$F_{ab}(x_r)$=achromatic receptor response, TUB model

$F_{ab}(x_r) = b\,\frac{10^{x_r/a'} - 10^{-x_r/a'}}{10^{x_r/a'} + 10^{-x_r/a'}}$ $10^{x_r}=e^{\ln(10)\,x_r}$, $10^{x_r/\ln(10)}=e^{x_r}$

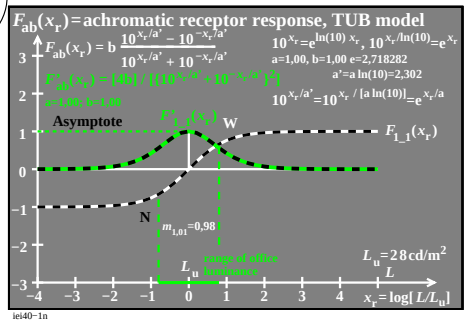
a=1,00, b=1,00 e=2,718282

a'=a ln(10)=2,302

$10^{x_r/a'}=10^{x_r\,/\,[a\ln(10)]}=e^{x_r/a}$

$F'_{ab}(x_r)=4b/[a(10^{x_r/a'}+10^{-x_r/a'})^2]$

a=1,00; b=1,00



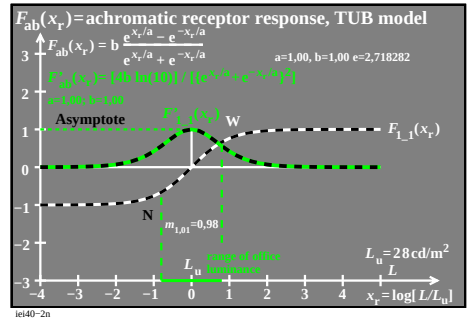
iei40−1n

$F_{ab}(x_r)$=achromatic receptor response, TUB model

$F_{ab}(x_r) = b\,\frac{e^{x_r/a} - e^{-x_r/a}}{e^{x_r/a} + e^{-x_r/a}}$ a=1,00, b=1,00 e=2,718282

$F'_{ab}(x_r)=4b\ln(10)/[(e^{x_r/a}+e^{-x_r/a})^2]$

a=1,00; b=1,00



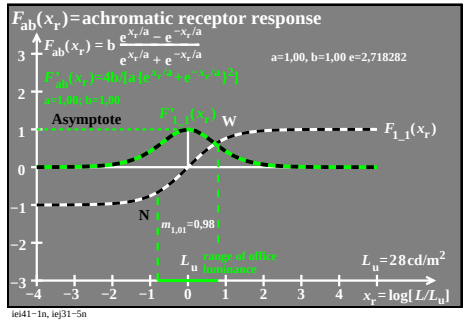
iei40−2n

$F_{ab}(x_r)$=achromatic receptor response

$F_{ab}(x_r) = b\,\frac{e^{x_r/a} - e^{-x_r/a}}{e^{x_r/a} + e^{-x_r/a}}$ a=1,00, b=1,00 e=2,718282

$F'_{ab}(x_r)=4b\ln(10)/[(e^{x_r/a}+e^{-x_r/a})^2]$

a=1,00; b=1,00



iei41−1n, iej31−5n

$F_{ab}(x_r)$=achromatic receptor response

$F_{ab}(x_r) = b\,\frac{10^{x_r/a'} - 10^{-x_r/a'}}{10^{x_r/a'} + 10^{-x_r/a'}}$ $10^{x_r}=e^{\ln(10)\,x_r}$, $10^{x_r/\ln(10)}=e^{x_r}$

