Achromatic colour vision with reliance luminance
Mathematical equations with potential functions

$F_{ab}(L_r,m) = b\tanh(x_r/a) = b\dfrac{L_r^m - L_r^{-m}}{L_r^m + L_r^{-m}}$ $\quad x_r = \ln(L_r/L_u)$, $L_r = L/L_u$, $x_r <= 0$ [1]

$\dfrac{dF_{ab}(L_r,m)}{dL_r} = \dfrac{4bm}{L_r[L_r^m + L_r^{-m}]^2}$ $\quad \dfrac{dx_r}{dL_r} = 1/(\ln(10)L_r)$, $m = 1/(\ln(10)a)$ [5]

$\dfrac{dF_{ab}(L_r,m)}{dL} = \dfrac{4bmL_u}{L_r[L_r^m + L_r^{-m}]^2}$ $\quad \dfrac{dL_r}{dL} = dL/L_u$, $\dfrac{dF_{ab}(L_r,m)}{dL_r} = 1$ [6]

$\dfrac{L}{dL} = \dfrac{4bmL_uL}{L_r[L_r^m + L_r^{-m}]^2}$ $\quad dL = \dfrac{L_r[L_r^m + L_r^{-m}]^2}{4bmL_u}$ [7]

ie101-1a bey10-1a

Achromatic colour vision with reliance luminance
Mathematical equations with potential functions

$F_{cb}(L_r,m) = b\tanh(x_r/c) = b\dfrac{L_r^n - L_r^{-n}}{L_r^n + L_r^{-n}}$ $\quad x_r = \ln(L_r/L_u)$, $L_r = L/L_u$, $x_r >= 0$ [1]

$\dfrac{dF_{cb}(L_r,n)}{dL_r} = \dfrac{4bn}{L_r[L_r^n + L_r^{-n}]^2}$ $\quad \dfrac{dx_r}{dL_r} = 1/(\ln(10)L_r)$, $n = 1/(\ln(10)c)$ [5]

$\dfrac{dF_{cb}(L_r,n)}{dL} = \dfrac{4bnL_u}{L_r[L_r^n + L_r^{-n}]^2}$ $\quad \dfrac{dL_r}{dL} = dL/L_u$, $\dfrac{dF_{cb}(L_r,n)}{dL_r} = 1$ [6]

$\dfrac{L}{dL} = \dfrac{4bnL_uL}{L_r[L_r^n + L_r^{-n}]^2}$ $\quad dL = \dfrac{L_r[L_r^n + L_r^{-n}]^2}{4bnL_u}$ [7]

ie101-2a bey10-2a

Achromatic colour vision with reliance luminance
Mathematical hyperbel and potential functions

$F_{ab}(x_r,a) = b\tanh(x_r/a) = b\dfrac{e^{x_r/a} - e^{-x_r/a}}{e^{x_r/a} + e^{-x_r/a}}$ $\quad x_r = \ln(L_r/L_u)$, $L_r = L/L_u$, $x_r <= 0$ [1]

$\dfrac{dF_{ab}(x_r,a)}{dx_r} = \dfrac{4b}{a[e^{x_r/a} + e^{-x_r/a}]^2}$ $\quad \dfrac{x_r}{dx_r} = \ln(L_r/\ln(10)a)$, $m = 1/(\ln(10)a)$ [5]

$\dfrac{L}{dL} = \dfrac{4bm}{[e^{x_r/a} + e^{-x_r/a}]^2}$ $\quad dL = \dfrac{[e^{x_r/a} + e^{-x_r/a}]^2L}{4bm}$ [7]

$\dfrac{L}{dL} = \dfrac{4bm}{[L_r^m + L_r^{-m}]^2}$ $\quad dL = \dfrac{[L_r^m + L_r^{-m}]^2L}{4bm}$ [8]

ie11-1a bey11-1a

Achromatic colour vision with reliance luminance
Mathematical hyperbel and potential functions

