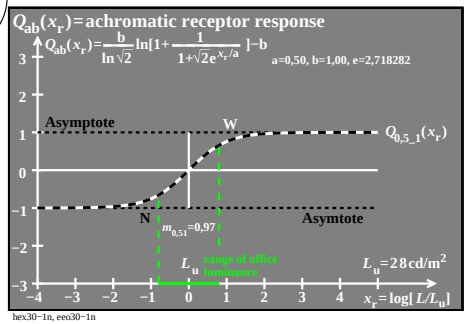
see similar files of the whole serie: http://farbe.li.tu-berlin.de/hexs.htm
technical information: http://farbe.li.tu-berlin.de or http://color.li.tu-berlin.de

TUB registration: 20241201-hex3/hex3l0np.pdf /.ps
application for evaluation and measurement of display or print output
TUB material: code=rha4ta



hex30−1n, eeo30−1n

Achromatic receptor-response function $Q_{ab}[x_r/a]$ for a=0,5 and b=1,0

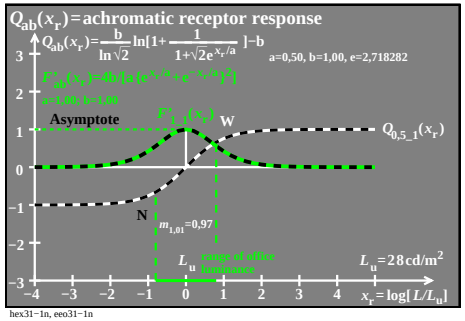
with x_r=log [L/L_u] (L=test luminance)
L_u=surround luminance

$$Q_{ab}[x_r/a]=\frac{b}{\ln\sqrt{2}}\ln[\frac{1}{1+\sqrt{2}e^{(x_r/a)}}]-b$$

function values for b=1 and any a>0 :

$Q_{a1}[x_r/a\to-\infty]$	=−1	$x=\log L$, $u=\log L_u$
$Q_{a1}[x_r/a=0]$	= 0	$x_r=\log[L/L_u]$
$Q_{a1}[x_r/a\to+\infty]$	=+1	$=x-u$

hex30−2n, eeo30−2n



hex31−1n, eeo31−1n

Mathematical equations of hyperbel functions

See: *Papula, L., (2003), Mathematische Formelsammlung, Vieweg*

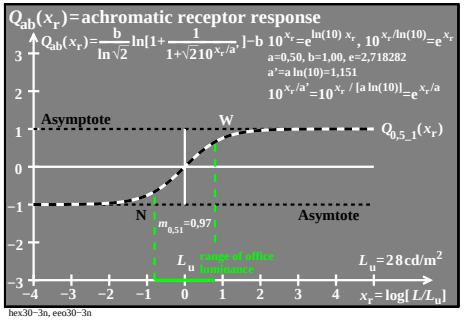
$$F(x)=\tanh(x)=\frac{e^x-e^{-x}}{e^x+e^{-x}}=\frac{u(x)}{v(x)}\quad u'(x)=v(x)\quad v'(x)=u(x)\quad [1]$$

$$F'(x)=\frac{u'(x)\,v(x)-u(x)\,v'(x)}{v^2(x)}=\frac{v^2(x)-u^2(x)}{v^2(x)}\quad [2]$$

$$F'(x)=\frac{[e^x+e^{-x}][e^x+e^{-x}]-[e^x-e^{-x}][e^x-e^{-x}]}{[e^x+e^{-x}]^2}\quad [3]$$

$$F'(x)=\frac{4}{[e^x+e^{-x}]^2}=\frac{1}{\cosh^2(x)}\quad [4]$$

hex31−2n, eeo31−2n



hex30−3n, eeo30−3n

Achromatic receptor-response function $Q_{ab}[x_r/a]$ for a=0,5 and b=1,0

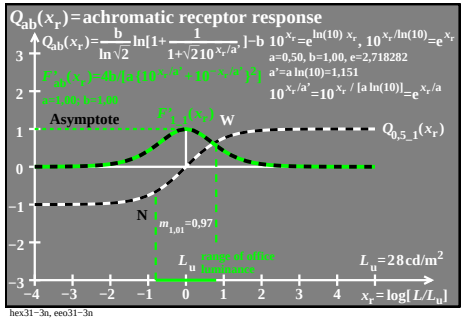
with x_r=log [L/L_u] (L=test luminance)
L_u=surround luminance

