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### Achromatisches Sehen mit relativer Leuchtdichte Mathematikgleichungen mit Hyperbelfunktionen

$$\begin{aligned}
& F_{ab}(x_r, a) = b \tanh(x_r/a) = b \frac{e^{x_r/a} - e^{-x_r/a}}{e^{x_r/a} + e^{-x_r/a}} = b \frac{e^{-x_r/a} - e^{x_r/a}}{e^{-x_r/a} + e^{x_r/a}} = b \frac{x_r}{x_r + e^{2x_r/a}} \\
& dF_{ab}(x_r, a) = \frac{4b}{dx_r} = \frac{4b}{a(e^{x_r/a} + e^{-x_r/a})^2} = \frac{4b}{a(1 + (10x_r)^2)} = \frac{4b}{a(1 + (10x_r)^2)} \\
& dF_{ab}(x_r, a) \frac{dx_r}{dx_T} = \frac{4bm}{[e^{x_r/a} + e^{-x_r/a}]^2 x_T} = \frac{4bm}{[e^{x_r/a} + e^{-x_r/a}]^2 x_T} = \frac{4bm}{[e^{x_r/a} + e^{-x_r/a}]^2 x_T} \\
& \frac{L}{dL} = \frac{4bm}{[e^{x_r/a} + e^{-x_r/a}]^2} = \frac{dL}{dL} = \frac{[e^{x_r/a} + e^{-x_r/a}]^2}{4bm} \quad [7]
\end{aligned}$$

$$\begin{aligned}
 & \text{Achromatisches Schen mit relativ Leuchtintensität} \\
 & \text{Mathematikgleichungen mit Hyperbolifunktionen} \\
 F_{ab}(x_r, a) &= b \tanh(x_r/a) = b \frac{e^{x_r/a} - e^{-x_r/a}}{e^{x_r/a} + e^{-x_r/a}} = \frac{x_r}{L_d + L_u} = \frac{\log(10)}{L_d + L_u} \\
 dF_{ab}/dx_r &= \frac{4b}{x_r[e^{x_r/a} + e^{-x_r/a}]^2} = \frac{x_r \cdot \ln(10)}{L_d + L_u} = \frac{1}{L_d + L_u} \cdot \frac{[x_r \cdot \ln(10)]^2}{[e^{x_r/a} + e^{-x_r/a}]^2} \\
 \frac{L_d}{L_u} &= \frac{4bm}{[e^{x_r/a} + e^{-x_r/a}]^2} = \frac{dL_u}{4bm} = \frac{[e^{x_r/a} + e^{-x_r/a}]^2 L_u}{4bm} \\
 \frac{L/dL_u}{(L/dL_u)_0} &= \frac{4}{[e^{x_r/a} + e^{-x_r/a}]^2}; \quad \frac{dL_u}{dL_u} = \frac{[e^{x_r/a} + e^{-x_r/a}]^2}{4L_u} \quad [8]
 \end{aligned}$$

$$\begin{aligned} & \text{Achromatisches Sehen mit relativer Leuchtdichte} \\ & \text{Mathematikgleichungen mit Hyperbelfunktionen} \\ f_{\text{ab}}(x_r, a) = b \tanh(x_r/a) = b \frac{e^{x_r/a} - e^{-x_r/a}}{e^{x_r/a} + e^{-x_r/a}}, \quad x_r = \ln(10) \\ \frac{df_{\text{ab}}(x_r, a)}{dx_r} = \frac{4b}{a[e^{x_r/a} + e^{-x_r/a}]^2}, \quad x_r = \ln(10), \quad m = 1/(10 \ln(10)) \\ \frac{dL/dL}{dL/dL_u} = \frac{4}{[e^{x_r/a} + e^{-x_r/a}]^2}, \quad dL_u = 4L_u \quad [8] \\ \frac{dL/dL_u}{(dL/dL_u)_{\text{u}}} = \frac{1}{[e^{x_r/a} + e^{-x_r/a}]^2}, \quad (dL/dL_u)_{\text{u}} = L_u \quad [9] \\ \frac{dL/dL_u}{(dL/dL_u)_{\text{u}}} = 1 \quad \text{für } x_r = 0, \quad \frac{dL/dL_u}{(dL/dL_u)_{\text{u}}} = 1 \quad \text{für } x_r = 0 \quad [9] \end{aligned}$$

