Line-element examples for grey samples ($0{,}2 \le x \le 5$)

$F(x)$ is called the line-element function of $f(x)$.
The following relations are valid for $x=Y/Y_u=Y/18$:

$$\frac{d[F(x)]}{dx} = f(x) \quad [1]$$

$$F(x) = \int \frac{f'(x)}{f(x)}\,dx \quad [2]$$

Example for the normalized tristimulus value $x=Y/Y_u$:

$$\frac{d[a\ln(1+bx)]}{dx} = \frac{ab}{1+bx} \quad [3]$$

$$a\ln(1+bx) = \int \frac{ab}{1+bx}\,dx \quad [4]$$

hex00−1n DEQ60−1N

Line-element examples for grey samples ($0{,}2 \le x \le 5$)

$F_u(x)$ is called the line-element function of $f_u(x)$.
Both functions are normalized to the surround value:

$$\frac{d[F_u(x)]}{dx} = f_u(x) \quad [1]$$

$$F_u(x) = \int \frac{f'_u(x)}{f_u(x)}\,dx \quad [2]$$

Example for the normalized functions with $x_u=1$:

$$F_u(x) = \frac{F(x)}{F(x_u)} = \frac{\ln(1+bx)}{\ln(1+b)} \quad [3]$$

$$f_u(x) = \frac{f(x)}{f(x_u)} = \frac{1+bx}{1+b} \quad [4]$$

hex00−2n DEQ60−2N

Line-element examples for grey samples ($0{,}2 \le Y_r \le 5$)

$F(Y_r)$ is called the line-element function of $f(Y_r)$.
The following relations are valid for $Y_r=Y/Y_u=Y/18$:

$$\frac{d[F(Y_r)]}{dY_r} = f(Y_r) \quad [1]$$

$$F(Y_r) = \int \frac{f'(Y_r)}{f(Y_r)}\,dY_r \quad [2]$$

Example for the normalized tristimulus value $Y_r=Y/Y_u$:

$$\frac{d[a\ln(1+bY_r)]}{dY_r} = \frac{ab}{1+bY_r} \quad [3]$$

$$a\ln(1+bY_r) = \int \frac{ab}{1+bY_r}\,dY_r \quad [4]$$

hex01−1n DEQ61−1N

Line-element examples for grey samples ($0{,}2 \le Y_r \le 5$)

$F_u(Y_r)$ is called the line-element function of $f_u(Y_r)$.
Both functions are normalized to the surround value:

$$\frac{d[F_u(Y_r)]}{dY_r} = f_u(Y_r) \quad [1]$$

$$F_u(Y_r) = \int \frac{f'_u(Y_r)}{f_u(Y_r)}\,dY_r \quad [2]$$

Example for the normalized functions with $Y_r=1$:

$$F_u(Y_r) = \frac{F(Y_r)}{F(1)} = \frac{\ln(1+bY_r)}{\ln(1+b)} \quad [3]$$

$$f_u(Y_r) = \frac{f(Y_r)}{f(1)} = \frac{1+bY_r}{1+b} \quad [4]$$

hex01−2n DEQ61−2N

Line-element examples for grey samples ($0{,}2 \le x \le 5$)

$F_u(x)$ is called the line-element function of $f_u(x)$.
Both functions are normalized to the surround value:

$$\frac{d[F_u(x)]}{dx} = f_u(x) \quad [1]$$

$$F_u(x) = \int \frac{f'_u(x)}{f_u(x)}\,dx = \int \frac{b}{1+bx}\,dx \quad [2]$$

Example for $L^*(x)$ & ΔY with $x=Y/Y_u$, $x_u=1$, b=6,141:

$$L^*_u(x) = \frac{L^*(x)}{L^*(x_u)} = \frac{\ln(1+bx)}{\ln(1+b)} \quad [3]$$

$$f_u(x) = \frac{\Delta Y}{\Delta Y_u} = \frac{1+bx}{1+b} \quad [4]$$

hex00−3n DEQ60−3N

Line-element equations according to CIE 230:2019

Colour-threshold (t) function $f_t(x) = \Delta Y_t = \Delta x\, Y_u$ [0]
$\Delta Y_t=(A_1+A_2Y)/A_0$ $A_0=1{,}5$, $A_1=0{,}0170$, $A_2=0{,}0058$

$$f_{tu}(x) = \frac{\Delta Y_t}{\Delta Y_{tu}} = \frac{1+bx}{1+b} \quad b=A_2Y_u/A_1 \quad x=Y/Y_u \quad [1]$$

$$F_{tu}(x) = \int \frac{f'_{tu}(x)}{f_{tu}(x)}\,dx = \int \frac{b}{1+bx}\,dx \quad [2]$$

