http://farbe.li.tu-berlin.de/iei5/iei5l0n1.txt /.ps; only vector graphic VG; start output
see separate figures of this output: http://farbe.li.tu-berlin.de/iei5/iei5.htm

Mathematical equations of hyperbolic functions
See: Handbook of Mathematical functions, NBS, USA, Sec. 4.5

$F_a(x_r) = a\frac{e^{x_r/a}}{e^{x_r/a}+e^{-x_r/a}} = a\frac{u(x_r,a)}{v(x_r,a)}$ [1]

$F'_a(x_r) = a\frac{u'(x_r,a)\,v(x_r,a) - v'(x_r,a)\,u(x_r,a)}{v^2(x_r,a)}$ [2]

$u'v = [(1/a)\,e^{x_r/a}]\,[e^{x_r/a} + e^{-x_r/a}]$ [3]

$v'u = [(1/a)\,e^{x_r/a} - (1/a)\,e^{-x_r/a}]\,[e^{x_r/a}]$ [4]

$u'v = [1/a]\,\{[e^{x_r/a}]^2 + 1\}$ [5]

$v'u = [1/a]\,\{[e^{x_r/a}]^2 - 1\}$ [6]

$u'(x_r,a)\,v(x_r,a) - v'(x_r,a)\,u(x_r,a) = 2/a$ [7]

iei50-1n

Mathematical equations of hyperbolic functions
See: Handbook of Mathematical functions, NBS, USA, Sec. 4.5

$F_c(x_r) = c\frac{-e^{-x_r/c}}{e^{x_r/c}+e^{-x_r/c}} = c\frac{u(x_r,c)}{v(x_r,c)}$ [1]

$F'_c(x_r) = c\frac{u'(x_r,c)\,v(x_r,c) - v'(x_r,c)\,u(x_r,c)}{v^2(x_r,c)}$ [2]

$u'v = [(1/c)\,e^{-x_r/c}]\,[e^{x_r/c} + e^{-x_r/c}]$ [3]

$v'u = [(1/c)\,e^{x_r/c} - (1/c)\,e^{-x_r/c}]\,[-e^{-x_r/c}]$ [4]

$u'v = [1/c]\,\{[e^{-x_r/c}]^2 + 1\}$ [5]

$v'u = [1/c]\,\{[e^{-x_r/c}]^2 - 1\}$ [6]

$u'(x_r,c)\,v(x_r,c) - v'(x_r,c)\,u(x_r,c) = 2/c$ [7]

iei50-2n

Achromatic colour vision with relative luminance
Mathematical hyperbel and potential functions

$F_{ab}(x_r,a) = b\tanh(x_r/a) = b\frac{e^{x_r/a}-e^{-x_r/a}}{e^{x_r/a}+e^{-x_r/a}}$; $x_r=\log(L_r)$, $L_r=L/L_u$, $x_r<=0$ [1]

$\frac{dF_{ab}(x_r,a)}{dx_r} = \frac{4b}{a[e^{x_r/a}+e^{-x_r/a}]^2}$; $x_r=\ln L_r/\ln(10)$, $dx_r/dL_r=1/(\ln(10)L_r)$, $m=1/(\ln(10)\,a)$ [5]

$\frac{L}{dL} = \frac{4bm}{[e^{x_r/a}+e^{-x_r/a}]^2}$; $dL = \frac{[e^{x_r/a}+e^{-x_r/a}]^2 L}{4bm}$ [7]

$\frac{L}{dL} = \frac{4bm}{[L_r^{m}+L_r^{-m}]^2}$; $dL = \frac{[L_r^{m}+L_r^{-m}]^2 L}{4bm}$ [8]

iei51-1n iei01-1n

Achromatic colour vision with relative luminance
Mathematical hyperbel and potential functions

$F_{cb}(x_r,c) = b\tanh(x_r/c) = b\frac{e^{x_r/c}-e^{-x_r/c}}{e^{x_r/c}+e^{-x_r/c}}$; $x_r=\log(L_r)$, $L_r=L/L_u$, $x_r>=0$ [1]

$\frac{dF_{cb}(x_r,c)}{dx_r} = \frac{4b}{c[e^{x_r/c}+e^{-x_r/c}]^2}$; $x_r=\ln L_r/\ln(10)$, $dx_r/dL_r=1/(\ln(10)L_r)$, $n=1/(\ln(10)\,c)$ [5]

$\frac{L}{dL} = \frac{4bn}{[e^{x_r/c}+e^{-x_r/c}]^2}$; $dL = \frac{[e^{x_r/c}+e^{-x_r/c}]^2 L}{4bn}$ [7]

$\frac{L}{dL} = \frac{4bn}{[L_r^{n}+L_r^{-n}]^2}$; $dL = \frac{[L_r^{n}+L_r^{-n}]^2 L}{4bn}$ [8]

iei51-2n iei01-2n

Mathematical equations of hyperbolic functions
See: Handbook of Mathematical functions, NBS, USA, Sec. 4.5

