

Mathematical equations of hyperbolic functions

See: *Handbook of mathematical functions, NBS, USA, Sec. 4.5*

$$F_{ab}(x/a) = b \tanh(x/a) = b \frac{e^{x/a} - e^{-x/a}}{e^{x/a} + e^{-x/a}} \quad [1]$$

$$\frac{dF_{ab}(x/a)}{dx} = \frac{4b}{a[e^{x/a} + e^{-x/a}]^2} = \frac{b}{a \cosh^2(x/a)} \quad [4]$$

$$\frac{dF_{ab}(x_r/a)}{dx_r} = \frac{4b}{a[e^{x_r/a} + e^{-x_r/a}]^2} \quad x_r = \log(L/L_u) \quad [5]$$

$dx_r/dL = \ln(10)/L$

$$\frac{dF_{ab}(x_r/a)}{dx_r} \frac{dx_r}{dL} = \frac{4b}{a[e^{x_r/a} + e^{-x_r/a}]^2} \frac{\ln(10)}{L} \quad [6]$$