TUB registration: 20241201-hex4l0np.pdf /.ps
application for evaluation and measurement of display or print output

TUB material: code=rha4ta

Mathematical equations of hyperbel functions
See: *Papula, L., (2003), Mathematische Formelsammlung, Vieweg*

$$F(x) = \tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{u(x)}{v(x)} \quad \begin{matrix} u'(x) = v(x) \\ v'(x) = u(x) \end{matrix} \quad [1]$$

$$F'(x) = \frac{u'(x)\,v(x) - u(x)\,v'(x)}{v^2(x)} = \frac{v^2(x) - u^2(x)}{v^2(x)} \quad [2]$$

$$F'(x) = \frac{[e^x + e^{-x}][e^x + e^{-x}] - [e^x - e^{-x}][e^x - e^{-x}]}{[e^x + e^{-x}]^2} \quad [3]$$

$$F'(x) = \frac{4}{[e^x + e^{-x}]^2} = \frac{1}{\cosh^2(x)} \quad [4]$$

hex41−2n, eeo31−2n

Mathematical equations of hyperbel functions
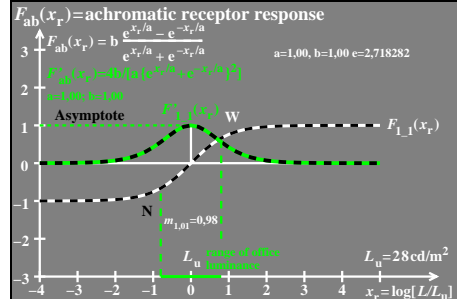See: *Papula, L., (2003), Mathematische Formelsammlung, Vieweg*

$$F(x/a) = \tanh(x/a) = \frac{e^{x/a} - e^{-x/a}}{e^{x/a} + e^{-x/a}} = \frac{u(x/a)}{v(x/a)} \quad [1]$$

$$F'(x/a) = \frac{u'(x/a)v(x/a) - u(x/a)v'(x/a)}{v^2(x/a)} \quad [2]$$

$$F'(x/a) = \frac{v^2(x/a) - u^2(x/a)}{a\,v^2(x/a)} \quad [3]$$

$$F'(x/a) = \frac{4}{a\,[e^{x/a} + e^{-x/a}]^2} = \frac{1}{a\,\cosh^2(x/a)} \quad [4]$$

hex41−4n, eeo31−4n

$F_{ab}(x_r)$ = achromatic receptor response

hex41−5n, eeo31−5n

Mathematical equations of hyperbel functions
See: *Papula, L., (2003), Mathematische Formelsammlung, Vieweg*

$$F_{ab}(x/a) = b\tanh(x/a) = b\frac{e^{x/a} - e^{-x/a}}{e^{x/a} + e^{-x/a}} = b\frac{u(x/a)}{v(x/a)} \quad [1]$$

$$F'_{ab}(x/a) = b\frac{u'(x/a)\,v(x/a) - u(x/a)\,v'(x/a)}{v^2(x/a)} \quad [2]$$

$$F'_{ab}(x/a) = b\frac{v^2(x/a) - u^2(x/a)}{a\,v^2(x/a)} \quad [3]$$

$$F'_{ab}(x/a) = \frac{4b}{a\,[e^{x/a} + e^{-x/a}]^2} = \frac{b}{a\,\cosh^2(x/a)} \quad [4]$$

hex41−2n eeo41−2n

Mathematical equations of hyperbel functions
See: *Papula, L., (2003), Mathematische Formelsammlung, Vieweg*

$$F_{1b}(x) = b\tanh(x) = b\frac{e^x - e^{-x}}{e^x + e^{-x}} = b\frac{u(x)}{v(x)} \quad [1]$$

