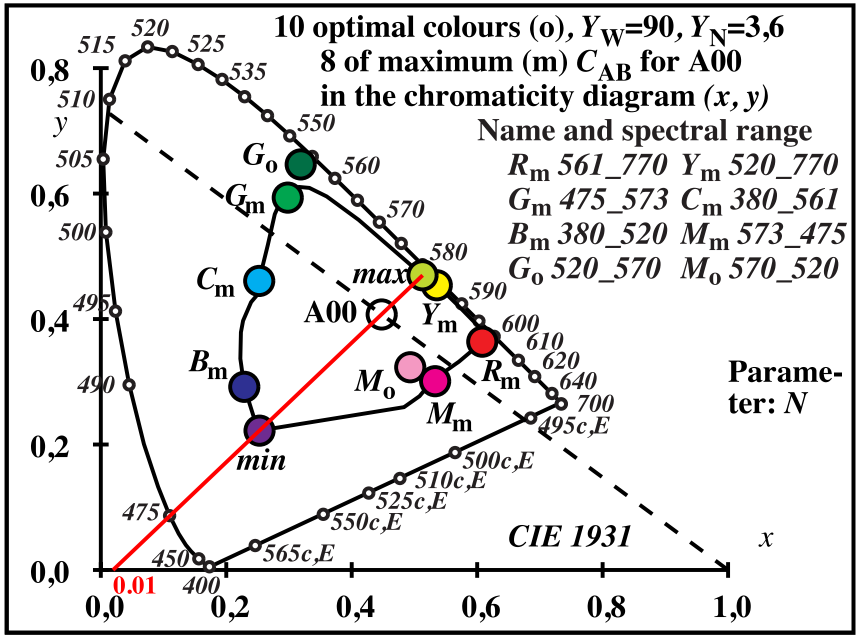


BEA01−3N



BEA01−4N

Figure 9: *Ostwald* optimal colours for the CIE standard illuminants D65 and A in *(x,y)*.

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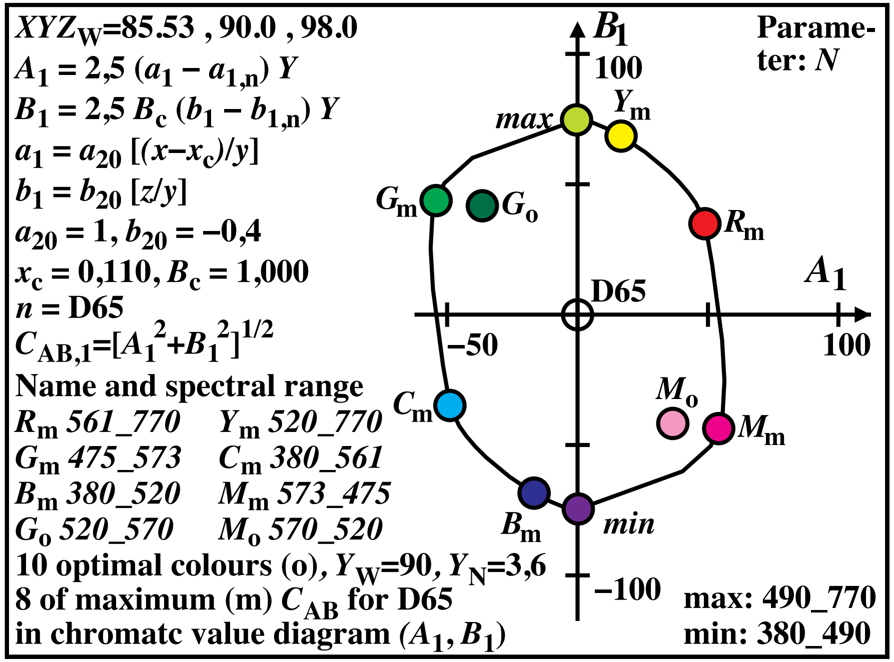
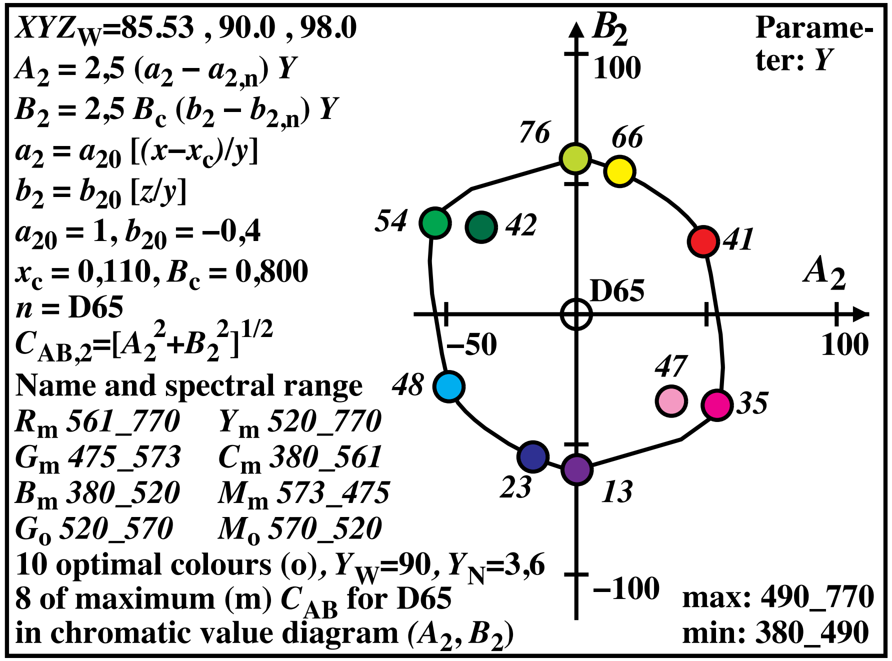


