Achromatic color vision with relative luminance
Mathematical equations with hyperbel functions

$F_{ab}(x_r,a)=b\tanh(x_r/a)=b\frac{e^{x_r/a}-e^{-x_r/a}}{e^{x_r/a}+e^{-x_r/a}}\quad\frac{x_r=\ln(L_r)}{L_r=L/L_u}\quad x_r<0$ [1]

$\frac{dF_{ab}(x_r,a)}{dx_r}=\frac{4b}{a[e^{x_r/a}+e^{-x_r/a}]^2}\quad\frac{x_r=\ln L_r/\ln(10)}{dx_r/dL_r=1/(\ln(10)L_r)}\quad m=1/(\ln(10)a)$ [5]

$\frac{dF_{ab}(x_r,a)}{dL_r}=\frac{4b\,m}{[e^{x_r/a}+e^{-x_r/a}]^2L_r}\quad\frac{L_r/dL_r=L/dL}{dF_{ab}(x_r,a)=1}$ [6]

$\frac{L}{dL}=\frac{4b\,m}{[e^{x_r/a}+e^{-x_r/a}]^2}\qquad dL=\frac{[e^{x_r/a}+e^{-x_r/a}]^2L}{4b\,m}$ [7]

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Achromatic color vision with relative luminance
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$\frac{L}{dL}=\frac{4}{(L/dL)_u[e^{x_r/a}+e^{-x_r/a}]^2};\quad\frac{dL}{dL_u}=\frac{a[e^{x_r/a}+e^{-x_r/a}]^2L}{4L_u}$ [8]

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Achromatic color vision with relative luminance
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$\frac{L}{dL}=\frac{4b\,m}{[L_r^{2m}+2+L_r^{-2m}]}\qquad dL=\frac{[L_r^{2m}+2+L_r^{-2m}]L}{4b\,m}$ [8]

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$\frac{L/dL}{(L/dL)_u}=\frac{4}{[e^{x_r/a}+e^{-x_r/a}]^2};\quad\frac{dL}{dL_u}=\frac{[e^{x_r/a}+e^{-x_r/a}]^2L}{4L_u}$ [8]

$\frac{L/dL}{(L/dL)_u}=1\text{ for }\begin{cases}L=L_u\\x_r=0\end{cases}\quad\frac{dL}{dL_u}=1\text{ for }\begin{cases}L=L_u\\x_r=0\end{cases}$ [9]

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$\frac{L/dL}{(L/dL)_u}=\frac{4}{[L_r^{2m}+2+L_r^{-2m}]};\quad\frac{dL}{dL_u}=\frac{[L_r^{2m}+2+L_r^{-2m}]L}{4L_u}$ [9]

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Achromatic color vision with relative luminance
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TUB-test chart hey0; Model of normalized receptor-response functions $F_{ab}(x_r)$ and $F_{cb}(x_r)$
Calculation of derivations $F'_{ab}(x_r)$, $F'_{cb}(x_r)$, of contrasts $L/\Delta L$, and discriminations $(\Delta L)_{ab}$, $(\Delta L)_{cb}$