Achromatic colour vision with relative luminance
Mathematical equations with potential functions

$F_{ab}(L_r,m)=b\tanh(x_r/a)=b\frac{L_r^m - L_r^{-m}}{L_r^m + L_r^{-m}}$ $x_r=\log(L_r)$, $L_r=L/L_u$, $x_r<=0$ [1]

$\frac{dF_{ab}(L_r,m)}{dL_r}=\frac{4bm}{L_r[L_r^m + L_r^{-m}]^2}$ $x_r=\ln L_r/\ln(10)$, $dx_r/dL_r=1/(\ln(10)L_r)$, $m=1/(\ln(10)a)$ [5]

$\frac{dF_{ab}(L_r,m)}{dL}=\frac{4bmL_u}{L_r[L_r^m + L_r^{-m}]^2}$ $dL_r=dL/L_u$, $dF_{ab}(L_r,m)=1$ [6]

$\frac{L}{dL}=\frac{4bmL_uL}{L_r[L_r^m + L_r^{-m}]^2}$ $dL=\frac{L_r[L_r^m + L_r^{-m}]^2}{4bmL_u}$ [7]

hey10−1n eer10−1n eeo41−8n

Achromatic colour vision with relative luminance
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$F_{cb}(L_r,n)=b\tanh(x_r/c)=b\frac{L_r^n - L_r^{-n}}{L_r^n + L_r^{-n}}$ $x_r=\log(L_r)$, $L_r=L/L_u$, $x_r>=0$ [1]

$\frac{dF_{cb}(L_r,n)}{dL_r}=\frac{4bn}{L_r[L_r^n + L_r^{-n}]^2}$ $x_r=\ln L_r/\ln(10)$, $dx_r/dL_r=1/(\ln(10)L_r)$, $n=1/(\ln(10)c)$ [5]

$\frac{dF_{cb}(L_r,n)}{dL}=\frac{4bnL_u}{L_r[L_r^n + L_r^{-n}]^2}$ $dL_r=dL/L_u$, $dF_{cb}(L_r,n)=1$ [6]

$\frac{L}{dL}=\frac{4bnL_uL}{L_r[L_r^n + L_r^{-n}]^2}$ $dL=\frac{L_r[L_r^n + L_r^{-n}]^2}{4bnL_u}$ [7]

hey10−2n eer10−2n eeo41−8n

Achromatic colour vision with relative luminance
Mathematical hyperbel and potential functions

$F_{ab}(x_r,a)=b\tanh(x_r/a)=b\frac{e^{x_r/a}-e^{-x_r/a}}{e^{x_r/a}+e^{-x_r/a}}$ $x_r=\log(L_r)$, $L_r=L/L_u$, $x_r<=0$ [1]

$\frac{dF_{ab}(x_r,a)}{dx_r}=\frac{4b}{a[e^{x_r/a}+e^{-x_r/a}]^2}$ $x_r=\ln L_r/\ln(10)$, $dx_r/dL_r=1/(\ln(10)L_r)$, $m=1/(\ln(10)a)$ [5]

$\frac{L}{dL}=\frac{4bm}{[e^{x_r/a}+e^{-x_r/a}]^2}$ $dL=\frac{[e^{x_r/a}+e^{-x_r/a}]^2L}{4bm}$ [7]

$\frac{L}{dL}=\frac{4bm}{[L_r^m+L_r^{-m}]^2}$ $dL=\frac{[L_r^m+L_r^{-m}]^2L}{4bm}$ [8]

hey11−1n eer11−1n

Achromatic colour vision with relative luminance
Mathematical hyperbel and potential functions

$F_{cb}(x_r,a)=b\tanh(x_r/c)=b\frac{e^{x_r/c}-e^{-x_r/c}}{e^{x_r/c}+e^{-x_r/c}}$ $x_r=\log(L_r)$, $L_r=L/L_u$, $x_r>=0$ [1]

$\frac{dF_{cb}(x_r,a)}{dx_r}=\frac{4b}{c[e^{x_r/c}+e^{-x_r/c}]^2}$ $x_r=\ln L_r/\ln(10)$, $dx_r/dL_r=1/(\ln(10)L_r)$, $n=1/(\ln(10)c)$ [5]

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hey11−2n eer11−2n

Achromatic colour vision with relative luminance
Mathematical equations with potential functions

$F_{ab}(L_r,m)=b\tanh(x_r/a)=b\frac{L_r^m - L_r^{-m}}{L_r^m + L_r^{-m}}$ $x_r=\log(L_r)$, $L_r=L/L_u$, $x_r<=0$ [1]