BEA01−5N



BEA01−6N

Figure 10: *Ostwald* optimal colours for the CIE standard illuminant D65 in (A_1,B_1) & (A_2,B_2)

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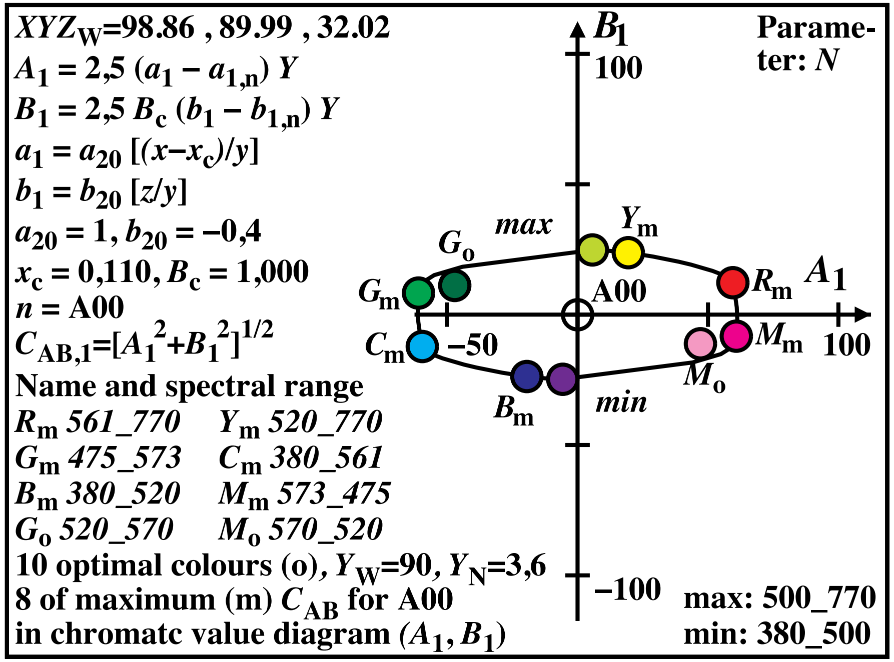
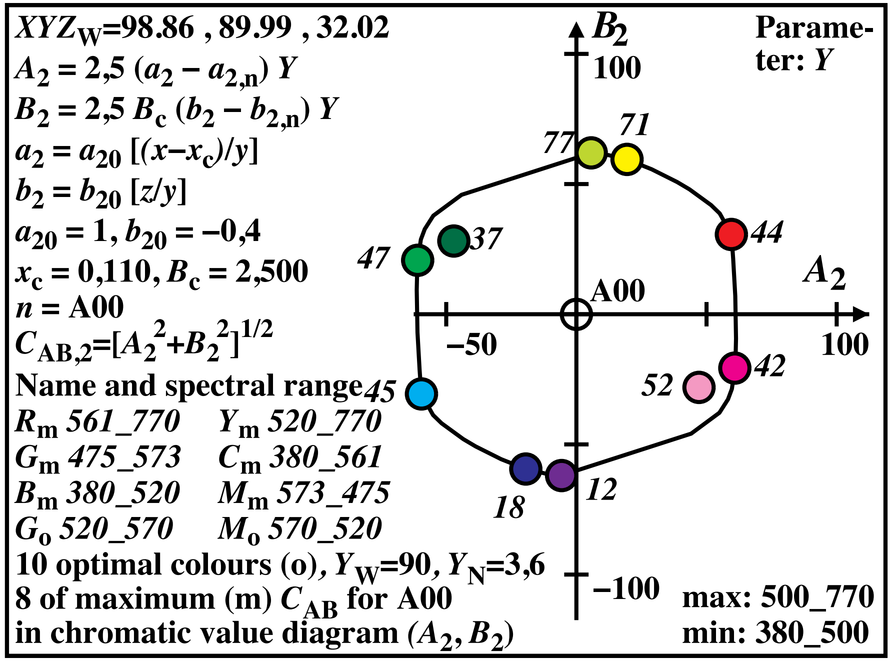


BEA01−7N



BEA01−8N

Figure 11: *Ostwald* optimal colours for the CIE standard illuminant A in (A_1,B_1) & (A_2,B_2)

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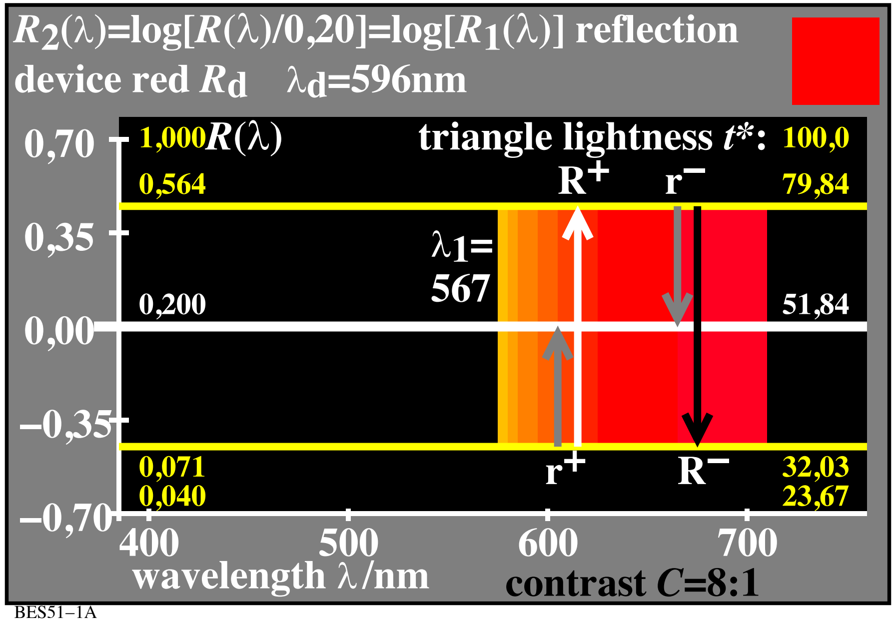

