

Mathematical equations of hyperbolic functions

See: *Handbook of mathematical functions, NBS, USA, Sec. 4.5*

$$F_{abu}(x_r/a) = \tanh(x_r/a) = \frac{e^{x_r/a} - e^{-x_r/a}}{e^{x_r/a} + e^{-x_r/a}} \quad [1u]$$

$$\frac{dF_{abu}(x_r/a)}{dx_r} = \frac{4}{a[e^{x_r/a} + e^{-x_r/a}]^2} \quad x_r = \log(L/L_u) \quad [5u]$$

$dx_r/dL = \ln(10)/L$

$$\frac{dF_{abu}(x_r/a)}{dx_r} \frac{dx_r}{dL} = \frac{4}{a[e^{x_r/a} + e^{-x_r/a}]^2} \frac{\ln(10)}{L} \quad [6u]$$

$$\frac{L}{dL} = \frac{4 \ln(10)}{a[e^{x_r/a} + e^{-x_r/a}]^2} \quad dL = \frac{a[e^{x_r/a} + e^{-x_r/a}]^2 L}{4 \ln(10)} \quad [7u]$$