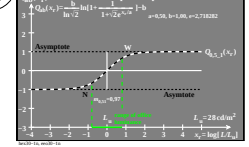


Achromatic receptor-response function $Q_{ab}[x_r/a]$ for a=0,5 and b=1,0

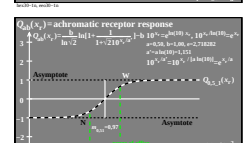
with $x_r = \log[L/L_z]$ (L=test luminance)
L_u=surround luminance

$$Q_{ab}[x_r/a] = \frac{b}{\ln\sqrt{2}} \ln\left[\frac{1}{1+\sqrt{2}e^{(x_r/a)}}\right] - b$$

function values for b=1 and any a>0 :

$Q_{a1}[x_r/a \to -\infty]$	= -1	$x = \log L_u$, $u = \log L_u$
$Q_{a1}[x_r/a = 0]$	= 0	$x_r = \log[L/L_u]$
$Q_{a1}[x_r/a \to +\infty]$	= +1	$= x - u$

hex3l0n 1a, mx3l=2a



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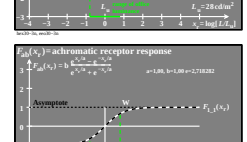
hex3l0n 2a, mx3l=4a



Mathematical equations of hyperbel functions

See: *Papula, L., (2003), Mathematische Formelsammlung, Vieweg*

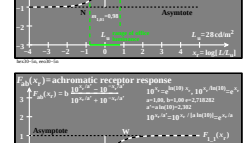
$$\sinh(x) = \frac{e^x - e^{-x}}{2} \quad [1], \quad \cosh(x) = \frac{e^x + e^{-x}}{2} \quad [2]$$

$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}} \quad [3]$$

$$\tanh(x/2) = \frac{\sinh(x)}{\cosh(x)+1} = \frac{\cosh(x)+1}{\sinh(x)} = \frac{e^{x/2} - e^{-x/2}}{e^{x/2} + e^{-x/2}} \quad [4]$$

$$\sinh^2(x) + \cosh^2(x) = 1 \quad [5]$$

hex3l0n 3a, mx3l=6a



Mathematical equations of hyperbel functions

See: *Papula, L., (2003), Mathematische Formelsammlung, Vieweg*

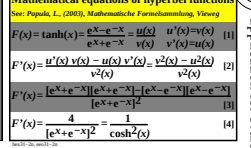
$$\sinh(x) = \frac{10^{x/a\ln(10)} - 10^{-x/a\ln(10)}}{2} \quad [1], \quad \cosh(x) = \frac{10^{x/a\ln(10)} + 10^{-x/a\ln(10)}}{2} \quad [2]$$

$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{10^{x/a\ln(10)} - 10^{-x/a\ln(10)}}{10^{x/a\ln(10)} + 10^{-x/a\ln(10)}} \quad [3]$$

$$\tanh(x/2) = \frac{\sinh(x)}{\cosh(x)+1} = \frac{\cosh(x)+1}{\sinh(x)} = \frac{10^{x/2a\ln(10)} - 10^{-x/2a\ln(10)}}{10^{x/2a\ln(10)} + 10^{-x/2a\ln(10)}} \quad [4]$$

$$\sinh^2(x) + \cosh^2(x) = 1 \quad [5]$$

hex3l0n 4a, mx3l=8a



Mathematical equations of hyperbel functions

See: *Papula, L., (2003), Mathematische Formelsammlung, Vieweg*

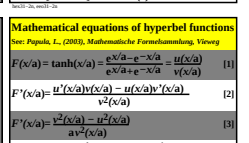
$$F_{1b}(x) = b\tanh(x) = b\frac{e^x - e^{-x}}{e^x + e^{-x}} = b\frac{u(x)}{v(x)} \quad [1]$$

$$F'_{1b}(x) = b\frac{u'(x)\,v(x) - u(x)\,v'(x)}{v^2(x)} \quad [2]$$

$$F'_{1b}(x) = b\frac{v^2(x) - u^2(x)}{a\,v^2(x)} \quad [3]$$

$$F'_{1b}(x) = \frac{4b}{a\left[e^x + e^{-x}\right]^2} = \frac{b}{a\cosh^2(x)} \quad [4]$$

hex3l0n 1a, mx3l=1a



Mathematical equations of hyperbel functions

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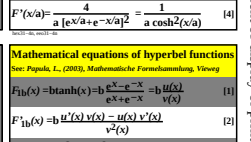

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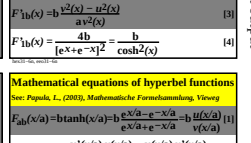

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hex3l0n 4a, mx3l=8a

TUB-test chart hex3; Model of two normalized response functions $F_{ab}(x_r)$ & $Q_{ab}(x_r)$ and derivation
Tangens hyperbolicus tanh(x_r) and modified functions with e^{x_r} and 10^{x_r}; $a^n = a^{1,0}$