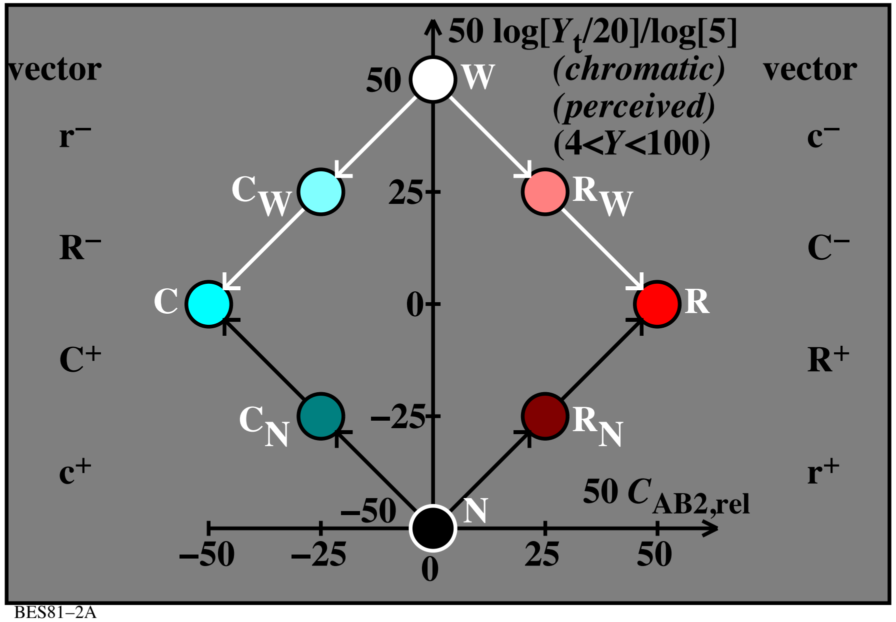

Figure 12: Logarithmic reflection for the contrast 8:1 and colour vectors.

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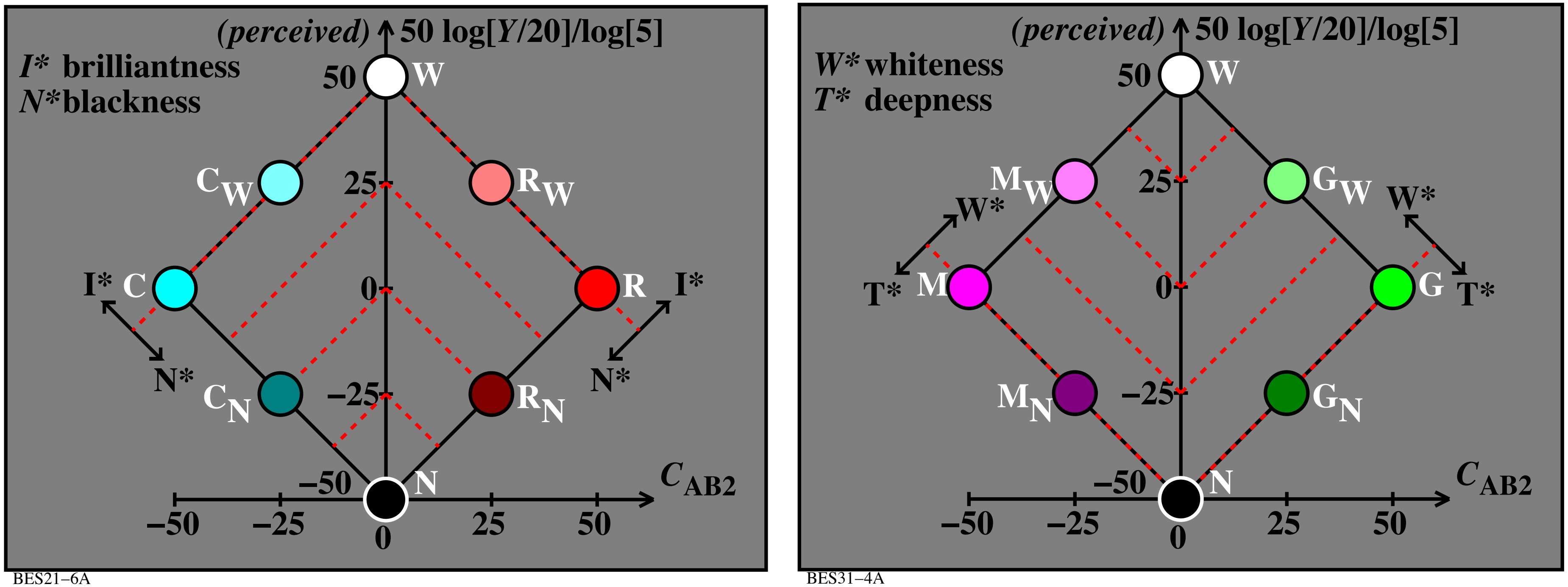

Figure 13: Antagonistic colour appearances blackness *N** and brillantness *I**
as well as whiteness *W** and colour deepness *T*.*

