

Line-element examples for grey samples (0,2≤x≤5)

$F(x)$ is called the line-element function of $f(x)$.
 The following relations are valid for $x=Y/Y_u=Y/18$:

$$\frac{d[F_u(x)]}{dx} = f(x) \quad [1]$$

$$F_u(x) = \int \frac{f'(x)}{f(x)} dx \quad [2]$$

Example for the normalized tristimulus value $x=Y/Y_u$:

$$\frac{d[\ln(1+b x)]}{dx} = \frac{ab}{1+b x} \quad [3]$$

$$a \ln(1+b x) = \int \frac{ab}{1+b x} dx \quad [4]$$

eeo00-1n DEQ60-1N

Line-element examples for grey samples (0,2≤x≤5)

$F_u(x)$ is called the line-element function of $f_u(x)$.
 Both functions are normalized to the surround value:

$$\frac{d[F_u(x)]}{dx} = f_u(x) \quad [1]$$

$$F_u(x) = \int \frac{f'_u(x)}{f_u(x)} dx \quad [2]$$

Example for the normalized functions with $x_u=1$:

$$F_u(x) = \frac{F(x)}{F(x_u)} = \frac{\ln(1+b x)}{\ln(1+b)} \quad [3]$$

$$f_u(x) = \frac{f(x)}{f(x_u)} = \frac{1+b x}{1+b} \quad [4]$$

eeo00-2n DEQ60-2N

Line-element examples for grey samples (0,2≤Y_r≤5)

$F(Y_r)$ is called the line-element function of $f(Y_r)$.
 The following relations are valid for $Y_r=Y/Y_u=Y/18$:

$$\frac{d[F(Y_r)]}{dY_r} = f(Y_r) \quad [1]$$

$$F(Y_r) = \int \frac{f'(Y_r)}{f(Y_r)} dY_r \quad [2]$$

Example for the normalized tristimulus value $Y_r=Y/Y_u$:

$$\frac{d[\ln(1+b Y_r)]}{dY_r} = \frac{ab}{1+b Y_r} \quad [3]$$

$$a \ln(1+b Y_r) = \int \frac{ab}{1+b Y_r} dY_r \quad [4]$$

eeo01-1n DEQ61-1N

Line-element examples for grey samples (0,2≤Y_r≤5)

$F_u(Y_r)$ is called the line-element function of $f_u(Y_r)$.
 Both functions are normalized to the surround value:

$$\frac{d[F_u(Y_r)]}{dY_r} = f_u(Y_r) \quad [1]$$

$$F_u(Y_r) = \int \frac{f'_u(Y_r)}{f_u(Y_r)} dY_r \quad [2]$$

Example for the normalized functions with $Y_r=1$:

$$F_u(Y_r) = \frac{F(Y_r)}{F(1)} = \frac{\ln(1+b Y_r)}{\ln(1+b)} \quad [3]$$

$$f_u(Y_r) = \frac{f(Y_r)}{f(1)} = \frac{1+b Y_r}{1+b} \quad [4]$$

eeo01-2n DEQ61-2N

Line-element equations according to CIE 230:2019

Colour-threshold (t) function $f_t(x) = \Delta Y_t = \Delta x Y_u$ [0]
 $\Delta Y_t = (A_1 + A_2 Y) / A_0$ $A_0=1.5, A_1=0.0170, A_2=0.0058$

$$\frac{d[F_u(x)]}{dx} = f_u(x) \quad [1]$$

$$F_u(x) = \int \frac{f'_u(x)}{f_u(x)} dx = \int \frac{b}{1+b x} dx \quad [2]$$

Example for $L^*(x)$ & ΔY with $x=Y/Y_u, x_u=1, b=6,141$:

$$L^*_u(x) = \frac{L^*(x)}{L^*(x_u)} = \frac{\ln(1+b x)}{\ln(1+b)} \quad [3]$$

$$f_u(x) = \frac{\Delta Y}{\Delta Y_u} = \frac{1+b x}{1+b} \quad [4]$$

eeo00-3n DEQ60-3N

Line-element equations according to CIE 230:2019

Colour-threshold (t) function $f_t(x) = \Delta Y_t = \Delta x Y_u$ [0]
 $\Delta Y_t = (A_1 + A_2 Y) / A_0$ $A_0=1.5, A_1=0.0170, A_2=0.0058$

$$f_{tu}(x) = \frac{\Delta Y_t}{\Delta Y_{tu}} = \frac{1+b x}{1+b} \quad b=A_2 Y_u / A_1 \quad x=Y/Y_u \quad [1]$$

$$F_{tu}(x) = \int \frac{f'_{tu}(x)}{f_{tu}(x)} dx = \int \frac{b}{1+b x} dx \quad [2]$$