$\frac{dF_{ab}(L_r,m)}{dL_r}=\frac{4bm}{L_r[L_r^m+L_r^{-m}]^2}$ $x_r=\ln L_r/\ln(10)$, $dx_r/dL_r=1/(\ln(10)L_r)$, $m=1/(\ln(10)a)$ [5]

$\frac{L}{dL}=\frac{4bmL_uL}{L_r[L_r^m + L_r^{-m}]^2}$ $dL=\frac{L_r[L_r^m + L_r^{-m}]^2}{4bmL_u}$ [7]

$\frac{L/dL}{(L/dL)_u}=\frac{4L}{L_r[L_r^m+L_r^{-m}]^2L_u}$; $\frac{dL}{dL_u}=\frac{L_r[L_r^m+L_r^{-m}]^2}{4}$ [8]

hey10−3n eer10−3n eeo41−8n,ch

Achromatic colour vision with relative luminance
Mathematical equations with potential functions

$F_{cb}(L_r,n)=b\tanh(x_r/c)=b\frac{L_r^n - L_r^{-n}}{L_r^n + L_r^{-n}}$ $x_r=\log(L_r)$, $L_r=L/L_u$, $x_r>=0$ [1]

$\frac{dF_{cb}(L_r,n)}{dL_r}=\frac{4bm}{L_r[L_r^n+L_r^{-n}]^2}$ $x_r=\ln L_r/\ln(10)$, $dx_r/dL_r=1/(\ln(10)L_r)$, $n=1/(\ln(10)c)$ [5]

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hey10−4n eer10−4n eeo41−8n,ch

Achromatic colour vision with relative luminance
Mathematical hyperbel and potential functions

$F_{ab}(x_r,a)=b\tanh(x_r/a)=b\frac{e^{x_r/a}-e^{-x_r/a}}{e^{x_r/a}+e^{-x_r/a}}$ $x_r=\log(L_r)$, $L_r=L/L_u$, $x_r<=0$ [1]

$\frac{dF_{ab}(x_r,a)}{dx_r}=\frac{4b}{a[e^{x_r/a}+e^{-x_r/a}]^2}$ $x_r=\ln L_r/\ln(10)$, $dx_r/dL_r=1/(\ln(10)L_r)$, $m=1/(\ln(10)a)$ [5]

$\frac{L}{dL}=\frac{4bm}{[e^{x_r/a}+e^{-x_r/a}]^2}$ $dL=\frac{[e^{x_r/a}+e^{-x_r/a}]^2L}{4bm}$ [7]

$\frac{L}{dL}=\frac{4bm}{[L_r^{2m}+2+L_r^{-2m}]}$ $dL=\frac{[L_r^{2m}+2+L_r^{-2m}]L}{4bm}$ [8]

hey11−3n eer11−3n

Achromatic colour vision with relative luminance
Mathematical hyperbel and potential functions

$F_{cb}(x_r,a)=b\tanh(x_r/c)=b\frac{e^{x_r/c}-e^{-x_r/c}}{e^{x_r/c}+e^{-x_r/c}}$ $x_r=\log(L_r)$, $L_r=L/L_u$, $x_r>=0$ [1]

$\frac{dF_{cb}(x_r,a)}{dx_r}=\frac{4b}{c[e^{x_r/c}+e^{-x_r/c}]^2}$ $x_r=\ln L_r/\ln(10)$, $dx_r/dL_r=1/(\ln(10)L_r)$, $n=1/(\ln(10)c)$ [5]

$\frac{L}{dL}=\frac{4bn}{[e^{x_r/c}+e^{-x_r/c}]^2}$ $dL=\frac{[e^{x_r/c}+e^{-x_r/c}]^2L}{4bn}$ [7]

$\frac{L}{dL}=\frac{4bn}{[L_r^{2n}+2+L_r^{-2n}]}$ $dL=\frac{[L_r^{2n}+2+L_r^{-2n}]L}{4bn}$ [8]

hey11−4n eer11−4n

Achromatic colour vision with relative luminance
Mathematical equations with potential functions

$F_{ab}(L_r,m)=b\tanh(x_r/a)=b\frac{L_r^m - L_r^{-m}}{L_r^m + L_r^{-m}}$ $x_r=\log(L_r)$, $L_r=L/L_u$, $x_r<=0$ [1]