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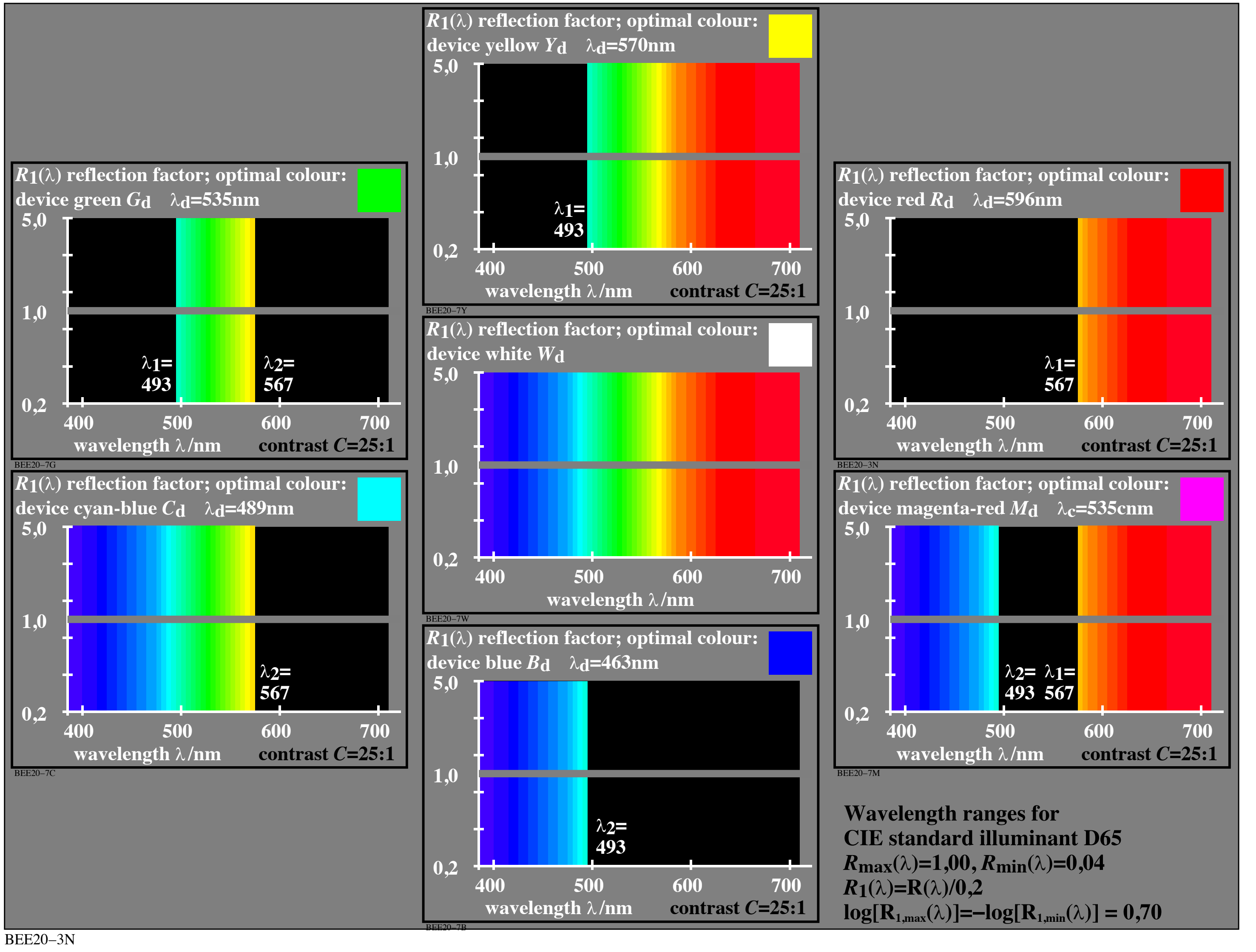

Figure 14: Normalized logarithmic reflection of optimal colours for the contrast 25:1.

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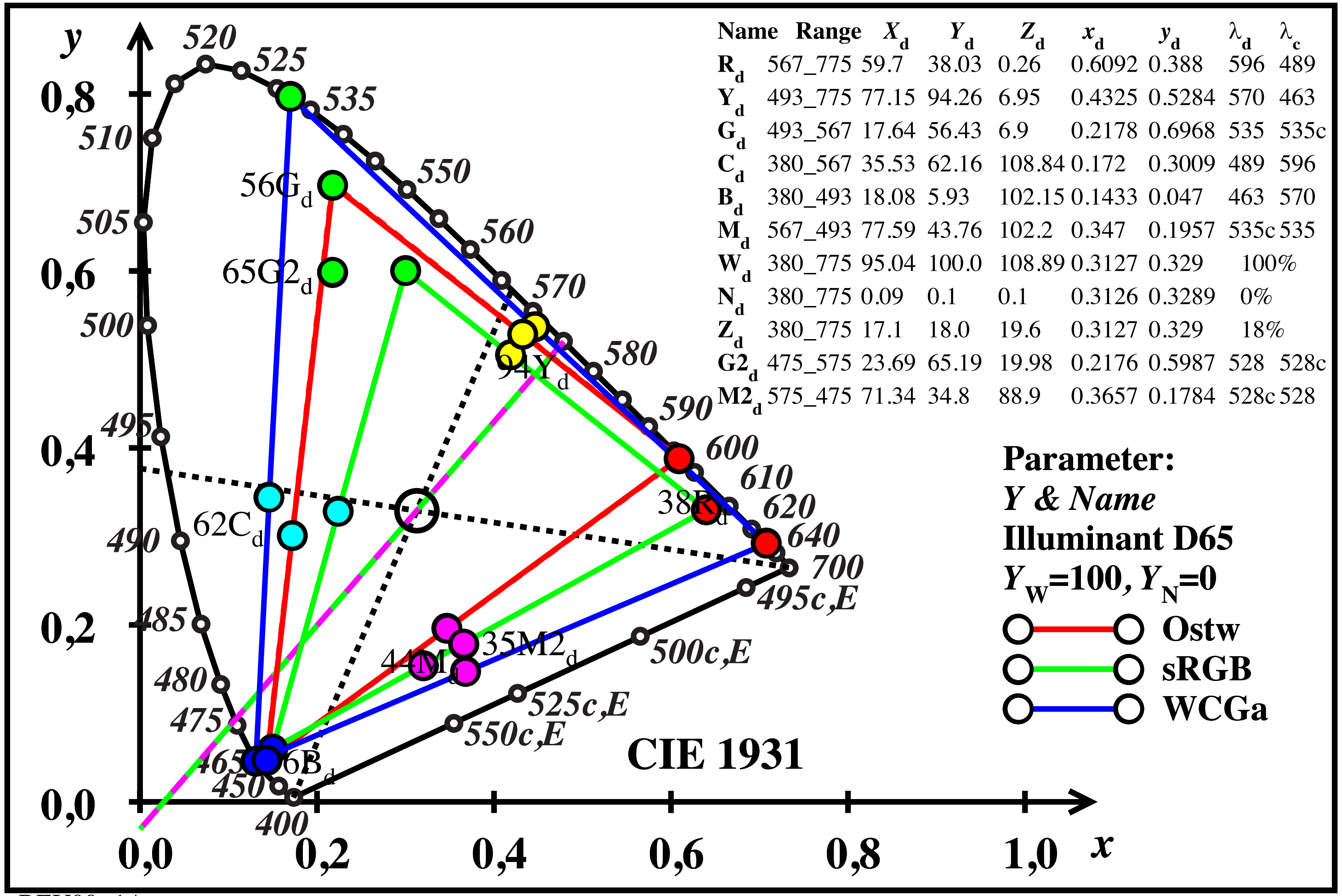