$F_{cb}(x_r,a) = b\tanh(x_r/c) = b\dfrac{e^{x_r/c} - e^{-x_r/c}}{e^{x_r/c} + e^{-x_r/c}}$ $\quad x_r = \ln(L_r/L_u)$, $L_r = L/L_u$, $x_r >= 0$ [1]

$\dfrac{dF_{cb}(x_r,a)}{dx_r} = \dfrac{4b}{c[e^{x_r/c} + e^{-x_r/c}]^2}$ $\quad \dfrac{x_r}{dx_r} = \ln(L_r/\ln(10)c)$, $n = 1/(\ln(10)c)$ [5]

$\dfrac{L}{dL} = \dfrac{4bn}{[e^{x_r/c} + e^{-x_r/c}]^2}$ $\quad dL = \dfrac{[e^{x_r/c} + e^{-x_r/c}]^2L}{4bn}$ [7]

$\dfrac{L}{dL} = \dfrac{4bn}{[L_r^n + L_r^{-n}]^2}$ $\quad dL = \dfrac{[L_r^n + L_r^{-n}]^2L}{4bn}$ [8]

ie11-2a bey11-2a

Achromatic colour vision with reliance luminance
Mathematical equations with potential functions

$F_{ab}(L_r,m) = b\tanh(x_r/a) = b\dfrac{L_r^m - L_r^{-m}}{L_r^m + L_r^{-m}}$ $\quad x_r = \ln(L_r/L_u)$, $L_r = L/L_u$, $x_r <= 0$ [1]

$\dfrac{dF_{ab}(L_r,m)}{dL_r} = \dfrac{4bm}{L_r[L_r^m + L_r^{-m}]^2}$ $\quad \dfrac{dx_r}{dL_r} = 1/(\ln(10)L_r)$, $m = 1/(\ln(10)a)$ [5]

$\dfrac{L}{dL} = \dfrac{4bmL}{L_r[L_r^m + L_r^{-m}]^2}$ $\quad dL = \dfrac{L_r[L_r^m + L_r^{-m}]^2}{4bmL_u}$ [7]

$\dfrac{L/dL}{(L/dL)_u} = \dfrac{4L}{L_r[L_r^m + L_r^{-m}]^2L_u}$; $\dfrac{dL}{dL_u} = \dfrac{L_r[L_r^m + L_r^{-m}]^2}{4}$ [8]

ie101-3a bey10-3a

Achromatic colour vision with reliance luminance
Mathematical equations with potential functions

$F_{cb}(L_r,m) = b\tanh(x_r/c) = b\dfrac{L_r^n - L_r^{-n}}{L_r^n + L_r^{-n}}$ $\quad x_r = \ln(L_r/L_u)$, $L_r = L/L_u$, $x_r >= 0$ [1]

$\dfrac{dF_{cb}(L_r,n)}{dL_r} = \dfrac{4bn}{L_r[L_r^n + L_r^{-n}]^2}$ $\quad \dfrac{dx_r}{dL_r} = 1/(\ln(10)L_r)$, $n = 1/(\ln(10)c)$ [5]

$\dfrac{L}{dL} = \dfrac{4bnL}{L_r[L_r^n + L_r^{-n}]^2}$ $\quad dL = \dfrac{L_r[L_r^n + L_r^{-n}]^2}{4bnL_u}$ [7]

$\dfrac{L/dL}{(L/dL)_u} = \dfrac{4L}{L_r[L_r^n + L_r^{-n}]^2L_u}$; $\dfrac{dL}{dL_u} = \dfrac{L_r[L_r^n + L_r^{-n}]^2}{4}$ [8]

ie101-3a bey10-3a

Achromatic colour vision with reliance luminance
Mathematical hyperbel and potential functions

$F_{ab}(x_r,a) = b\tanh(x_r/a) = b\dfrac{e^{x_r/a} - e^{-x_r/a}}{e^{x_r/a} + e^{-x_r/a}}$ $\quad x_r = \ln(L_r/L_u)$, $L_r = L/L_u$, $x_r <= 0$ [1]