$$F'_{1b}(x) = b\frac{u'(x)\,v(x) - u(x)\,v'(x)}{v^2(x)} \quad [2]$$

$$F'_{1b}(x) = b\frac{v^2(x) - u^2(x)}{a\,v^2(x)} \quad [3]$$

$$F'_{1b}(x) = \frac{4b}{[e^x + e^{-x}]^2} = \frac{b}{\cosh^2(x)} \quad [4]$$

hex41−6n, eeo31−6n

Mathematical equations of hyperbel functions
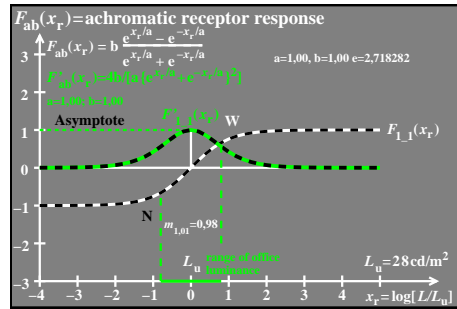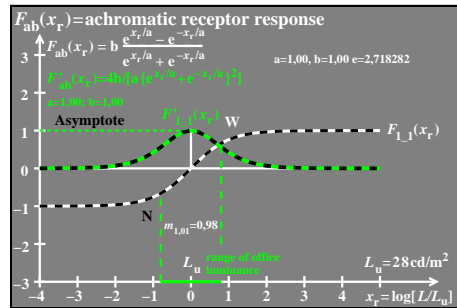See: *Papula, L., (2003), Mathematische Formelsammlung, Vieweg*

$$F_{ab}(x/a) = b\tanh(x/a) = b\frac{e^{x/a} - e^{-x/a}}{e^{x/a} + e^{-x/a}} = b\frac{u(x/a)}{v(x/a)} \quad [1]$$

$$F'_{ab}(x/a) = b\frac{u'(x/a)\,v(x/a) - u(x/a)\,v'(x/a)}{v^2(x/a)} \quad [2]$$

$$F'_{ab}(x/a) = b\frac{v^2(x/a) - u^2(x/a)}{a\,v^2(x/a)} \quad [3]$$

$$F'_{ab}(x/a) = \frac{4b}{a\,[e^{x/a} + e^{-x/a}]^2} = \frac{b}{a\,\cosh^2(x/a)} \quad [4]$$

hex41−8n, eeo31−8n

$F_{ab}(x_r)$ = achromatic receptor response

hex41−5n, eeo31−5n

Mathematical equations of hyperbel functions
See: *Papula, L., (2003), Mathematische Formelsammlung, Vieweg*

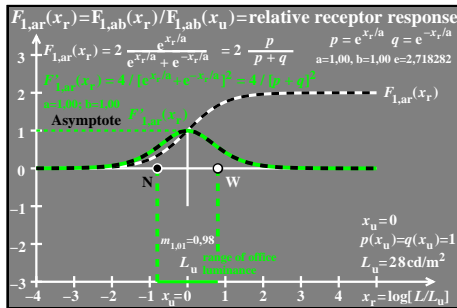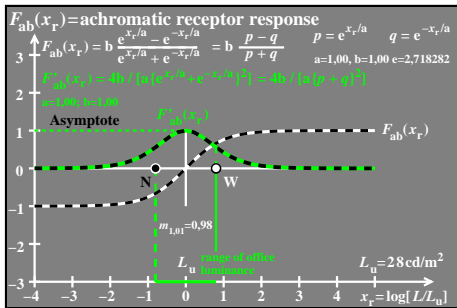$$F_{ab}(x/a) = b\tanh(x/a) = b\frac{e^{x/a} - e^{-x/a}}{e^{x/a} + e^{-x/a}} = b\frac{u(x/a)}{v(x/a)} \quad [1]$$

$$F'_{ab}(x/a) = b\frac{u'(x/a)\,v(x/a) - u(x/a)\,v'(x/a)}{v^2(x/a)} \quad [2]$$

$$F'_{ab}(x/a) = b\frac{v^2(x/a) - u^2(x/a)}{a\,v^2(x/a)} \quad [3]$$

$$F'_{ab}(x/a) = \frac{4b}{a\,[e^{x/a} + e^{-x/a}]^2} = \frac{b}{a\,\cosh^2(x/a)} \quad [4]$$

