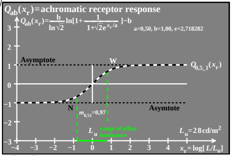


hey20−1n eer20−1n

Achromatic receptor-response function $Q_{ab}[x_r/a]$ for a=0,5 and b=1,0

with x_r=log [L/L_u] (L=test luminance)

L_u=surround luminance

$Q_{ab}[x_r/a] = \frac{b}{\ln\sqrt{2}} \ln[\frac{1}{1+\sqrt{2}\,e^{(x_r/a)}}] - b$

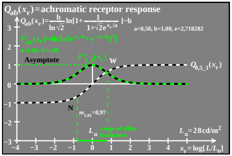
function values for b=1 and any a>0 :

$Q_{a1}[x_r/a \to -\infty] = -1$ $x = \log L$, $u = \log L_u$

$Q_{a1}[x_r/a = 0] = 0$ $x_r = \log[L/L_u]$

$Q_{a1}[x_r/a \to +\infty] = +1$ $= x - u$

hey20−2n eer20−2n



hey21−1n eer21−1n

Achromatic colour vision with relative luminance
Mathematical equations with hyperbel functions

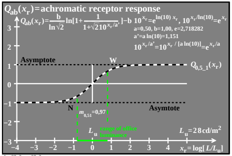
$F(x) = \tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{u(x)}{v(x)}$ $u'(x) = v(x)$, $v'(x) = u(x)$ [1]

$\frac{dF(x)}{dx} = \frac{u'(x)\,v(x) - u(x)\,v'(x)}{v^2(x)} = \frac{v^2(x) - u^2(x)}{v^2(x)}$ [2]

$\frac{dF(x)}{dx} = \frac{[e^x+e^{-x}][e^x+e^{-x}] - [e^x-e^{-x}][e^x-e^{-x}]}{[e^x+e^{-x}]^2}$ [3]

$\frac{dF(x)}{dx} = \frac{4}{[e^x+e^{-x}]^2} = \frac{1}{\cosh^2(x)}$ [4]

hey21−2n eer21−2n



hey20−3n eer20−3n

Achromatic receptor-response function $Q_{ab}[x_r/a]$ for a=0,5 and b=1,0

with x_r=log [L/L_u] (L=test luminance)

L_u=surround luminance

$Q_{ab}[x_r/a] = \frac{b}{\ln\sqrt{2}} \ln[\frac{1}{1+\sqrt{2}\,e^{(x_r/a)}}] - b$

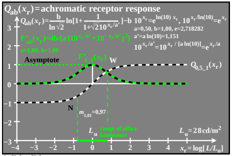
function values for b=1 and any a>0 :

$Q_{a1}[x_r/a \to -\infty] = -1$ $x = \log L$, $u = \log L_u$

$Q_{a1}[x_r/a = 0] = 0$ $x_r = \log[L/L_u]$

$Q_{a1}[x_r/a \to +\infty] = +1$ $= x - u$

hey20−4n eer20−4n



hey21−3n eer21−3n

Achromatic colour vision with relative luminance
Mathematical equations with hyperbel functions

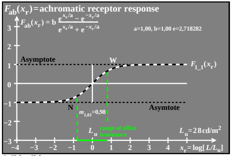
$F(x,a) = \tanh(x/a) = \frac{e^{x/a} - e^{-x/a}}{e^{x/a} + e^{-x/a}} = \frac{u(x/a)}{v(x/a)}$ [1]

$\frac{dF(x,a)}{dx} = \frac{u'(x/a)v(x/a) - u(x/a)v'(x/a)}{v^2(x/a)}$ [2]

$\frac{dF(x,a)}{dx} = \frac{v^2(x/a) - u^2(x/a)}{a\,v^2(x/a)}$ [3]

$\frac{dF(x,a)}{dx} = \frac{4}{a\,[e^{x/a}+e^{-x/a}]^2} = \frac{1}{a\cosh^2(x/a)}$ [4]

hey21−4n eer21−4n



hey20−5n eer20−5n

Mathematical equations of hyperbel functions

See: *Handbook of mathematical functions, NBS, USA, Sec. 4.5*

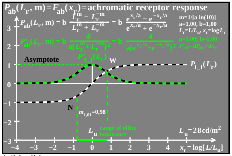
$\sinh(x) = \frac{e^x - e^{-x}}{2}$ [1], $\cosh(x) = \frac{e^x + e^{-x}}{2}$ [2]

$\tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ [3]