$\dfrac{dF_{ab}(x_r,a)}{dx_r} = \dfrac{4b}{a[e^{x_r/a} + e^{-x_r/a}]^2}$ $\quad \dfrac{x_r}{dx_r} = \ln(L_r/\ln(10)a)$, $m = 1/(\ln(10)a)$ [5]

$\dfrac{L/dL}{(L/dL)_u} = \dfrac{4}{[e^{x_r/a} + e^{-x_r/a}]^2}$; $\dfrac{dL}{dL_u} = \dfrac{[e^{x_r/a} + e^{-x_r/a}]^2L}{4L_u}$ [8]

$\dfrac{L/dL}{(L/dL)_u} = \dfrac{4}{[L_r^m + L_r^{-m}]^2}$; $\dfrac{dL}{dL_u} = \dfrac{[L_r^m + L_r^{-m}]^2L}{4L_u}$ [9]

ie11-3a bey11-3a

Achromatic colour vision with reliance luminance
Mathematical hyperbel and potential functions

$F_{cb}(x_r,a) = b\tanh(x_r/c) = b\dfrac{e^{x_r/c} - e^{-x_r/c}}{e^{x_r/c} + e^{-x_r/c}}$ $\quad x_r = \ln(L_r/L_u)$, $L_r = L/L_u$, $x_r >= 0$ [1]

$\dfrac{dF_{cb}(x_r,a)}{dx_r} = \dfrac{4b}{c[e^{x_r/c} + e^{-x_r/c}]^2}$ $\quad \dfrac{x_r}{dx_r} = \ln(L_r/\ln(10)c)$, $n = 1/(\ln(10)c)$ [5]

$\dfrac{L/dL}{(L/dL)_u} = \dfrac{4}{[e^{x_r/c} + e^{-x_r/c}]^2}$; $\dfrac{dL}{dL_u} = \dfrac{[e^{x_r/c} + e^{-x_r/c}]^2L}{4L_u}$ [8]

$\dfrac{L/dL}{(L/dL)_u} = \dfrac{4}{[L_r^n + L_r^{-n}]^2}$; $\dfrac{dL}{dL_u} = \dfrac{[L_r^n + L_r^{-n}]^2L}{4L_u}$ [9]

ie11-3a bey11-3a

Achromatic colour vision with reliance luminance
Mathematical equations with potential functions

$F_{ab}(L_r,m) = b\tanh(x_r/a) = b\dfrac{L_r^m - L_r^{-m}}{L_r^m + L_r^{-m}}$ $\quad x_r = \ln(L_r/L_u)$, $L_r = L/L_u$, $x_r <= 0$ [1]

$\dfrac{dF_{ab}(L_r,m)}{dL_r} = \dfrac{4bm}{L_r[L_r^m + L_r^{-m}]^2}$ $\quad \dfrac{dx_r}{dL_r} = 1/(\ln(10)L_r)$, $m = 1/(\ln(10)a)$ [5]

$\dfrac{L/dL}{(L/dL)_u} = \dfrac{4L}{L_r[L_r^m + L_r^{-m}]^2L_u}$; $\dfrac{dL}{dL_u} = \dfrac{L_r[L_r^m + L_r^{-m}]^2}{4}$ [8]

$\dfrac{L/dL}{(L/dL)_u} = 1$ for $\begin{cases} L=L_u \\ x_r=0 \end{cases}$; $\dfrac{dL}{dL_u} = 1$ for $\begin{cases} L=L_u \\ x_r=0 \end{cases}$ [9]

ie101-3a bey10-3a

Achromatic colour vision with reliance luminance
Mathematical equations with potential functions

$F_{cb}(L_r,m) = b\tanh(x_r/c) = b\dfrac{L_r^n - L_r^{-n}}{L_r^n + L_r^{-n}}$ $\quad x_r = \ln(L_r/L_u)$, $L_r = L/L_u$, $x_r >= 0$ [1]

$\dfrac{dF_{cb}(L_r,n)}{dL_r} = \dfrac{4bn}{L_r[L_r^n + L_r^{-n}]^2}$ $\quad \dfrac{dx_r}{dL_r} = 1/(\ln(10)L_r)$, $n = 1/(\ln(10)c)$ [5]