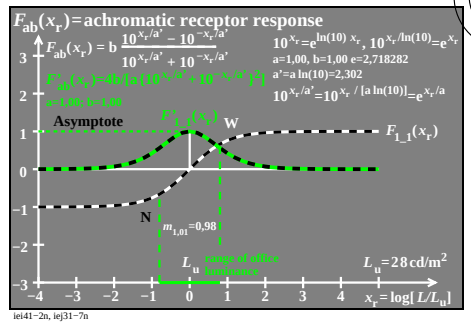
a=1,00, b=1,00 e=2,718282

a'=a ln(10)=2,302

$10^{x_r/a'}=10^{x_r\,/\,[a\ln(10)]}=e^{x_r/a}$

$F'_{ab}(x_r)=4b/[a(10^{x_r/a'}+10^{-x_r/a'})^2]$

a=1,00; b=1,00



iei41−2n, iej31−7n

$F_{ab}(x_r)$=achromatic receptor response, TUB model

$F_{ab}(x_r) = b\,\frac{L_r^{1/a'} - L_r^{-1/a'}}{L_r^{1/a'} + L_r^{-1/a'}}$ $10^{x_r}=e^{\ln(10)\,x_r}$, $10^{x_r/\ln(10)}=e^{x_r}$

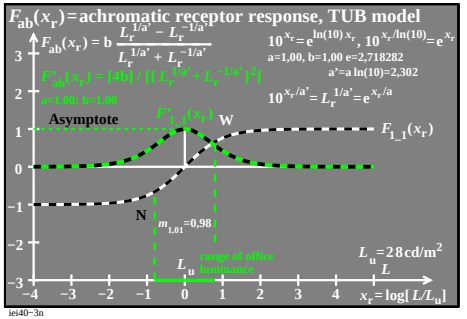
a=1,00, b=1,00 e=2,718282

a'=a ln(10)=2,302

$10^{x_r/a'}=L_r^{1/a'}=e^{x_r/a}$

$F'_{ab}(x_r)=4b/[a(L_r^{1/a'}+L_r^{-1/a'})^2]$

a=1,00; b=1,00



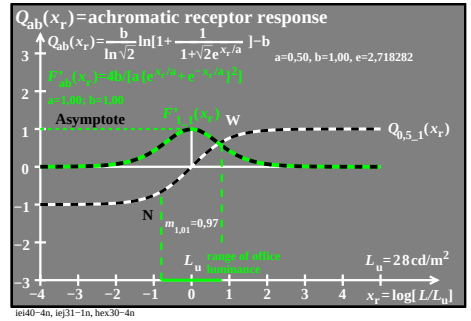
iei40−3n

$Q_{ab}(x_r)$=achromatic receptor response

$Q_{ab}(x_r)=\frac{b}{\ln\sqrt{2}}\ln[1+\frac{1}{1+\sqrt{2}e^{x_r/a}}]-b$ a=0,50, b=1,00, e=2,718282

$F'_{ab}(x_r)=4b/[a(e^{x_r/a}+e^{-x_r/a})^2]$

a=1,00; b=1,00



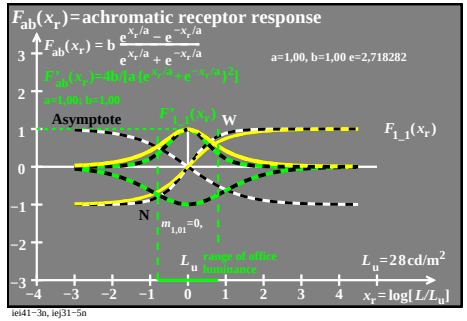
iei40−4n, iej31−1n, hex30−4n

$F_{ab}(x_r)$=achromatic receptor response

$F_{ab}(x_r) = b\,\frac{e^{x_r/a} - e^{-x_r/a}}{e^{x_r/a} + e^{-x_r/a}}$ a=1,00, b=1,00 e=2,718282

$F'_{ab}(x_r)=4b/[a(e^{x_r/a}+e^{-x_r/a})^2]$

a=1,00; b=1,00



iei41−3n, iej31−5n

Mathematical equations of hyperbel functions

See: *Papula, L., (2003), Mathematische Formelsammlung, Vieweg*

$$F_{ab}(x_r/a)=b\,\frac{e^{x_r/a}-e^{-x_r/a}}{e^{x_r/a}+e^{-x_r/a}}=b\,\frac{u(x_r/a)}{v(x_r/a)} \quad [1]$$

$$F'_{ab}(x_r/a)=b\,\frac{u'(x_r/a)\,v(x_r/a)-u(x_r/a)\,v'(x_r/a)}{v^2(x_r/a)} \quad [2]$$