Name	Range	X_d	Y_d	Z_d	x_d	y_d	λ_d	λ_c
R_d	567_775	59.7	38.03	0.26	0.6092	0.388	596	489
Y_d	493_775	77.15	94.26	6.95	0.4325	0.5284	570	463
G_d	493_567	17.64	56.43	6.9	0.2178	0.6968	535	535c
C_d	380_567	35.53	62.16	108.84	0.172	0.3009	489	596
B_d	380_493	18.08	5.93	102.15	0.1433	0.047	463	570
M_d	567_493	77.59	43.76	102.2	0.347	0.1957	535c	535
W_d	380_775	95.04	100.0	108.89	0.3127	0.329	100%	
N_d	380_775	0.09	0.1	0.1	0.3126	0.3289	0%	
Z_d	380_775	17.1	18.0	19.6	0.3127	0.329	18%	
$G2_d$	475_575	23.69	65.19	19.98	0.2176	0.5987	528	528c
$M2_d$	575_475	71.34	34.8	88.9	0.3657	0.1784	528c	528

BEU00−1A

Figure 15: *Ostwald,* sRGB, and WCGa colours in the chromaticity diagram *(x, y).*

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XYZ_W=95.04 , 100.0 , 108.89

$A_2 = 2{,}5\ (a_2 - a_{2,n})\ Y$

$B_2 = 2{,}5\ B_c\ (b_2 - b_{2,n})\ Y$

$a_2 = a_{20}\ [\ (\ x - x_c\ )\ /\ y]$

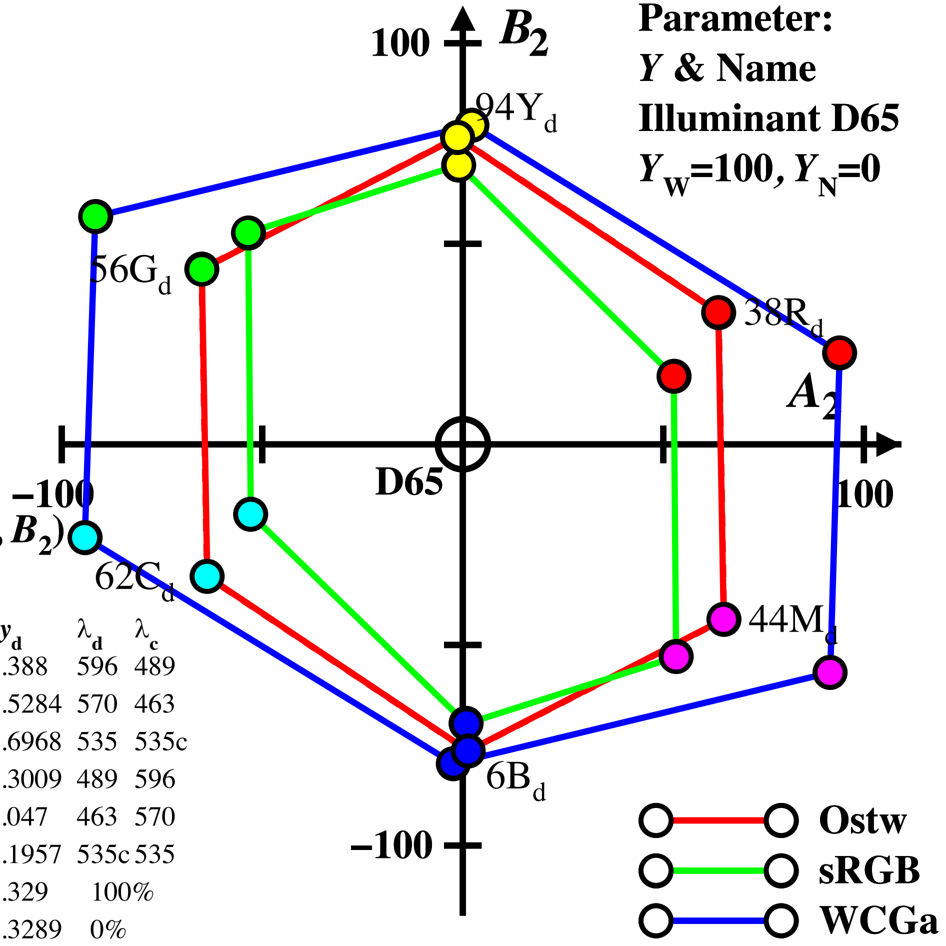
$b_2 = b_{20}\ [\ z\ /\ y]$

$a_{20} = 1,\ \ b_{20} = -0{,}4$

$x_c = 0{,}110,\ \ B_c = 0{,}800$

$C_{AB2}=[A_2^2+B_2^2]^{1/2}$

6 *Ostwald* colours (o)
of maximum (m) C_{AB} in
chromatic value diagram (A_2, B_2)
Illumin. D65, Y_W=100, Y_N=0