$$Q_{ab}[x_r/a]=\frac{b}{\ln\sqrt{2}}\ln[\frac{1}{1+\sqrt{2}e^{(x_r/a)}}]-b$$

function values for b=1 and any a>0 :

$Q_{a1}[x_r/a\to-\infty]$	=−1	$x=\log L$, $u=\log L_u$
$Q_{a1}[x_r/a=0]$	= 0	$x_r=\log[L/L_u]$
$Q_{a1}[x_r/a\to+\infty]$	=+1	$=x-u$

hex30−4n, eeo30−4n



hex31−3n, eeo31−3n

Mathematical equations of hyperbel functions

See: *Papula, L., (2003), Mathematische Formelsammlung, Vieweg*

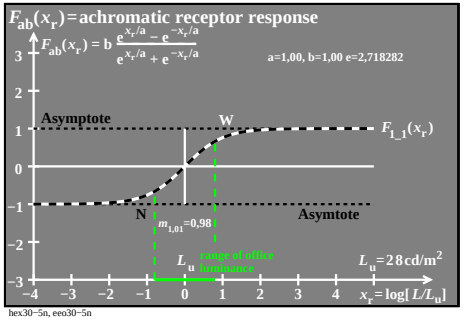
$$F(x/a)=\tanh(x/a)=\frac{e^{x/a}-e^{-x/a}}{e^{x/a}+e^{-x/a}}=\frac{u(x/a)}{v(x/a)}\quad [1]$$

$$F'(x/a)=\frac{u'(x/a)v(x/a)-u(x/a)v'(x/a)}{v^2(x/a)}\quad [2]$$

$$F'(x/a)=\frac{v^2(x/a)-u^2(x/a)}{a\,v^2(x/a)}\quad [3]$$

$$F'(x/a)=\frac{4}{a\,[e^{x/a}+e^{-x/a}]^2}=\frac{1}{a\cosh^2(x/a)}\quad [4]$$

hex31−4n, eeo31−4n



hex30−5n, eeo30−5n

Mathematical equations of hyperbel functions

See: *Papula, L., (2003), Mathematische Formelsammlung, Vieweg*

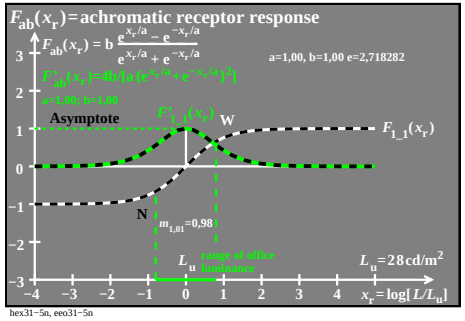
$$\sinh(x)=\frac{e^x-e^{-x}}{2}\quad [1],\quad \cosh(x)=\frac{e^x+e^{-x}}{2}\quad [2]$$

$$\tanh(x)=\frac{\sinh(x)}{\cosh(x)}=\frac{e^x-e^{-x}}{e^x+e^{-x}}\quad [3]$$

$$\tanh(x/2)=\frac{\sinh(x)}{\cosh(x)+1}=\frac{\cosh(x)+1}{\sinh(x)}=\frac{e^{x/2}-e^{-x/2}}{e^{x/2}+e^{-x/2}}\quad [4]$$