$\frac{dF_{ab}(L_r,m)}{dL_r}=\frac{4bm}{L_r[L_r^m+L_r^{-m}]^2}$ $x_r=\ln L_r/\ln(10)$, $dx_r/dL_r=1/(\ln(10)L_r)$, $m=1/(\ln(10)a)$ [5]

$\frac{L/dL}{(L/dL)_u}=\frac{4L}{L_r[L_r^m+L_r^{-m}]^2L_u}$; $\frac{dL}{dL_u}=\frac{L_r[L_r^m+L_r^{-m}]^2}{4}$ [8]

$\frac{L/dL}{(L/dL)_u}=1$ for $\{L=L_u, x_r=0\}$ $\frac{dL}{dL_u}=1$ for $\{L=L_u, x_r=0\}$ [9]

hey10−5n eer10−5n eeo41−8n,ch

Achromatic colour vision with relative luminance
Mathematical equations with potential functions

$F_{cb}(L_r,n)=b\tanh(x_r/c)=b\frac{L_r^n - L_r^{-n}}{L_r^n + L_r^{-n}}$ $x_r=\log(L_r)$, $L_r=L/L_u$, $x_r>=0$ [1]

$\frac{dF_{cb}(L_r,n)}{dL_r}=\frac{4bm}{L_r[L_r^n+L_r^{-n}]^2}$ $x_r=\ln L_r/\ln(10)$, $dx_r/dL_r=1/(\ln(10)L_r)$, $n=1/(\ln(10)c)$ [5]

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hey10−6n eer10−6n eeo41−8n,ch

Achromatic colour vision with relative luminance
Mathematical hyperbel and potential functions

$F_{ab}(x_r,a)=b\tanh(x_r/a)=b\frac{e^{x_r/a}-e^{-x_r/a}}{e^{x_r/a}+e^{-x_r/a}}$ $x_r=\log(L_r)$, $L_r=L/L_u$, $x_r<=0$ [1]

$\frac{dF_{ab}(x_r,a)}{dx_r}=\frac{4b}{a[e^{x_r/a}+e^{-x_r/a}]^2}$ $x_r=\ln L_r/\ln(10)$, $dx_r/dL_r=1/(\ln(10)L_r)$, $m=1/(\ln(10)a)$ [5]

$\frac{L/dL}{(L/dL)_u}=\frac{4}{[e^{x_r/a}+e^{-x_r/a}]^2}$; $\frac{dL}{dL_u}=\frac{[e^{x_r/a}+e^{-x_r/a}]^2L}{4L_u}$ [8]

$\frac{L/dL}{(L/dL)_u}=\frac{4}{[L_r^m+L_r^{-m}]^2}$; $\frac{dL}{dL_u}=\frac{[L_r^m+L_r^{-m}]^2L}{4L_u}$ [9]

hey11−5n eer11−5n

Achromatic colour vision with relative luminance
Mathematical hyperbel and potential functions

$F_{cb}(x_r,a)=b\tanh(x_r/c)=b\frac{e^{x_r/c}-e^{-x_r/c}}{e^{x_r/c}+e^{-x_r/c}}$ $x_r=\log(L_r)$, $L_r=L/L_u$, $x_r>=0$ [1]

$\frac{dF_{cb}(x_r,a)}{dx_r}=\frac{4b}{c[e^{x_r/c}+e^{-x_r/c}]^2}$ $x_r=\ln L_r/\ln(10)$, $dx_r/dL_r=1/(\ln(10)L_r)$, $n=1/(\ln(10)c)$ [5]

$\frac{L/dL}{(L/dL)_u}=\frac{4}{[e^{x_r/c}+e^{-x_r/c}]^2}$; $\frac{dL}{dL_u}=\frac{[e^{x_r/c}+e^{-x_r/c}]^2L}{4L_u}$ [8]

$\frac{L/dL}{(L/dL)_u}=\frac{4}{[L_r^n+L_r^{-n}]^2}$; $\frac{dL}{dL_u}=\frac{[L_r^n+L_r^{-n}]^2L}{4L_u}$ [9]

hey11−6n eer11−6n

Achromatic colour vision with relative luminance
Mathematical equations with potential functions

$F_{ab}(L_r,m)=b\tanh(x_r/a)=b\frac{L_r^m - L_r^{-m}}{L_r^m + L_r^{-m}}$ $x_r=\log(L_r)$, $L_r=L/L_u$, $x_r<=0$ [1]