$F_a(x_r) = a\frac{e^{x_r/a}}{e^{x_r/a}+e^{-x_r/a}} = a\frac{u(x_r,a)}{v(x_r,a)}$ [1]

$F'_a(x_r) = a\frac{u'(x_r,a)\,v(x_r,a) - v'(x_r,a)\,u(x_r,a)}{v^2(x_r,a)}$ [2]

$u'v = [(1/a)\,e^{x_r/a}]\,[e^{x_r/a} + e^{-x_r/a}]$ [3]

$v'u = [(1/a)\,e^{x_r/a} - (1/a)\,e^{-x_r/a}]\,[e^{x_r/a}]$ [4]

$u'v = [1/a]\,\{[e^{x_r/a}]^2 + 1\}$ [5]

$v'u = [1/a]\,\{[e^{x_r/a}]^2 - 1\}$ [6]

$F'_a(x_r) = 2\,/\,[e^{x_r/a}+e^{-x_r/a}]^2$ [7]

iei50-3n

Mathematical equations of hyperbolic functions
See: Handbook of Mathematical functions, NBS, USA, Sec. 4.5

$F_c(x_r) = c\frac{-e^{-x_r/c}}{e^{x_r/c}+e^{-x_r/c}} = c\frac{u(x_r,c)}{v(x_r,c)}$ [1]

$F'_c(x_r) = c\frac{u'(x_r,c)\,v(x_r,c) - v'(x_r,c)\,u(x_r,c)}{v^2(x_r,c)}$ [2]

$u'v = [(1/c)\,e^{-x_r/c}]\,[e^{x_r/c} + e^{-x_r/c}]$ [3]

$v'u = [(1/c)\,e^{x_r/c} - (1/c)\,e^{-x_r/c}]\,[-e^{-x_r/c}]$ [4]

$u'v = [1/c]\,\{[e^{-x_r/c}]^2 + 1\}$ [5]

$v'u = [1/c]\,\{[e^{-x_r/c}]^2 - 1\}$ [6]

$F'_c(x_r) = 2\,/\,[e^{x_r/c}+e^{-x_r/c}]^2$ [7]

iei50-4n

Achromatic colour vision with relative luminance
Mathematical hyperbel and potential functions

$F_{ab}(x_r,a) = b\tanh(x_r/a) = b\frac{e^{x_r/a}-e^{-x_r/a}}{e^{x_r/a}+e^{-x_r/a}}$; $x_r=\log(L_r)$, $L_r=L/L_u$, $x_r<=0$ [1]

$\frac{dF_{ab}(x_r,a)}{dx_r} = \frac{4b}{a[e^{x_r/a}+e^{-x_r/a}]^2}$; $x_r=\ln L_r/\ln(10)$, $dx_r/dL_r=1/(\ln(10)L_r)$, $m=1/(\ln(10)\,a)$ [5]

$\frac{L}{dL} = \frac{4bm}{[e^{x_r/a}+e^{-x_r/a}]^2}$; $dL = \frac{[e^{x_r/a}+e^{-x_r/a}]^2 L}{4bm}$ [7]

$\frac{L}{dL} = \frac{4bm}{[L_r^{2m}+2+L_r^{-2m}]}$; $dL = \frac{[L_r^{2m}+2+L_r^{-2m}] L}{4bm}$ [8]

iei51-3n iei01-3n

Achromatic colour vision with relative luminance
Mathematical hyperbel and potential functions

$F_{cb}(x_r,c) = b\tanh(x_r/c) = b\frac{e^{x_r/c}-e^{-x_r/c}}{e^{x_r/c}+e^{-x_r/c}}$; $x_r=\log(L_r)$, $L_r=L/L_u$, $x_r>=0$ [1]

$\frac{dF_{cb}(x_r,c)}{dx_r} = \frac{4b}{c[e^{x_r/c}+e^{-x_r/c}]^2}$; $x_r=\ln L_r/\ln(10)$, $dx_r/dL_r=1/(\ln(10)L_r)$, $n=1/(\ln(10)\,c)$ [5]

$\frac{L}{dL} = \frac{4bn}{[e^{x_r/c}+e^{-x_r/c}]^2}$; $dL = \frac{[e^{x_r/c}+e^{-x_r/c}]^2 L}{4bn}$ [7]

$\frac{L}{dL} = \frac{4bn}{[L_r^{2n}+2+L_r^{-2n}]}$; $dL = \frac{[L_r^{2n}+2+L_r^{-2n}] L}{4bn}$ [8]

iei51-4n iei01-4n

Mathematical equations of hyperbolic functions
See: Handbook of Mathematical functions, NBS, USA, Sec. 4.5