hex41−4n eeo41−4n

$F_{ab}(x_r)$ = achromatic receptor response

hex40−5n eeo40−5n

$F_{1,ar}(x_r) = F_{1,ab}(x_r)/F_{1,ab}(x_u)$ = relative receptor response

hex40−6n eeo40−6n

$F_{ab}(x_r)$ = achromatic receptor response

hex41−5n, eeo31−5n

Mathematical equations of hyperbel functions
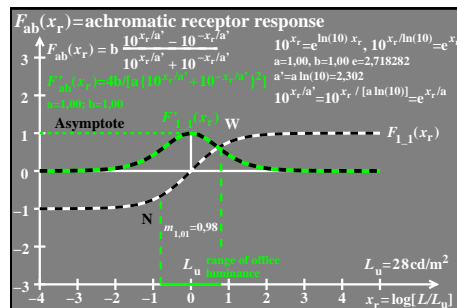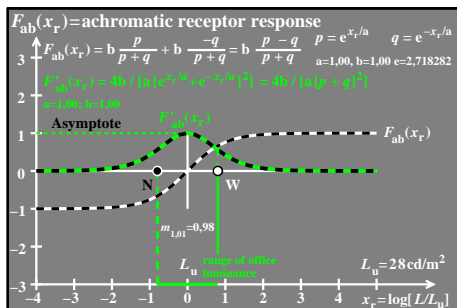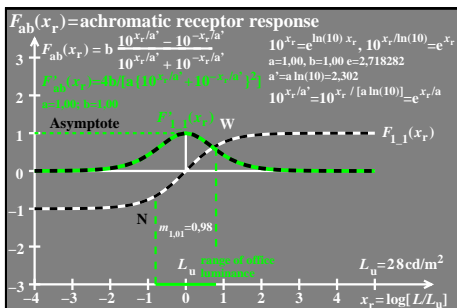See: *Papula, L., (2003), Mathematische Formelsammlung, Vieweg*

$$F_{ab}(x/a) = b\tanh(x/a) = b\frac{e^{x/a} - e^{-x/a}}{e^{x/a} + e^{-x/a}} \quad [1]$$

$$\frac{dF_{ab}(x/a)}{dx} = \frac{4b}{a[e^{x/a} + e^{-x/a}]^2} = \frac{b}{a\,\cosh^2(x/a)} \quad [4]$$

$$\frac{dF_{ab}(x_r/a)}{dx_r} = \frac{4b}{a[e^{x_r/a} + e^{-x_r/a}]^2} \quad \begin{matrix} x_r = \log(L/L_u) \\ dx_r/dL = \ln(10)/L \end{matrix} \quad [5]$$

$$\frac{dF_{ab}(x_r/a)}{dx_r}\,\frac{dx_r}{dL} = \frac{4b}{a[e^{x_r/a} + e^{-x_r/a}]^2}\,\frac{\ln(10)}{L} \quad [6]$$

hex41−6n eeo41−6n

$F_{ab}(x_r)$ = achromatic receptor response

hex41−7n

$F_{ab}(x_r)$ = achromatic receptor response

hex40−8n eeo40−8n

$F_{ab}(x_r)$ = achromatic receptor response

hex41−7n, eeo31−7n

Mathematical equations of hyperbel functions
See: *Papula, L., (2003), Mathematische Formelsammlung, Vieweg*

$$F_{ab}(x_r/a) = b\tanh(x_r/a) = b\frac{e^{x_r/a} - e^{-x_r/a}}{e^{x_r/a} + e^{-x_r/a}} \quad [1]$$

$$\frac{dF_{ab}(x_r/a)}{dx_r} = \frac{4b}{a[e^{x_r/a} + e^{-x_r/a}]^2} \quad \begin{matrix} x_r = \log(L/L_u) \\ dx_r/dL = \ln(10)/L \end{matrix} \quad [5]$$

$$\frac{dF_{ab}(x_r/a)}{dx_r}\,\frac{dx_r}{dL} = \frac{4b}{a[e^{x_r/a} + e^{-x_r/a}]^2}\,\frac{\ln(10)}{L} \quad [6]$$

$$\frac{L}{dL} = \frac{4b\ln(10)}{a[e^{x_r/a} + e^{-x_r/a}]^2} \qquad dL = \frac{a[e^{x_r/a} + e^{-x_r/a}]^2 L}{4b\ln(10)} \quad [7]$$

hex41−8n eeo41−8n

TUB-test chart hex4; Model of normalized response function $F_{ab}(x_r)$ and derivation $F'_{ab}(x_r)$
Mathematical calculation of the derivation $F'_{ab}(x_r)$, of the contrast $L/\Delta L$, and the discrimination ΔL