$\frac{dF_{ab}(L_r,m)}{dL_r}=\frac{4bm}{L_r[L_r^m+L_r^{-m}]^2}$ $x_r=\ln L_r/\ln(10)$, $dx_r/dL_r=1/(\ln(10)L_r)$, $m=1/(\ln(10)a)$ [5]

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$\frac{L/dL}{(L/dL)_u}=1$ for $\{L_r=1, x_r=0\}$ $\frac{dL}{dL_u}=1$ for $\{L_r=1, x_r=0\}$ [9]

hey10−7n eer10−7n eeo41−8n,ch

Achromatic colour vision with relative luminance
Mathematical equations with potential functions

$F_{cb}(L_r,n)=b\tanh(x_r/c)=b\frac{L_r^n - L_r^{-n}}{L_r^n + L_r^{-n}}$ $x_r=\log(L_r)$, $L_r=L/L_u$, $x_r>=0$ [1]

$\frac{dF_{cb}(L_r,n)}{dL_r}=\frac{4bm}{L_r[L_r^n+L_r^{-n}]^2}$ $x_r=\ln L_r/\ln(10)$, $dx_r/dL_r=1/(\ln(10)L_r)$, $n=1/(\ln(10)c)$ [5]

$\frac{L/dL}{(L/dL)_u}=\frac{4L}{L_r[L_r^n+L_r^{-n}]^2L_u}$; $\frac{dL}{dL_u}=\frac{L_r[L_r^n+L_r^{-n}]^2}{4}$ [8]

$\frac{L/dL}{(L/dL)_u}=1$ for $\{L_r=1, x_r=0\}$ $\frac{dL}{dL_u}=1$ for $\{L_r=1, x_r=0\}$ [9]

hey10−8n eer10−8n eeo41−8n,ch

Achromatic colour vision with relative luminance
Mathematical hyperbel and potential functions

$F_{ab}(x_r,a)=b\tanh(x_r/a)=b\frac{e^{x_r/a}-e^{-x_r/a}}{e^{x_r/a}+e^{-x_r/a}}$ $x_r=\log(L_r)$, $L_r=L/L_u$, $x_r<=0$ [1]

$\frac{dF_{ab}(x_r,a)}{dx_r}=\frac{4b}{a[e^{x_r/a}+e^{-x_r/a}]^2}$ $x_r=\ln L_r/\ln(10)$, $dx_r/dL_r=1/(\ln(10)L_r)$, $m=1/(\ln(10)a)$ [5]

$\frac{L/dL}{(L/dL)_u}=\frac{4}{[e^{x_r/a}+e^{-x_r/a}]^2}$; $\frac{dL}{dL_u}=\frac{[e^{x_r/a}+e^{-x_r/a}]^2L}{4L_u}$ [8]

$\frac{L/dL}{(L/dL)_u}=\frac{4}{L_r^{2m}+2+L_r^{-2m}}$; $\frac{dL}{dL_u}=\frac{(L_r^{2m}+2+L_r^{-2m})L}{4L_u}$ [9]

hey11−7n eer11−7n

Achromatic colour vision with relative luminance
Mathematical hyperbel and potential functions

$F_{cb}(x_r,a)=b\tanh(x_r/c)=b\frac{e^{x_r/c}-e^{-x_r/c}}{e^{x_r/c}+e^{-x_r/c}}$ $x_r=\log(L_r)$, $L_r=L/L_u$, $x_r>=0$ [1]

$\frac{dF_{cb}(x_r,a)}{dx_r}=\frac{4b}{c[e^{x_r/c}+e^{-x_r/c}]^2}$ $x_r=\ln L_r/\ln(10)$, $dx_r/dL_r=1/(\ln(10)L_r)$, $n=1/(\ln(10)c)$ [5]

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$\frac{L/dL}{(L/dL)_u}=\frac{4}{L_r^{2n}+2+L_r^{-2n}}$; $\frac{dL}{dL_u}=\frac{(L_r^{2n}+2+L_r^{-2n})L}{4L_u}$ [9]

hey11−8n eer11−8n

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TUB registration: 20241201-hey1/hey1l0np.pdf /.ps
application for evaluation and measurement of display or print output
TUB material: code=rha4ta

TUB-test chart hey1; Model of normalized receptor-response functions $F_{ab}(L_r)$ and $F_{cb}(L_r)$
Calculation of derivations $F'_{ab}(L_r)$, $F'_{cb}(L_r)$, of contrasts $L/\Delta L$, and discriminations $(\Delta L)_{ab}$, $(\Delta L)_{cb}$