$F_{ab}(x/a) = b\tanh(x/a) = b\frac{e^{x/a}-e^{-x/a}}{e^{x/a}+e^{-x/a}} = b\frac{u(x/a)}{v(x/a)}$ [1]

$F'_{ab}(x/a) = b\frac{u'(x/a)\,v(x/a) - u(x/a)\,v'(x/a)}{v^2(x/a)}$ [2]

$F'_{ab}(x/a) = b\frac{v^2(x/a) - u^2(x/a)}{a\,v^2(x/a)}$ [3]

$F'_{ab}(x/a) = \frac{4b}{a\,[e^{x/a})+e^{-x/a}]^2} = \frac{b}{a\cosh^2(x/a)}$ [4]

iei50-5n

Mathematical equations of hyperbolic functions
See: Handbook of Mathematical functions, NBS, USA, Sec. 4.5

$F_{ab}(x_r/a) = b\frac{e^{x_r/a}-e^{-x_r/a}}{e^{x_r/a}+e^{-x_r/a}} = b\frac{u(x_r/a)}{v(x_r/a)}$ [1]

$F'_{ab}(x_r/a) = b\frac{v^2(x_r/a) - u^2(x_r/a)}{a\,v^2(x_r/a)}$ [2]

$F'_{ab}(x_r/a) = \frac{4b}{a\,[e^{x_r/a}+e^{-x_r/a}]^2}$ [3]

$x_r = \log(L/L_u) = \log(L_r)$ [4]

$e^{x_r/a} = 10^{x_r/a'} = L_r^{1/a'}$ [5] $a' = a\ln(10)$ [6]

iei50-6n

Achromatic colour vision with relative luminance
Mathematical hyperbel and potential functions

$F_{ab}(x_r,a) = b\tanh(x_r/a) = b\frac{e^{x_r/a}-e^{-x_r/a}}{e^{x_r/a}+e^{-x_r/a}}$; $x_r=\log(L_r)$, $L_r=L/L_u$, $x_r<=0$ [1]

$\frac{dF_{ab}(x_r,a)}{dx_r} = \frac{4b}{a[e^{x_r/a}+e^{-x_r/a}]^2}$; $x_r=\ln L_r/\ln(10)$, $dx_r/dL_r=1/(\ln(10)L_r)$, $m=1/(\ln(10)\,a)$ [5]

$\frac{L/dL}{(L/dL)_u} = \frac{4}{[e^{x_r/a}+e^{-x_r/a}]^2}$; $\frac{dL}{dL_u} = \frac{[e^{x_r/a}+e^{-x_r/a}]^2 L}{4L_u}$ [8]

$\frac{L/dL}{(L/dL)_u} = \frac{4}{[L_r^{m}+L_r^{-m}]^2}$; $\frac{dL}{dL_u} = \frac{[L_r^{m}+L_r^{-m}]^2 L}{4L_u}$ [9]

iei51-5n iei01-5n

Achromatic colour vision with relative luminance
Mathematical hyperbel and potential functions

$F_{cb}(x_r,c) = b\tanh(x_r/c) = b\frac{e^{x_r/c}-e^{-x_r/c}}{e^{x_r/c}+e^{-x_r/c}}$; $x_r=\log(L_r)$, $L_r=L/L_u$, $x_r>=0$ [1]

$\frac{dF_{cb}(x_r,c)}{dx_r} = \frac{4b}{c[e^{x_r/c}+e^{-x_r/c}]^2}$; $x_r=\ln L_r/\ln(10)$, $dx_r/dL_r=1/(\ln(10)L_r)$, $n=1/(\ln(10)\,c)$ [5]

$\frac{L/dL}{(L/dL)_u} = \frac{4}{[e^{x_r/c}+e^{-x_r/c}]^2}$; $\frac{dL}{dL_u} = \frac{[e^{x_r/c}+e^{-x_r/c}]^2 L}{4L_u}$ [8]

$\frac{L/dL}{(L/dL)_u} = \frac{4}{[L_r^{n}+L_r^{-n}]^2}$; $\frac{dL}{dL_u} = \frac{[L_r^{n}+L_r^{-n}]^2 L}{4L_u}$ [9]

iei51-6n iei01-6n

Mathematical equations of hyperbolic functions
See: Handbook of Mathematical functions, NBS, USA, Sec. 4.5

$F_{abc}(x_r) = b\frac{e^{x_r/a}-e^{-x_r/c}}{e^{x_r/a}+e^{-x_r/c}} = b\frac{u(z,a,c)}{v(z,a,c)}$ $z=x_r$ [1]

$F'_{abc}(z) = b\frac{u'(z,a,c)\,v(z,a,c) - v'(z,a,c)\,u(z,a,c)}{v^2(z,a,c)}$ [2]