Example for $L^*_{tu}(x)$, ΔY_t with $x=Y/Y_u, x_u=1, b=6,141$:

$$L^*_{tu}(x) = \frac{L^*(x)}{L^*(x_u)} = \frac{\ln(1+b x)}{\ln(1+b)} \quad [3]$$

$$f_{tu}(x) = \frac{\Delta Y_t}{\Delta Y_{tu}} = \frac{1+b x}{1+b} \quad [4]$$

eeo00-4n DEQ60-4N

Line-element examples for grey samples (0,2≤Y_r≤5)

$F_u(Y_r)$ is called the line-element function of $f_u(Y_r)$.
 Both functions are normalized to the surround value:

$$\frac{d[F_u(Y_r)]}{dY_r} = f_u(Y_r) \quad [1]$$

$$F_u(Y_r) = \int \frac{f'_u(Y_r)}{f_u(Y_r)} dY_r = \int \frac{b}{1+b Y_r} dY_r \quad [2]$$

Example for $L^*(Y_r)$ & ΔY_r with $Y_r=1, b=6,141$:

$$L^*_u(Y_r) = \frac{L^*(Y_r)}{L^*(Y_{ru})} = \frac{\ln(1+b Y_r)}{\ln(1+b)} \quad [3]$$

$$f_u(Y_r) = \frac{\Delta Y_r}{\Delta Y_{ru}} = \frac{1+b Y_r}{1+b} \quad [4]$$

eeo01-3n DEQ61-3N

Line-element equations according to CIE 230:2019

Colour-threshold (t) function $f_t(Y_r) = \Delta Y_t = \Delta Y_r Y_u$ [0]
 $\Delta Y_t = (A_1 + A_2 Y) / A_0$ $A_0=1.5, A_1=0.0170, A_2=0.0058$

$$f_{tu}(Y_r) = \frac{\Delta Y_t}{\Delta Y_{tu}} = \frac{1+b Y_r}{1+b} \quad b=A_2 Y_u / A_1 \quad Y_r=Y/Y_u \quad [1]$$

$$F_{tu}(Y_r) = \int \frac{f'_{tu}(Y_r)}{f_{tu}(Y_r)} dY_r = \int \frac{b}{1+b Y_r} dY_r \quad [2]$$

Example for $L^*_{tu}(Y_r)$, ΔY_t with $Y_r=Y/Y_u, Y_{ru}=1, b=6,141$:

$$L^*_{tu}(Y_r) = \frac{L^*(Y_r)}{L^*(Y_{ru})} = \frac{\ln(1+b Y_r)}{\ln(1+b)} \quad [3]$$

$$f_{tu}(Y_r) = \frac{\Delta Y_t}{\Delta Y_{tu}} = \frac{1+b Y_r}{1+b} \quad [4]$$

eeo01-4n DEQ61-4N

Line-element equations according to CIE 230:2019

Colour-discrimination function $f(x) = \Delta Y = \Delta x Y_u$ [0]
 $\Delta Y = 1 / [(1+x)(2+x)] = 1 / [1+x] - 1 / [2+x]$ $x = \sqrt{2} e^{k(u-u_0)}$

$$f_u(x) = \frac{\Delta Y}{\Delta Y_u} = \frac{1+b x}{1+b} \quad b=A_2 Y_u / A_1 \quad x=Y/Y_u \quad [1]$$

$$F_u(x) = \int \frac{f'_u(x)}{f_u(x)} dx = \int \frac{b}{1+b x} dx \quad [2]$$

Example for $L^*(x)$ & ΔY with $x=Y/Y_u, x_u=1, b=6,141$:

$$L^*_u(x) = \frac{L^*(x)}{L^*(x_u)} = \frac{\ln(1+b x)}{\ln(1+b)} \quad [3]$$

$$f_u(x) = \frac{\Delta Y}{\Delta Y_u} = \frac{1+b x}{1+b} \quad [4]$$

eeo00-5n DEQ60-5N

Line-element equations for thresholds and scaling

Colour-discrimination function $f(x) = \Delta Y = \Delta x Y_u$ [0]
 $\Delta Y = 1 / [(1+x)(2+x)] = 1 / [1+x] - 1 / [2+x]$ $x = \sqrt{2} e^{k(u-u_0)}$

$$f_u(x) = \frac{\Delta Y}{\Delta Y_u} = \frac{1+b x}{1+b} - \frac{1+0,5b x}{1+0,5b} \quad b=1, x=Y/Y_u \quad [1]$$

$$F_u(x) = \int \frac{f'_u(x)}{f_u(x)} dx = \int \frac{b}{1+b x} dx - \int \frac{0,5b}{1+0,5b x} dx \quad [2]$$