Example for $L^*_{tu}(x)$, ΔY_t with $x=Y/Y_u$, $x_u=1$, b=6,141:

$$L^*_{tu}(x) = \frac{L^*_t(x)}{L^*_{tu}(x)} = \frac{\ln(1+bx)}{\ln(1+b)} \quad [3]$$

$$f_{tu}(x) = \frac{\Delta Y_t}{\Delta Y_{tu}} = \frac{1+bx}{1+b} \quad [4]$$

hex00−4n DEQ60−4N

Line-element examples for grey samples ($0{,}2 \le Y_r \le 5$)

$F_u(Y_r)$ is called the line-element function of $f_u(Y_r)$.
Both functions are normalized to the surround value:

$$\frac{d[F_u(Y_r)]}{dY_r} = f_u(Y_r) \quad [1]$$

$$F_u(Y_r) = \int \frac{f'_u(Y_r)}{f_u(Y_r)}\,dY_r = \int \frac{b}{1+bY_r}\,dY_r \quad [2]$$

Example for $L^*(Y_r)$ & ΔY_r with $Y_{ru}=1$, b=6,141:

$$L^*_u(Y_r) = \frac{L^*(Y_r)}{L^*(Y_{ru})} = \frac{\ln(1+bY_r)}{\ln(1+b)} \quad [3]$$

$$f_u(Y_r) = \frac{\Delta Y_r}{\Delta Y_{ru}} = \frac{1+bY_r}{1+b} \quad [4]$$

hex01−3n DEQ61−3N

Line-element equations according to CIE 230:2019

Colour-threshold (t) function $f_t(Y_r) = \Delta Y_t = \Delta Y_r\, Y_u$ [0]
$\Delta Y_t=(A_1+A_2Y)/A_0$ $A_0=1{,}5$, $A_1=0{,}0170$, $A_2=0{,}0058$

$$f_{tu}(Y_r) = \frac{\Delta Y_t}{\Delta Y_{tu}} = \frac{1+bY_r}{1+b} \quad b=A_2Y_u/A_1 \quad Y_r=Y/Y_u \quad [1]$$

$$F_{tu}(Y_r) = \int \frac{f'_{tu}(Y_r)}{f_{tu}(Y_r)}\,dY_r = \int \frac{b}{1+bY_r}\,dY_r \quad [2]$$

Example for $L^*_{tu}(Y_r)$, ΔY_t with $Y_{ru}=Y_r/Y_u=1$, b=6,141:

$$L^*_{tu}(Y_r) = \frac{L^*_t(Y_r)}{L^*_t(Y_{ru})} = \frac{\ln(1+bY_r)}{\ln(1+b)} \quad [3]$$

$$f_{tu}(Y_r) = \frac{\Delta Y_t}{\Delta Y_{tu}} = \frac{1+bY_r}{1+b} \quad [4]$$

hex01−4n DEQ61−4N

Line-element equations according to CIE 230:2019

Colour-discrimination function $f(x) = \Delta Y = \Delta x\, Y_u$ [0]
$\Delta Y=(A_1+A_2Y)/A_0$ $A_0=1{,}5$, $A_1=0{,}0170$, $A_2=0{,}0058$

$$f_u(x) = \frac{\Delta Y}{\Delta Y_u} = \frac{1+bx}{1+b} \quad b=A_2Y_u/A_1 \quad x=Y/Y_u \quad [1]$$

$$F_u(x) = \int \frac{f'_u(x)}{f_u(x)}\,dx = \int \frac{b}{1+bx}\,dx \quad [2]$$

Example for $L^*(x)$ & ΔY with $x=Y/Y_u$, $x_u=1$, b=6,141:

$$L^*_u(x) = \frac{L^*(x)}{L^*(x_u)} = \frac{\ln(1+bx)}{\ln(1+b)} \quad [3]$$

$$f_u(x) = \frac{\Delta Y}{\Delta Y_u} = \frac{1+bx}{1+b} \quad [4]$$

hex00−5n DEQ60−5N

Line-element equations for thresholds and scaling

Colour-discrimination function $f(x) = \Delta Y = \Delta x\, Y_u$ [0]
$\Delta Y=1/[(1+x)(2+x)]=1/[1+x]-1/[2+x]$ $x=\sqrt{2}\,e^{k(u-u_0)}$

$$f_u(x) = \frac{\Delta Y}{\Delta Y_u} = \frac{1+bx}{1+b} - \frac{1+0{,}5bx}{1+0{,}5b} \quad b=1,\ x=Y/Y_u \quad [1]$$

$$F_u(x) = \int \frac{f'_u(x)}{f_u(x)}\,dx = \int \frac{b}{1+bx}\,dx - \int \frac{0{,}5b}{1+0{,}5bx}\,dx \quad [2]$$