$\dfrac{L/dL}{(L/dL)_u} = \dfrac{4L}{L_r[L_r^n + L_r^{-n}]^2L_u}$; $\dfrac{dL}{dL_u} = \dfrac{L_r[L_r^n + L_r^{-n}]^2}{4}$ [8]

$\dfrac{L/dL}{(L/dL)_u} = 1$ for $\begin{cases} L=L_u \\ x_r=0 \end{cases}$; $\dfrac{dL}{dL_u} = 1$ for $\begin{cases} L=L_u \\ x_r=0 \end{cases}$ [9]

ie101-3a bey10-3a

Achromatic colour vision with reliance luminance
Mathematical hyperbel and potential functions

$F_{ab}(x_r,a) = b\tanh(x_r/a) = b\dfrac{e^{x_r/a} - e^{-x_r/a}}{e^{x_r/a} + e^{-x_r/a}}$ $\quad x_r = \ln(L_r/L_u)$, $L_r = L/L_u$, $x_r <= 0$ [1]

$\dfrac{dF_{ab}(x_r,a)}{dx_r} = \dfrac{4b}{a[e^{x_r/a} + e^{-x_r/a}]^2}$ $\quad \dfrac{x_r}{dx_r} = \ln(L_r/\ln(10)a)$, $m = 1/(\ln(10)a)$ [5]

$\dfrac{L/dL}{(L/dL)_u} = \dfrac{4}{[e^{x_r/a} + e^{-x_r/a}]^2}$; $\dfrac{dL}{dL_u} = \dfrac{[e^{x_r/a} + e^{-x_r/a}]^2L}{4L_u}$ [8]

$\dfrac{L/dL}{(L/dL)_u} = \dfrac{4}{[L_r^{2m} + 2 + L_r^{-2m}]}$; $\dfrac{dL}{dL_u} = \dfrac{[L_r^{2m} + 2 + L_r^{-2m}]L}{4L_u}$ [9]

ie11-3a bey11-3a

Achromatic colour vision with reliance luminance
Mathematical hyperbel and potential functions

$F_{cb}(x_r,a) = b\tanh(x_r/c) = b\dfrac{e^{x_r/c} - e^{-x_r/c}}{e^{x_r/c} + e^{-x_r/c}}$ $\quad x_r = \ln(L_r/L_u)$, $L_r = L/L_u$, $x_r >= 0$ [1]

$\dfrac{dF_{cb}(x_r,a)}{dx_r} = \dfrac{4b}{c[e^{x_r/c} + e^{-x_r/c}]^2}$ $\quad \dfrac{x_r}{dx_r} = \ln(L_r/\ln(10)c)$, $n = 1/(\ln(10)c)$ [5]

$\dfrac{L/dL}{(L/dL)_u} = \dfrac{4}{[e^{x_r/c} + e^{-x_r/c}]^2}$; $\dfrac{dL}{dL_u} = \dfrac{[e^{x_r/c} + e^{-x_r/c}]^2L}{4L_u}$ [8]

$\dfrac{L/dL}{(L/dL)_u} = \dfrac{4}{[L_r^{2n} + 2 + L_r^{-2n}]}$; $\dfrac{dL}{dL_u} = \dfrac{[L_r^{2n} + 2 + L_r^{-2n}]L}{4L_u}$ [9]

ie11-3a bey11-3a

TUB-test chart iei1; $F_{ab/cb}(L_r)$ colour vision models for receptor responses $F_{ab/cb}(L_r)$
Derivations $F'_{ab/cb}(L_r)$ with power functions $L_r{}^m$ and $L_r{}^n$ for contrast $L/\Delta L$ and discrimination ΔL

see similar files of the whole serie: http://farbe.li.tu-berlin.de/jeis.htm
technical information: http://farbe.li.tu-berlin.de or http://color.li.tu-berlin.de

TUB registration: 20253031-ie1/ie110n1.txt /.ps
application for evaluation and measurement of display or print output
TUB materials: code=rha4a