Name	Range	X_d	Y_d	Z_d	x_d	y_d	λ_d	λ_c
R_d	567_775	59.7	38.03	0.26	0.6092	0.388	596	489
Y_d	493_775	77.15	94.26	6.95	0.4325	0.5284	570	463
G_d	493_567	17.64	56.43	6.9	0.2178	0.6968	535	535c
C_d	380_567	35.53	62.16	108.84	0.172	0.3009	489	596
B_d	380_493	18.08	5.93	102.15	0.1433	0.047	463	570
M_d	567_493	77.59	43.76	102.2	0.347	0.1957	535c	535
W_d	380_775	95.04	100.0	108.89	0.3127	0.329	100%	
N_d	380_775	0.09	0.1	0.1	0.3126	0.3289	0%	
Z_d	380_775	17.1	18.0	19.6	0.3127	0.329	18%	

BEU00−1A

Figure 16: *Ostwald,* sRGB and WCGa colours in the chromatic value diagram *(A_2, B_2).*

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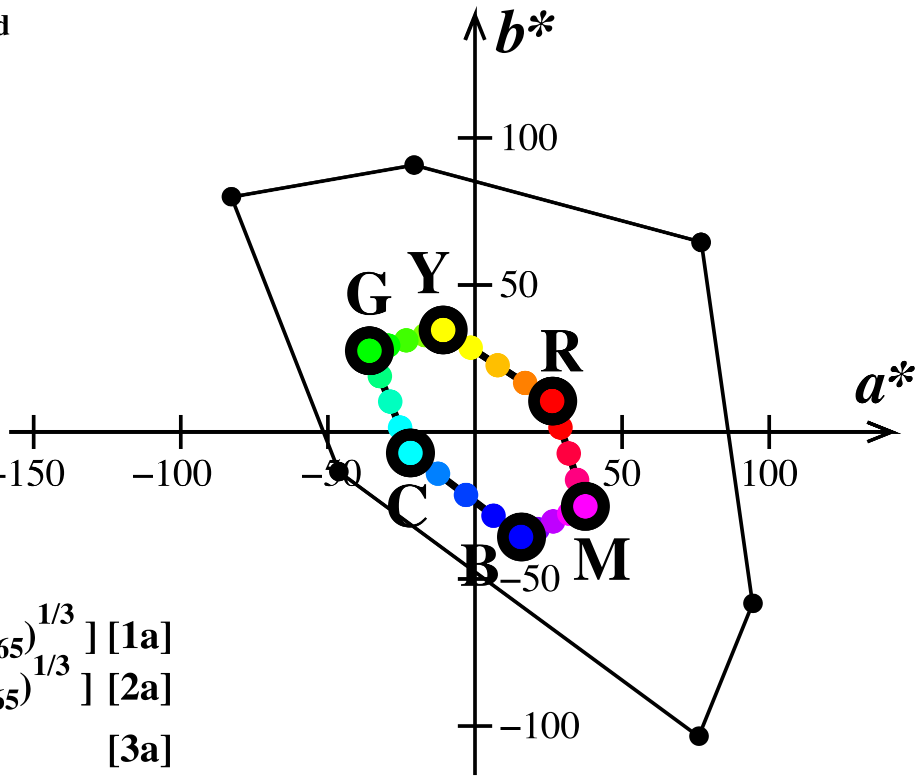
sRGB data *rgb**, *XYZxy*, and *LabC*h*$_{ab}$ in the CIELAB-colour space

Tristimulus values of black and white: Y_{Nn}=40,3, Y_{Wn}=88,6, Y_{Wa}=88,6.

	rgb^*_d	L^*_d	a^*_d	b^*_d	$C^*_{ab,d}$	$h_{ab,d}$
R_d	**1 0 0**	76	26	10	28	**21**
Y_d	**1 1 0**	93	−10	34	36	**107**
G_d	**0 1 0**	89	−35	27	45	**142**
C_d	**0 1 1**	90	−21	−7	23	**197**
B_d	**0 0 1**	72	15	−35	38	**293**
M_d	**1 0 1**	78	37	−25	45	**326**
N_d	**0 0 0**	69	0	0	0	**0**
W_d	**1 1 1**	95	0	0	0	**0**



$$a^* = 500\,[(X / X_{D65})^{1/3} - (Y / Y_{D65})^{1/3}] \quad [1a]$$

$$b^* = 200\,[(Y / Y_{D65})^{1/3} - (Z / Z_{D65})^{1/3}] \quad [2a]$$

$$C^*_{ab} = [\,a^{*2} + b^{*2}\,]^{0,5} \quad [3a]$$

$$h_{ab} = \text{atan}\,[\,b^* / a^*\,] \quad [4a]$$

BEE31−2N

Figure 17: WCGa colours in the CIELAB-chroma diagram *(a*, b*)* for the contrasts 2:1 and >288:1.

