

**Achromatic colour vision with relative luminance**

Mathematical equations with potential functions

$$F_{ab}(L_T, m) = b \tanh(x_T/a) = b \frac{L^m - L_u^{-m}}{L^m + L_u^{-m}} \quad x_T = \log(L_T/L_u) \quad x_T >= 0 \quad (1)$$

$$\frac{dF_{ab}(L_T, m)}{dL_T} = \frac{4bm}{L_T(L^m + L_u^{-m})^2} \frac{dx_T/dL_T \cdot \ln(10) L_u}{m=1/\ln(10)a} \quad (5)$$

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$$\frac{L}{dL} = \frac{4bmL_u L}{L_T(L^m + L_u^{-m})^2} \quad dL = \frac{L_T(L^m + L_u^{-m})^2}{4bmL_u} \quad (7)$$

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TUB-test chart hey1; Model of normalized receptor-response functions  $F_{ab}(L_T)$  and  $F_{cb}(L_T)$

Calculation of derivations  $F'_{ab}(L_T)$ ,  $F'_{cb}(L_T)$ , of contrasts  $\Delta L/\Delta L$ , and discriminations  $(\Delta L/\Delta L)_{cb}$