$$F'_{ab}(x_r/a)=b\,\frac{v^2(x_r/a)-u^2(x_r/a)}{a\,v^2(x_r/a)} \quad [3]$$

$$F'_{ab}(x_r/a)=\frac{4b}{a\,[e^{x_r/a}+e^{-x_r/a}]^2} \quad [4]$$

iei41−4n, inj31−8n mod

Mathematical equations of hyperbolic functions

See: *Handbook of Mathematical functions, NBS, USA, Sec. 4.5*

$$\sinh(x)=\frac{e^x-e^{-x}}{2} \quad [1],\quad \cosh(x)=\frac{e^x+e^{-x}}{2} \quad [2]$$

$$\tanh(x)=\frac{\sinh(x)}{\cosh(x)}=\frac{e^x-e^{-x}}{e^x+e^{-x}} \quad [3]$$

$$\tanh(x/2)=\frac{\sinh(x)}{\cosh(x)+1}=\frac{\cosh(x)+1}{\sinh(x)}=\frac{e^{x/2}-e^{-x/2}}{e^{x/2}+e^{-x/2}} \quad [4]$$

$$\sinh^2(x)+\cosh^2(x)=1 \quad [5]$$

iei40−5n, iej30−6n

Mathematical equations of hyperbolic functions

See: *Handbook of Mathematical functions, NBS, USA, Sec. 4.5*

$$F_{1b}(x)=b\tanh(x)=b\,\frac{e^x-e^{-x}}{e^x+e^{-x}}=b\,\frac{u(x)}{v(x)} \quad [1]$$

$$F'_{1b}(x)=b\,\frac{u'(x)\,v(x)-u(x)\,v'(x)}{v^2(x)} \quad [2]$$

$$F'_{1b}(x)=b\,\frac{v^2(x)-u^2(x)}{a\,v^2(x)} \quad [3]$$

$$F'_{1b}(x)=\frac{4b}{[e^x+e^{-x}]^2}=\frac{b}{\cosh^2(x)} \quad [4]$$

iei40−6n, iej31−6n

Mathematical equations of hyperbolic functions

See: *Handbook of Mathematical functions, NBS, USA, Sec. 4.5*

$$F_{ab}(x/a)=b\tanh(x/a)=b\,\frac{e^{x/a}-e^{-x/a}}{e^{x/a}+e^{-x/a}}=b\,\frac{u(x/a)}{v(x/a)} \quad [1]$$

$$F'_{ab}(x/a)=b\,\frac{u'(x/a)\,v(x/a)-u(x/a)\,v'(x/a)}{v^2(x/a)} \quad [2]$$

$$F'_{ab}(x/a)=b\,\frac{v^2(x/a)-u^2(x/a)}{a\,v^2(x/a)} \quad [3]$$

$$F'_{ab}(x/a)=\frac{4b}{a\,[e^{x/a}+e^{-x/a}]^2}=\frac{b}{a\cosh^2(x/a)} \quad [4]$$

iei41−5n, inj31−8n mod

Mathematical equations of hyperbolic functions

See: *Handbook of Mathematical functions, NBS, USA, Sec. 4.5*

$$F_{ab}(x_r/a)=b\,\frac{e^{x_r/a}-e^{-x_r/a}}{e^{x_r/a}+e^{-x_r/a}}=b\,\frac{u(x_r/a)}{v(x_r/a)} \quad [1]$$

$$F'_{ab}(x_r/a)=b\,\frac{v^2(x_r/a)-u^2(x_r/a)}{a\,v^2(x_r/a)} \quad [2]$$

$$F'_{ab}(x_r/a)=\frac{4b}{a\,[e^{x_r/a}+e^{-x_r/a}]^2} \quad [3]$$

$$x_r=\log(L/L_u)=\log(L_r) \quad [4]$$

$$e^{x_r/a}=10^{x_r/a'}=L_r{}^{1/a'} \quad [5] \qquad a'=a\ln(10) \quad [6]$$

iei41−6n, inj31−8n mod

$F_{ab}(x_r)$=achromatic receptor response

$F_{ab}(x_r) = b\,\frac{10^{x_r/a'} - 10^{-x_r/a'}}{10^{x_r/a'} + 10^{-x_r/a'}}$ $10^{x_r}=e^{\ln(10)\,x_r}$, $10^{x_r/\ln(10)}=e^{x_r}$