Example for $L^*(x)$ & ΔY with $x=Y/Y_u$, $x_u=1$, b=1:

$$L^*_u(x) = \frac{L^*(x)}{L^*(x_u)} = \frac{\ln(1+bx)}{\ln(1+b)} - \frac{\ln(1+0{,}5bx)}{\ln(1+0{,}5b)} \quad [3]$$

$$f_u(x) = \frac{\Delta Y}{\Delta Y_u} = \frac{1+bx}{1+b} - \frac{1+0{,}5bx}{1+0{,}5b} \quad [4]$$

see K. Richter (1996), Computer Graphic and Colorimetry, p. 113−127
http://color.li.tu-berlin.de/BUA4BF.PDF

hex00−6n DEQ60−6N

Line-element equations according to CIE 230:2019

Colour-discrimination function $f(Y_r) = \Delta Y_r$ [0]
$\Delta Y_r=(A_1+A_2Y_r)/A_0$ $A_0=1{,}5$, $A_1=0{,}0170$, $A_2=0{,}0058$

$$f_u(Y_r) = \frac{\Delta Y_r}{\Delta Y_{ru}} = \frac{1+bY_r}{1+b} \quad b=A_2Y_u/A_1 \quad Y_r=Y/Y_u \quad [1]$$

$$F_u(Y_r) = \int \frac{f'_u(Y_r)}{f_u(Y_r)}\,dY_r = \int \frac{b}{1+bY_r}\,dY_r \quad [2]$$

Example for $L^*(Y_r)$ & ΔY_r with $Y_{ru}=Y_r/Y_u=1$, b=6,141:

$$L^*_u(Y_r) = \frac{L^*(Y_r)}{L^*(Y_{ru})} = \frac{\ln(1+bY_r)}{\ln(1+b)} \quad [3]$$

$$f_u(Y_r) = \frac{\Delta Y_r}{\Delta Y_{ru}} = \frac{1+bY_r}{1+b} \quad [4]$$

hex01−5n DEQ61−5N

Line-element equations for thresholds and scaling

Colour-discrimination function $f(Y_r)=\Delta Y_r$, $u_r=\ln Y_r$ [0]
$\Delta Y_r=1/[(1+Y_r)(2+Y_r)]=1/[1+Y_r]-1/[2+Y_r]$ $Y_r=\sqrt{2}\,e^{ku_r}$

$$f_u(Y_r) = \frac{\Delta Y_u}{\Delta Y_{ru}} = \frac{1+bY_r}{1+b} - \frac{1+0{,}5bY_r}{1+0{,}5b} \quad b=1,\ Y_r=Y/Y_u \quad [1]$$

$$F_u(Y_r) = \int \frac{f'_u(Y_r)}{f_u(Y_r)}\,dY_r = \int \frac{b\,dY_r}{1+bY_r} - \int \frac{0{,}5b\,dY_r}{1+0{,}5bY_r} \quad [2]$$

Example for $L^*(Y_r)$ & ΔY with $Y_r=Y/Y_u$, $Y_{ru}=1$, b=1:

$$L^*_u(Y_r) = \frac{L^*(Y_u)}{L^*(Y_{ru})} = \frac{\ln(1+bY_r)}{\ln(1+b)} - \frac{\ln(1+0{,}5bY_r)}{\ln(1+0{,}5b)} \quad [3]$$

$$f_u(Y_r) = \frac{\Delta Y_r}{\Delta Y_{ru}} = \frac{1+bY_r}{1+b} - \frac{1+0{,}5bY_r}{1+0{,}5b} \quad [4]$$

see K. Richter (1996), Computer Graphic and Colorimetry, p. 113−127
http://color.li.tu-berlin.de/BUA4BF.PDF

hex01−6n DEQ61−6N

Line-element equations for thresholds and scaling

Colour-discrimination function $f(x) = \Delta Y = \Delta x\, Y_u$ [0]
$\Delta Y=1/[(1+x)(2+x)]=1/[1+x]-1/[2+x]$ $x=\sqrt{2}\,e^{k(u-u_0)}$

$$f_u(x) = \frac{\Delta Y}{\Delta Y_u} = \frac{1+x}{2} - \frac{2+x}{3} \quad x=Y/Y_u \quad [1]$$

$$F_u(x) = \int \frac{f'_u(x)}{f_u(x)}\,dx = \int \frac{1}{1+x}\,dx - \int \frac{1}{2+x}\,dx \quad [2]$$