Example for $L^*(x)$ & ΔY with $x=Y/Y_u, x_u=1, b=1$:

$$L^*_u(x) = \frac{L^*(x)}{L^*(x_u)} = \frac{\ln(1+b x)}{\ln(1+b)} - \frac{\ln(1+0,5b x)}{\ln(1+0,5b)} \quad [3]$$

$$f_u(x) = \frac{\Delta Y}{\Delta Y_u} = \frac{1+b x}{1+b} - \frac{1+0,5b x}{1+0,5b} \quad [4]$$

see K. Richter (1996), Computer Graphic and Colorimetry, p. 113-127
<http://color.li.tu-berlin.de/BUA4BF.PDF>

eeo00-6n DEQ60-6N

Line-element equations according to CIE 230:2019

Colour-discrimination function $f(Y_r) = \Delta Y_r$ [0]
 $\Delta Y_r = (A_1 + A_2 Y) / A_0$ $A_0=1.5, A_1=0.0170, A_2=0.0058$

$$f_u(Y_r) = \frac{\Delta Y_r}{\Delta Y_{ru}} = \frac{1+b Y_r}{1+b} \quad b=A_2 Y_u / A_1 \quad Y_r=Y/Y_u \quad [1]$$

$$F_u(Y_r) = \int \frac{f'_u(Y_r)}{f_u(Y_r)} dY_r = \int \frac{b}{1+b Y_r} dY_r \quad [2]$$

Example for $L^*(Y_r)$ & ΔY_r with $Y_r=Y/Y_u, Y_{ru}=1, b=6,141$:

$$L^*_u(Y_r) = \frac{L^*(Y_r)}{L^*(Y_{ru})} = \frac{\ln(1+b Y_r)}{\ln(1+b)} \quad [3]$$

$$f_u(Y_r) = \frac{\Delta Y_r}{\Delta Y_{ru}} = \frac{1+b Y_r}{1+b} \quad [4]$$

eeo01-5n DEQ61-5N

Line-element equations for thresholds and scaling

Colour-discrimination function $f(Y_r) = \Delta Y_r, u_r = \ln Y_r$ [0]
 $\Delta Y_r = 1 / [(1+Y_r)(2+Y_r)] = 1 / [1+Y_r] - 1 / [2+Y_r]$ $Y_r = \sqrt{2} e^{ku}$

$$f_u(Y_r) = \frac{\Delta Y_r}{\Delta Y_{ru}} = \frac{1+b Y_r}{1+b} - \frac{1+0,5b Y_r}{1+0,5b} \quad b=1, Y_r=Y/Y_u \quad [1]$$

$$F_u(Y_r) = \int \frac{f'_u(Y_r)}{f_u(Y_r)} dY_r = \int \frac{b}{1+b Y_r} dY_r - \int \frac{0,5b}{1+0,5b Y_r} dY_r \quad [2]$$

Example for $L^*(Y_r)$ & ΔY_r with $Y_r=Y/Y_u, Y_{ru}=1, b=1$:

$$L^*_u(Y_r) = \frac{L^*(Y_r)}{L^*(Y_{ru})} = \frac{\ln(1+b Y_r)}{\ln(1+b)} - \frac{\ln(1+0,5b Y_r)}{\ln(1+0,5b)} \quad [3]$$

$$f_u(Y_r) = \frac{\Delta Y_r}{\Delta Y_{ru}} = \frac{1+b Y_r}{1+b} - \frac{1+0,5b Y_r}{1+0,5b} \quad [4]$$

see K. Richter (1996), Computer Graphic and Colorimetry, p. 113-127
<http://color.li.tu-berlin.de/BUA4BF.PDF>

eeo01-6n DEQ61-6N

Line-element equations for thresholds and scaling

Colour-discrimination function $f(x) = \Delta Y = \Delta x Y_u$ [0]
 $\Delta Y = 1 / [(1+y)(1+y)] = 1 / (1+y) - 1 / (1+y)$ $y = (1 + \sqrt{2}) e^{k(u-u_0)}$

$$f_u(y) = \frac{\Delta Y}{\Delta Y_u} = \frac{1+y}{2} - \frac{2+y}{3} \quad x=Y/Y_u \quad [1]$$

$$F_u(x) = \int \frac{f'_u(x)}{f_u(x)} dx = \int \frac{1}{1+x} dx - \int \frac{1}{2+x} dx \quad [2]$$