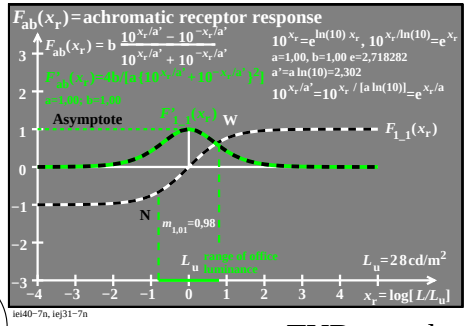
a=1,00, b=1,00 e=2,718282

a'=a ln(10)=2,302

$10^{x_r/a'}=10^{x_r\,/\,[a\ln(10)]}=e^{x_r/a}$

$F'_{ab}(x_r)=4b/[a(10^{x_r/a'}+10^{-x_r/a'})^2]$

a=1,00; b=1,00



iei40−7n, iej31−7n

Achromatic receptor-response function $Q_{ab}[x_r/a]$ for a=0,5 and b=1,0

with $x_r=\log[L/L_u]$ (L=test luminance)

L_u=surround luminance

$$Q_{ab}[x_r/a]=\frac{b}{\ln\sqrt{2}}\ln[\frac{1}{1+\sqrt{2}e^{(x_r/a)}}]-b$$

function values for b=1 and any a>0 :

$Q_{a1}[x_r/a\to-\infty]=-1$ $\quad x=\log L,\ u=\log L_u$

$Q_{a1}[x_r/a=0]=0$ $\quad x_r=\log[L/L_u]$

$Q_{a1}[x_r/a\to+\infty]=+1$ $\quad =x-u$

iei40−8n, iej30−2n

Mathematical equations of hyperbolic functions

See: *Handbook of Mathematical functions, NBS, USA, Sec. 4.5*

$$F_{ab}(x/a)=b\tanh(x/a)=b\,\frac{e^{x/a}-e^{-x/a}}{e^{x/a}+e^{-x/a}}=b\,\frac{u(x/a)}{v(x/a)} \quad [1]$$

$$F'_{ab}(x/a)=b\,\frac{u'(x/a)\,v(x/a)-u(x/a)\,v'(x/a)}{v^2(x/a)} \quad [2]$$

$$u'(x/a)=1/a\,[e^{x/a}-e^{-x/a}]=(1/a)v(x/a) \quad [3]$$

$$v'(x/a)=1/a\,[e^{x/a}+e^{-x/a}]=(1/a)u(x/a) \quad [4]$$

$$F'_{ab}(x/a)=b\,\frac{v^2(x/a)-u^2(x/a)}{a\,v^2(x/a)} \quad [5]$$

iei41−7n, inj31−8n mod mod

Mathematical equations of hyperbolic functions

See: *Handbook of Mathematical functions, NBS, USA, Sec. 4.5*

$$F_{ab}(x_r/a)=b\,\frac{e^{x_r/a}-e^{-x_r/a}}{e^{x_r/a}+e^{-x_r/a}} \quad [1]$$

$$F'_{ab}(x_r/a)=\frac{dF_{ab}(x_r/a)}{d(x_r/a)}=\frac{4b}{a\,[e^{x_r/a}+e^{-x_r/a}]^2} \quad [2]$$

$$x_r=\log(L/L_u)=\log(L_r) \quad [3]$$

$$e^{x_r/a}=10^{x_r/a'}=L_r{}^{1/a'} \quad [4] \qquad a'=a\ln(10) \quad [5]$$

$$F'_{ab}(x_r/a)=\frac{4b}{a\,[L_r{}^{1/a'}+L_r{}^{-1/a'}]^2} \quad [7]$$

iei41−8n mod, inj31−8n mod

TUB-test chart iei4; Model of normalized response function $F_{ab}(x_r)$ and derivation $F'_{ab}(x_r)$

Mathematical calculation of the derivation $F'_{ab}(x_r)$, of the contrast $L/\Delta L$, and the discrimination ΔL