Example for $L^*(x)$ & ΔY with $x=Y/Y_u$, $x_u=1$:

$$L^*_u(x) = \frac{L^*(x)}{L^*(x_u)} = \frac{\ln(1+x)}{\ln(2)} - \frac{\ln(1+0{,}5x)}{\ln(1{,}5)} \quad [3]$$

$$f_u(x) = \frac{\Delta Y}{\Delta Y_u} = \frac{1+x}{2} - \frac{1+0{,}5x}{1{,}5} \quad [4]$$

see K. Richter (1996), Computer Graphic and Colorimetry, p. 113−127
http://color.li.tu-berlin.de/BUA4BF.PDF

hex00−7n DEQ60−7N

Line-element equations for thresholds and scaling

Colour-discrimination function $f(y) = \Delta Y = \Delta y\, Y_u$ [0]
$\Delta Y=1/[y(1+y)]=1/y-1/(1+y)$ $y=(1+\sqrt{2}\,e^{k(u-u_0)}$

$$f_u(y) = \frac{\Delta Y}{\Delta Y_u} = \frac{y}{2} - \frac{1+y}{3} \quad y=1+Y/Y_u,\ dy=dx \quad [1]$$

$$F_u(y) = \int \frac{f'_u(y)}{f_u(y)}\,dy = \int \frac{1}{y}\,dy - \int \frac{1}{1+y}\,dy \quad [2]$$

Example for $L^*(y)$ & ΔY with $y=1+Y/Y_u$, $y_u=2$:

$$L^*_u(y) = \frac{L^*(y)}{L^*(y_u)} = \frac{\ln(y)}{\ln(2)} - \frac{\ln(1+y)}{\ln(3)} \quad [3]$$

$$f_u(x) = \frac{\Delta Y}{\Delta Y_u} = \frac{1+x}{2} - \frac{1+0{,}5x}{1{,}5} \quad [4]$$

see K. Richter (1996), Computer Graphic and Colorimetry, p. 113−127
http://color.li.tu-berlin.de/BUA4BF.PDF

hex00−8n DEQ60−8N

Line-element equations for thresholds and scaling

Colour-discrimination function $f(Y_r)=\Delta Y_r$, $u_r=\ln Y_r$ [0]
$\Delta Y_r=1/[(1+Y_r)(2+Y_r)]=1/[1+Y_r]-1/[2+Y_r]$ $Y_r=\sqrt{2}\,e^{ku_r}$

$$f_u(Y_u) = \frac{\Delta Y_r}{\Delta Y_{ru}} = \frac{1+Y_r}{2} - \frac{2+Y_r}{3} \quad Y_r=Y/Y_u \quad [1]$$

$$F_u(Y_r) = \int \frac{f'_u(Y_r)}{f_u(Y_r)}\,dY_r = \int \frac{dY_r}{1+Y_r} - \int \frac{dY_r}{2+Y_r} \quad [2]$$

Example for $L^*(Y_r)$ & ΔY_r with $Y_r=Y/Y_u=1$:

$$L^*_u(Y_r) = \frac{L^*(Y_r)}{L^*(Y_{ru})} = \frac{\ln(1+Y_r)}{\ln(2)} - \frac{\ln(1+0{,}5Y_r)}{\ln(1{,}5)} \quad [3]$$

$$f_u(Y_r) = \frac{\Delta Y_r}{\Delta Y_{ru}} = \frac{1+Y_r}{2} - \frac{1+0{,}5Y_r}{1{,}5} \quad [4]$$

see K. Richter (1996), Computer Graphic and Colorimetry, p. 113−127
http://color.li.tu-berlin.de/BUA4BF.PDF

hex01−7n DEQ61−7N

Line-element equations for thresholds and scaling

Colour-discrimination function $f(y) = \Delta Y = \Delta y\, Y_u$ [0]
$\Delta Y=1/[y(1+y)]=1/y-1/(1+y)$ $y=1+\sqrt{2}\,e^{k(u_r)}$, $u_r=\ln Y_r$

$$f_u(y) = \frac{\Delta Y}{\Delta Y_u} = \frac{y}{2} - \frac{1+y}{3} \quad y=1+Y/Y_u,\ dy=dx \quad [1]$$

$$F_u(y) = \int \frac{f'_u(y)}{f_u(y)}\,dy = \int \frac{1}{y}\,dy - \int \frac{1}{1+y}\,dy \quad [2]$$

Example for $L^*(y)$ & ΔY with $y=1+Y/Y_u$, $y_u=2$:

$$L^*_u(y) = \frac{L^*(y)}{L^*(y_u)} = \frac{\ln(y)}{\ln(2)} - \frac{\ln(1+y)}{\ln(3)} \quad [3]$$

$$f_u(y) = \frac{\Delta Y}{\Delta Y_u} = \frac{1+Y_r}{2} - \frac{1+0{,}5Y_r}{1{,}5} \quad [4]$$

see K. Richter (1996), Computer Graphic and Colorimetry, p. 113−127
http://color.li.tu-berlin.de/BUA4BF.PDF

hex01−8n DEQ61−8N

TUB-test chart hex0; LABJND & CIELAB colour-difference, CIE 230 & ISO/CIE 11664-4
log & lin[lightness L^*, threshold ΔY, sensitivity $\Delta Y / Y$, contrast $Y / \Delta Y$, normalized for grey U]