Example for $L^*(x)$ & ΔY with $x=Y/Y_u, x_u=1$:

$$L^*_u(x) = \frac{L^*(x)}{L^*(x_u)} = \frac{\ln(1+x)}{\ln(2)} - \frac{\ln(1+0,5 x)}{\ln(1,5)} \quad [3]$$

$$f_u(x) = \frac{\Delta Y}{\Delta Y_u} = \frac{1+x}{2} - \frac{1+0,5 x}{1,5} \quad [4]$$

see K. Richter (1996), Computer Graphic and Colorimetry, p. 113-127
<http://color.li.tu-berlin.de/BUA4BF.PDF>

eeo00-7n DEQ60-7N

Line-element equations for thresholds and scaling

Colour-discrimination function $f(y) = \Delta Y = \Delta y Y_u$ [0]
 $\Delta Y = 1 / [(1+y)(1+y)] = 1 / (1+y) - 1 / (1+y)$ $y = (1 + \sqrt{2}) e^{k(u-u_0)}$

$$f_u(y) = \frac{\Delta Y}{\Delta Y_u} = \frac{y}{2} - \frac{1+y}{3} \quad y=1+Y/Y_u, dy=dx \quad [1]$$

$$F_u(y) = \int \frac{f'_u(y)}{f_u(y)} dy = \int \frac{1}{y} dy - \int \frac{1}{1+y} dy \quad [2]$$

Example for $L^*(y)$ & ΔY with $y=1+Y/Y_u, y_u=2$:

$$L^*_u(y) = \frac{L^*(y)}{L^*(y_u)} = \frac{\ln(1+y)}{\ln(2)} - \frac{\ln(1+y)}{\ln(3)} \quad [3]$$

$$f_u(y) = \frac{\Delta Y}{\Delta Y_u} = \frac{1+x}{2} - \frac{1+0,5 x}{1,5} \quad [4]$$

see K. Richter (1996), Computer Graphic and Colorimetry, p. 113-127
<http://color.li.tu-berlin.de/BUA4BF.PDF>

eeo00-8n DEQ60-8N

Line-element equations for thresholds and scaling

Colour-discrimination function $f(Y_r) = \Delta Y_r, u_r = \ln Y_r$ [0]
 $\Delta Y_r = 1 / [(1+Y_r)(2+Y_r)] = 1 / [1+Y_r] - 1 / [2+Y_r]$ $Y_r = \sqrt{2} e^{ku}$

$$f_u(Y_r) = \frac{\Delta Y_r}{\Delta Y_{ru}} = \frac{1+Y_r}{2} - \frac{2+Y_r}{3} \quad Y_r=Y/Y_u \quad [1]$$

$$F_u(Y_r) = \int \frac{f'_u(Y_r)}{f_u(Y_r)} dY_r = \int \frac{dY_r}{1+Y_r} - \int \frac{dY_r}{2+Y_r} \quad [2]$$

Example for $L^*(Y_r)$ & ΔY_r with $Y_r=Y/Y_u=1$:

$$L^*_u(Y_r) = \frac{L^*(Y_r)}{L^*(Y_{ru})} = \frac{\ln(1+Y_r)}{\ln(2)} - \frac{\ln(1+0,5 Y_r)}{\ln(1,5)} \quad [3]$$

$$f_u(Y_r) = \frac{\Delta Y_r}{\Delta Y_{ru}} = \frac{1+Y_r}{2} - \frac{1+0,5 Y_r}{1,5} \quad [4]$$

see K. Richter (1996), Computer Graphic and Colorimetry, p. 113-127
<http://color.li.tu-berlin.de/BUA4BF.PDF>

eeo01-7n DEQ61-7N

Line-element equations for thresholds and scaling

Colour-discrimination function $f(y) = \Delta Y = \Delta y Y_u$ [0]
 $\Delta Y = 1 / [(1+y)(1+y)] = 1 / (1+y) - 1 / (1+y)$ $y = 1 + \sqrt{2} e^{k(u-u_0)}, u_r = \ln Y_r$

$$f_u(y) = \frac{\Delta Y}{\Delta Y_u} = \frac{y}{2} - \frac{1+y}{3} \quad y=1+Y/Y_u, dy=dx \quad [1]$$

$$F_u(y) = \int \frac{f'_u(y)}{f_u(y)} dy = \int \frac{1}{y} dy - \int \frac{1}{1+y} dy \quad [2]$$

Example for $L^*(y)$ & ΔY with $y=1+Y/Y_u, y_u=2$:

$$L^*_u(y) = \frac{L^*(y)}{L^*(y_u)} = \frac{\ln(1+y)}{\ln(2)} - \frac{\ln(1+y)}{\ln(3)} \quad [3]$$

$$f_u(y) = \frac{\Delta Y}{\Delta Y_u} = \frac{1+y}{2} - \frac{1+0,5 y}{1,5} \quad [4]$$

see K. Richter (1996), Computer Graphic and Colorimetry, p. 113-127
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eeo01-8n DEQ61-8N