$$\sinh^2(x)+\cosh^2(x)=1\quad [5]$$

hex30−6n, eeo30−6n



hex31−5n, eeo31−5n

Mathematical equations of hyperbel functions

See: *Papula, L., (2003), Mathematische Formelsammlung, Vieweg*

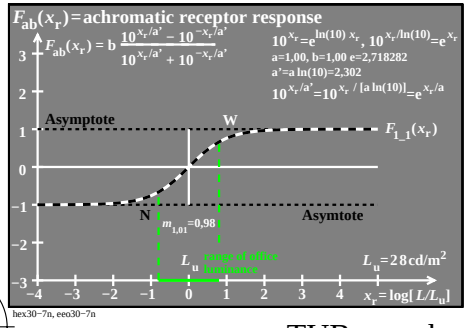
$$F_{1b}(x)=b\tanh(x)=b\frac{e^x-e^{-x}}{e^x+e^{-x}}=b\frac{u(x)}{v(x)}\quad [1]$$

$$F'_{1b}(x)=b\frac{u'(x)\,v(x)-u(x)\,v'(x)}{v^2(x)}\quad [2]$$

$$F'_{1b}(x)=b\frac{v^2(x)-u^2(x)}{a\,v^2(x)}\quad [3]$$

$$F'_{1b}(x)=\frac{4b}{[e^x+e^{-x}]^2}=\frac{b}{\cosh^2(x)}\quad [4]$$

hex31−6n, eeo31−6n



hex30−7n, eeo30−7n

Mathematical equations of hyperbel functions

See: *Papula, L., (2003), Mathematische Formelsammlung, Vieweg*

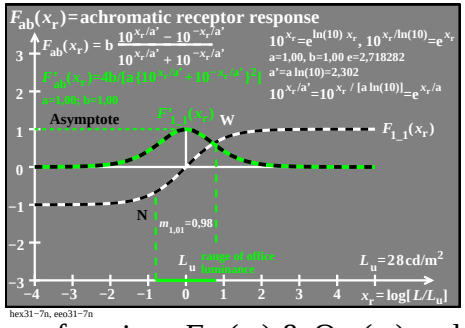
$$\sinh(x)=\frac{10^{x_r/a'}-10^{-x_r/a'}}{2}\quad [1],\quad \cosh(x)=\frac{10^{x_r/a'}+10^{-x_r/a'}}{2}\quad [2]$$

$$\tanh(x)=\frac{\sinh(x)}{\cosh(x)}=\frac{10^{x_r/a'}-10^{-x_r/a'}}{10^{x_r/a'}+10^{-x_r/a'}}\quad [3]$$

$$\tanh(x/2)=\frac{\sinh(x)}{\cosh(x)+1}=\frac{\cosh(x)+1}{\sinh(x)}=\frac{10^{x_r/2a'}-10^{-x_r/2a'}}{10^{x_r/2a'}+10^{-x_r/2a'}}\quad [4]$$

$$\sinh^2(x)+\cosh^2(x)=1\quad [5]$$

hex30−8n, eeo30−8n



hex31−7n, eeo31−7n

Mathematical equations of hyperbel functions

See: *Papula, L., (2003), Mathematische Formelsammlung, Vieweg*

$$F_{ab}(x/a)=b\tanh(x/a)=b\frac{e^{x/a}-e^{-x/a}}{e^{x/a}+e^{-x/a}}=b\frac{u(x/a)}{v(x/a)}\quad [1]$$

$$F'_{ab}(x/a)=b\frac{u'(x/a)\,v(x/a)-u(x/a)\,v'(x/a)}{v^2(x/a)}\quad [2]$$

$$F'_{ab}(x/a)=b\frac{v^2(x/a)-u^2(x/a)}{a\,v^2(x/a)}\quad [3]$$

$$F'_{ab}(x/a)=\frac{4b}{a\,[e^{x/a}+e^{-x/a}]^2}=\frac{b}{a\cosh^2(x/a)}\quad [4]$$

hex31−8n, eeo31−8n

TUB-test chart hex3; Model of two normalized response functions $F_{ab}(x_r)$ & $Q_{ab}(x_r)$ and derivation
Tangens hyperbolicus $\tanh(x_r)$ and modified functions with e^{xr} and 10^{xr}; $a^n=a^{1,0}$