$\tanh(x/2) = \frac{\sinh(x)}{\cosh(x)+1} = \frac{\cosh(x)+1}{\sinh(x)} = \frac{e^{x/2}-e^{-x/2}}{e^{x/2}+e^{-x/2}}$ [4]

$\sinh^2(x) + \cosh^2(x) = 1$ [5]

hey20−6n eer20−6n



hey21−5n eer21−5n

Achromatic colour vision with relative luminance
Mathematical equations with potential functions

$F(L) = \frac{L^m - L^{-m}}{L^m + L^{-m}} = \frac{u(L)}{v(L)}$ $u'(L) = m[L^{m-1} + L^{-m-1}]$, $v'(L) = m[L^{m-1} - L^{-m-1}]$ [1]

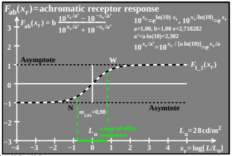
$\frac{dF(L)}{dL} = \frac{u'(L)v(L) - u(L)v'(L)}{v^2(L)}$ [2]

$u'(L)v(L) - v'(L)u(L) = m\{[L^{m-1}+L^{-m-1}][L^m+L^{-m}] - [L^{m-1}-L^{-m-1}][L^m-L^{-m}]\}$ [3]

$= m\{L^{2m-1}+L^{-1}+L^{-1}+L^{-2m-1} - L^{2m-1}+L^{-1}+L^{-1}-L^{-2m-1}\} = 4m/L^{-1}$

$\frac{dF(L)}{dL} = \frac{4m}{L[L^m+L^{-m}]^2}$ [4]

hey21−6n eer21−6n



hey20−7n eer20−7n

Mathematical equations of hyperbel functions

See: *Handbook of mathematical functions, NBS, USA, Sec. 4.5*

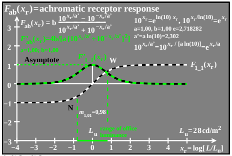
$\sinh(x) = \frac{10^{x_r/a'} - 10^{-x_r/a'}}{2}$ [1], $\cosh(x) = \frac{10^{x_r/a'} + 10^{-x_r/a'}}{2}$ [2]

$\tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{10^{x_r/a'} - 10^{-x_r/a'}}{10^{x_r/a'} + 10^{-x_r/a'}}$ [3]

$\tanh(x/2) = \frac{\sinh(x)}{\cosh(x)+1} = \frac{\cosh(x)+1}{\sinh(x)} = \frac{10^{x_r/2a'} - 10^{-x_r/2a'}}{10^{x_r/2a'} + 10^{-x_r/2a'}}$ [4]

$\sinh^2(x) + \cosh^2(x) = 1$ [5]

hey20−8n eer20−8n



hey21−7n eer21−7n

Achromatic colour vision with relative luminance
Equations with hyperbel and potential functions

$F_{ab}(x_r,a) = b\tanh(x_r/a) = b\frac{e^{x_r/a} - e^{-x_r/a}}{e^{x_r/a} + e^{-x_r/a}}$ $x_r = \log(L_r)$, $L_r = L/L_u$ [1a]

$\frac{dF_{ab}(x_r,a)}{dx_r} = \frac{4b}{a[e^{x_r/a} + e^{-x_r/a}]^2}$ $x_r = \ln L_r/\ln(10)$, $dx_r/dL_r = 1/(\ln(10)L_r)$, $m = 1/(a\ln(10))$ [5a]

$F_{ab}(L_r, m) = b\tanh(x_r/a) = b\frac{L_r^m - L_r^{-m}}{L_r^m + L_r^{-m}}$ $x_r = \log(L_r)$, $L_r = L/L_u$ [1b]

$\frac{dF_{ab}(L_r, m)}{dL_r} = \frac{4bm}{L_r[L_r^m + L_r^{-m}]^2}$ $x_r = \ln L_r/\ln(10)$, $dx_r/dL_r = 1/(\ln(10)L_r)$, $dx_r = (am/L_r)dL_r$ [5b]

hey21−8n eer21−8n

TUB-test chart hey2; Model of normalized response functions $F_{ab}(x_r)$, $Q_{ab}(x_r)$ & $P_{ab}(L_r)$, and derivation, Tangens hyperbolicus $\tanh(x_r)$ and modified functions with e^{x_r}, 10^{x_r}, and L_r

see similar files of the whole serie: http://farbe.li.tu-berlin.de/heys.htm
technical information: http://farbe.li.tu-berlin.de or http://color.li.tu-berlin.de

TUB registration: 20241201-hey2/hey2l0n1.txt /.ps
application for evaluation and measurement of display or print output

TUB material: code=rha4ta