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higher colour metric (color data: nonlinear relation to CIE 1931 data)		
nonlinear color terms	**name and relationship with tristimulues or chromaticity values**	**notes**
lightness	$L^* = 116\,(Y/100)^{1/3} - 16 \quad (Y > 0{,}8)$ approximation: $L^*=100\,(Y/100)^{1/2,4} \; (Y>0)$	CIELAB 1976
chroma	*nonlinear transform chromatic values A, B*	
red–green	$a^* = 500\,[\,(X/X_n)^{1/3} - (Y/Y_n)^{1/3}\,]$ $= 500\,(a' - a'_n)\,Y^{1/3}$	CIELAB 1976
yellow–blue	$b^* = 200\,[\,(Y/Y_n)^{1/3} - (Z/Z_n)^{1/3}\,]$ $= 500\,(b' - b'_n)\,Y^{1/3}$	CIELAB 1976 *n=D65*
radial	$C^*_{ab} = [\,a^{*2} + b^{*2}\,]^{1/2}$	*(background)*
chromaticity	*nonlinear transform chromaticities x/y, z/y*	*compare to log cone excitation*
red–green	$a' = (1/X_n)^{1/3}\,(x/y)^{1/3}$ $= 0{,}2191\,(x/y)^{1/3}$ for D65	$\log[L/(L+M)]$
yellow–blue	$b' = -0{,}4\,(1/Z_n)^{1/3}\,(z/y)^{1/3}$ $= -0{,}08376\,(z/y)^{1/3}$ for D65	$\log[S/(L+M)]$
radial	$c'_{ab} = [\,(a' - a'_n)^2 + (b' - b'_n)^2\,]^{1/2}$	

AES01−7N

Figure 18: Chromaticity diagram (cube root diagram) *(a‘, b‘)* of CIELAB 1976

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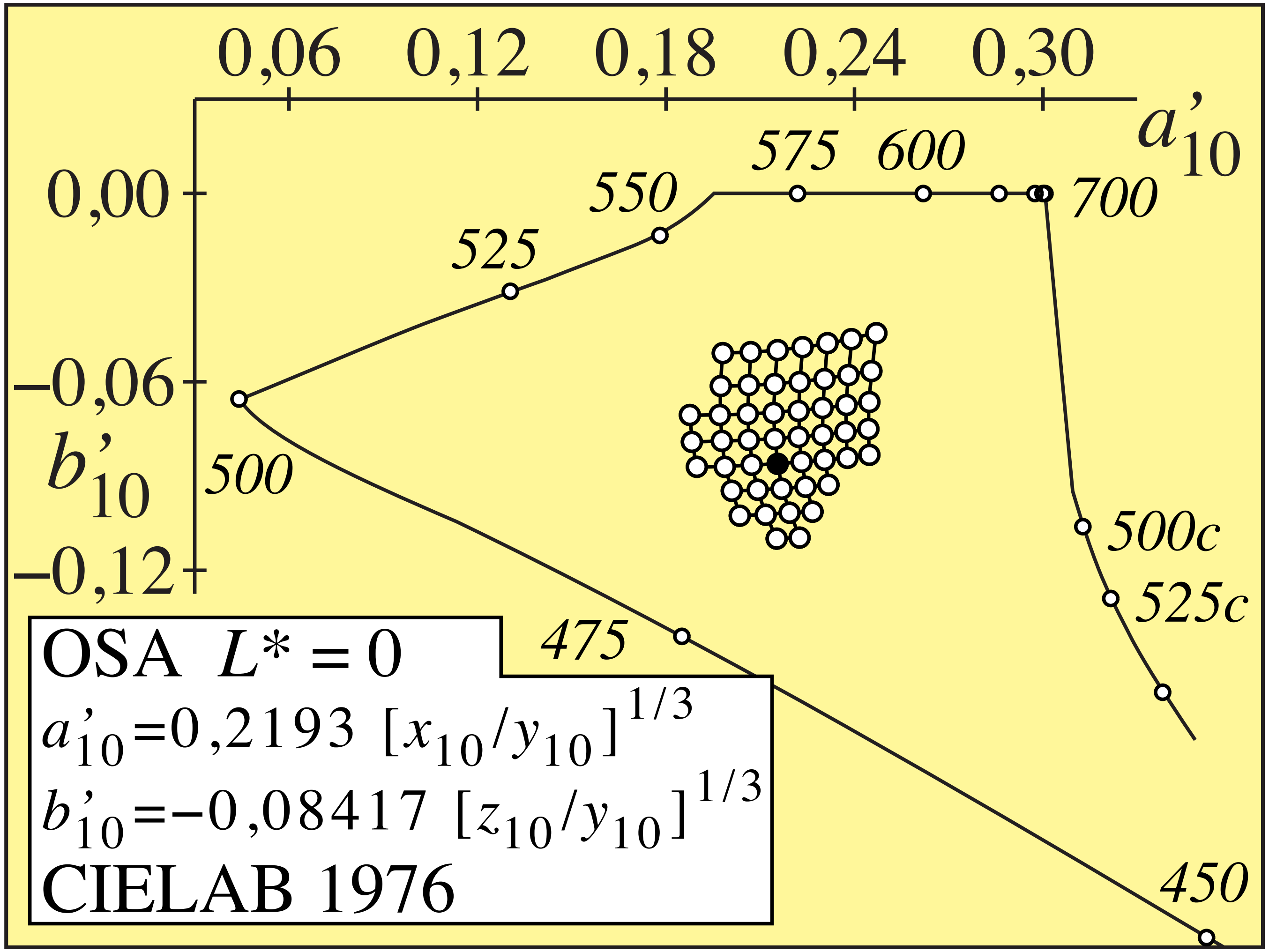

ME150−7, B4_14_2

Figure 19: OSA colours in the in chromaticity diagram *(a‘, b‘)* of CIELAB 1976

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Summary
Our daily environment requires a *logarithmic* and *antagonistic* lightness scaling, and in addition an antisymmetric metric compared to mean grey with Y_u=18. The lightness *L** has an asymptote for a bright white and a deep black (it is capped). This is also true for the chroma range yellow–blue.