$u'v = [(1/a)\,e^{z/a} + (1/c)\,e^{-z/c}]\,[e^{z/a} + e^{-z/c}]$ [3]

$v'u = [(1/a)\,e^{z/a} - (1/c)\,e^{-z/c}]\,[e^{z/a} - e^{-z/c}]$ [4]

$u'(z,a,c)\,v(z,a,c) - v'(z,a,c)\,u(z,a,c) = 2\,[1/a + 1/c]\,[e^{x_r/a}\,.\,e^{-x_r/c}]$ [5]

iei50-7n

Mathematical equations of hyperbolic functions
See: Handbook of Mathematical functions, NBS, USA, Sec. 4.5

$F_{abc}(x_r) = b\frac{e^{x_r/a}-e^{-x_r/c}}{e^{x_r/a}+e^{-x_r/c}} = b\frac{u(z,a,c)}{v(z,a,c)}$ $z=x_r$ [1]

$F'_{abc}(z) = b\frac{u'(z,a,c)\,v(z,a,c) - u(z,a,c)\,v'(z,a,c)}{v^2(z,a,c)}$ [2]

$u'(z,a,c)\,v(z,a,c) - v'(z,a,c)\,u(z,a,c) = 2\,[1/a + 1/c]\,[e^{z/a}\,.\,e^{-z/c}]$ [3]

$F'_{abc}(x_r) = 2\,b\,[1/a + 1/c]\,\frac{e^{x_r/a}\,.\,e^{-x_r/c}}{[e^{x_r/a}+e^{-x_r/c}]^2}$ [4]

iei50-8n, iej30-8n mod

Achromatic colour vision with relative luminance
Mathematical hyperbel and potential functions

$F_{ab}(x_r,a) = b\tanh(x_r/a) = b\frac{e^{x_r/a}-e^{-x_r/a}}{e^{x_r/a}+e^{-x_r/a}}$; $x_r=\log(L_r)$, $L_r=L/L_u$, $x_r<=0$ [1]

$\frac{dF_{ab}(x_r,a)}{dx_r} = \frac{4b}{a[e^{x_r/a}+e^{-x_r/a}]^2}$; $x_r=\ln L_r/\ln(10)$, $dx_r/dL_r=1/(\ln(10)L_r)$, $m=1/(\ln(10)\,a)$ [5]

$\frac{L/dL}{(L/dL)_u} = \frac{4}{[e^{x_r/a}+e^{-x_r/a}]^2}$; $\frac{dL}{dL_u} = \frac{[e^{x_r/a}+e^{-x_r/a}]^2 L}{4L_u}$ [8]

$\frac{L/dL}{(L/dL)_u} = \frac{4}{L_r^{2m}+2+L_r^{-2m}}$; $\frac{dL}{dL_u} = \frac{(L_r^{2m}+2+L_r^{-2m}) L}{4L_u}$ [9]

iei51-7n iei01-7n

Achromatic colour vision with relative luminance
Mathematical hyperbel and potential functions

$F_{cb}(x_r,c) = b\tanh(x_r/c) = b\frac{e^{x_r/c}-e^{-x_r/c}}{e^{x_r/c}+e^{-x_r/c}}$; $x_r=\log(L_r)$, $L_r=L/L_u$, $x_r>=0$ [1]

$\frac{dF_{cb}(x_r,c)}{dx_r} = \frac{4b}{c[e^{x_r/c}+e^{-x_r/c}]^2}$; $x_r=\ln L_r/\ln(10)$, $dx_r/dL_r=1/(\ln(10)L_r)$, $n=1/(\ln(10)\,c)$ [5]

$\frac{L/dL}{(L/dL)_u} = \frac{4}{[e^{x_r/c}+e^{-x_r/c}]^2}$; $\frac{dL}{dL_u} = \frac{[e^{x_r/c}+e^{-x_r/c}]^2 L}{4L_u}$ [8]

$\frac{L/dL}{(L/dL)_u} = \frac{4}{L_r^{2n}+2+L_r^{-2n}}$; $\frac{dL}{dL_u} = \frac{(L_r^{2n}+2+L_r^{-2n}) L}{4L_u}$ [9]

iei51-8n iei01-8n

TUB-test chart iei5; Colour vision model for normalized receptor responses $F_{a/c/ab/cb}(x_r)$
Derivations $F'_{a/c/ab/cb}(x_r)$ with hyberbel functions $e^{(x_r/a)}$ and $e^{(x_r/c)}$ for contrast $L/\Delta L$ and ΔL

see similar files of the whole serie: http://farbe.li.tu-berlin.de/iei5/iei5.htm
technical information: http://farbe.li.tu-berlin.de or http://color.li.tu-berlin.de

TUB registration: 20250301-iei5/iei5l0n1.txt /.ps
application for evaluation and measurement of display or print output
TUB material: code=rha4ta