In information technology the metric of the chromaticity and lightness scaling changes with the contrast *white : black*, see ISO 9241-306:2018 and the corresponding ISO-test charts
http://standards.iso.org/iso/9241/306/ed-2/index.html

For example it is valid *n*=1/1,2 (nearly linear) for a small contrast Y_W:Y_N=2:1 via *n*=1/2,4 (CIELAB and sRGB colour space according to IEC 61966-2-1) for the standard contrast Y_W:Y_N=25:1 up to *n*=1/3,6 for the high contrast Y_W:Y_N larger as 288:1. Also the metric of the antagonistic colour-appearance attributes blackness *N** and brilliantness (brilliance) *I*,* as well as whiteness *W** and colour deepness *T** changes with the exponent *n*.

In 1976 a proposed cube-root chromaticity diagram *(a‘, b‘)* for CIELAB 1976 was not accepted. However, chromatic adaptation is approximately a linear translation in *(a’, b’)*. This is confirmed by haploscopic experiments, see *K. Richter* (1980) in CR&A. Similar a linear translation is used in *(u‘, v‘)* of CIELUV 1976 for chromatic adaptation. However, the equally spaced OSA colours of Fig. 19 in the diagram *(a’, b’)* are much less equally spaced in the diagram *(u‘, v‘).*

The chromaticity *a‘* is approximately a linear function of the chromaticities *x* and *y.*
The chromaticiy *b‘* changes with the scene contrast. It is valid *n*=1/1,2, 1/2,4 (CIELAB), and 1/3,6 for the scene contrasts 2:1, 36:1 (ISO 9241-306) and >288:1, for example for the Wide Colour Gamut Displays (WCGa). Equal yellow-blue chroma, for example for D65 and A, require equal chromaticity differences b'-b'_{n}.. For constant *Y* this corresponds to a simple translation.

For information about colorimetric data in many languages, see http://farbe.li.tu-berlin.de/index.html
For other papers of the author and his research group, see http://farbe.li.tu-berlin.de/XY91FEN.html

Annex A:

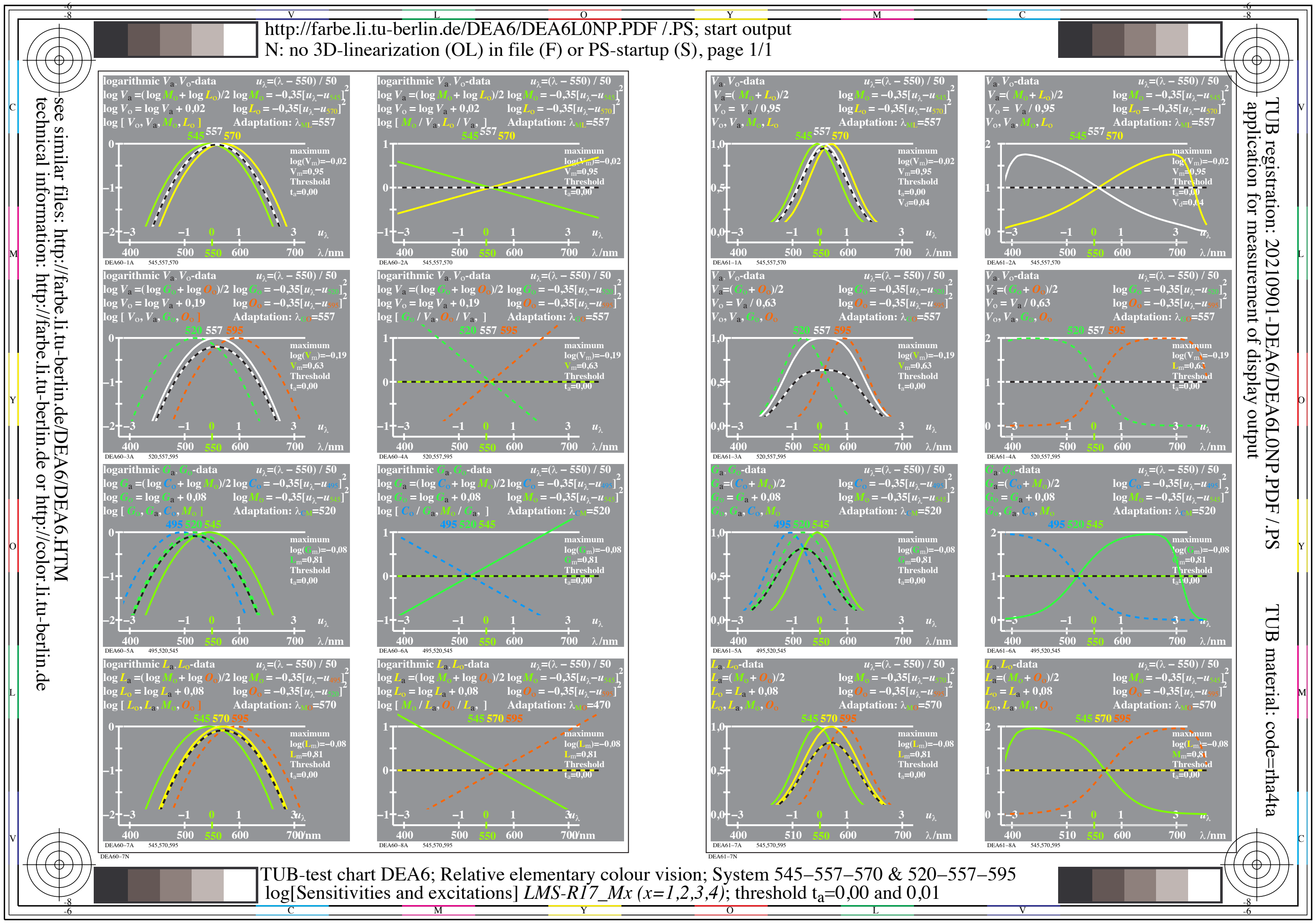

Figure A.1: Sums and differences of *L* und *M* receptor-colour values in log and linear plots

For the download of this figure: http://farbe.li.tu-berlin.de/DEA6/DEA6L0NP.PDF