$$\begin{aligned} & \text{Achromatisches Sehen mit relativer Leuchtdichte} \\ & \text{Mathematikgleichungen mit Hyperbelfunktionen} \\ F_{ab}(x_r, a) &= b \tanh(x_r/a) = b \frac{e^{x_r/a} - e^{-x_r/a}}{e^{x_r/a} + e^{-x_r/a}} = b \frac{\sinh(x_r/a)}{\cosh(x_r/a)} < 1 [1] \\ \frac{dF_{ab}}{dx_r}(x_r, a) &= \frac{4b}{a[e^{x_r/a} + e^{-x_r/a}]^2} = \frac{4b}{a[\cosh(x_r/a)]^2} = \frac{4b}{a[\cosh(x_r/a)]^2} < 1 [2] \\ \frac{dL}{dL_u} &= \frac{4}{(L/L_u)[e^{x_r/a} + e^{-x_r/a}]^2} = \frac{4L_u}{(L/L_u)[e^{x_r/a} + e^{-x_r/a}]^2} = \frac{4L_u}{(L/L_u)\cosh^2(x_r/a)} < 1 [3] \\ \frac{dL/L_u}{dL_u} &= 1 \text{ für } \left\{ \begin{array}{l} L/L_u = 1 \\ x_r = 0 \end{array} \right. \quad \frac{dL}{dL_u} = 1 \text{ für } \left\{ \begin{array}{l} L/L_u = 1 \\ x_r = 0 \end{array} \right. [4] \end{aligned}$$

Achromatisches Sehen mit relativer Leuchtdichte  
Mathematikgleichungen mit Hyperbelfunktionen

$$\begin{aligned}
F_{ch}(x_r, c) &= b \tanh(x_r/c) = b \frac{x_r/c - e^{-x_r/c}}{1 - e^{-x_r/c}} = b \frac{x_r/c - e^{-x_r/c}}{e^{x_r/c} - e^{-x_r/c}} = b \frac{x_r/c - e^{-x_r/c}}{L_r + L_r e^{-x_r/c}} & [1] \\
H_{ch}(x_r, c) &= \frac{4b}{dx_r} = \frac{x_r - \ln(e^{x_r/c} + 1)}{dx_r dL_r / (1/\ln(10)L_r)} = \frac{1}{1/\ln(10)L_r} \frac{dx_r}{e^{x_r/c} + e^{-x_r/c}} & [2] \\
H'_{ch}(x_r, c) &= \frac{dx_r}{dx_r} = \frac{4bm}{dL_r} = \frac{L_r dL_r / dL_r}{dL_r (x_r/c + e^{-x_r/c})^2 L_r} & [6] \\
\frac{L_r}{L_r} &= \frac{4bm}{dL_r} = \frac{dL_r}{dL_r} = \frac{[e^{x_r/c} + e^{-x_r/c}]^2 L_r}{4bm} & [7]
\end{aligned}$$

$$\begin{aligned}
& \text{Achromatisches Sehen mit relativer Leuchtdichte} \\
& \text{Mathematische Gleichungen mit Hyperbelfunktionen} \\
F_{\text{ch}}(x_r, c) &= b \tanh(x_r/c) = b \frac{x_r/e^c - e^{-x_r}/c}{e^{x_r/c} + e^{-x_r}/c} = b \frac{x_r - \ln(10)}{e^{x_r/c} + \ln(10)} \quad [5] \\
F_{\text{ch}}'(x_r, c) &= \frac{4b}{dx_r} = \frac{4b}{c(e^{x_r/c} + e^{-x_r}/c)^2} = \frac{4b}{c \cdot 1/(10e^{x_r/c})} = \frac{40be^{x_r/c}}{c} = \frac{40b}{c} e^{x_r/c} \quad [5] \\
L &= \frac{4bn}{dL} = \frac{4bn}{[e^{x_r/c} + e^{-x_r}/c]^2} = \frac{4bn}{[e^{x_r/c} + e^{-x_r}/c]^2 L} \quad [7] \\
\frac{LdL}{d(LdL)_U} &= \frac{4}{[e^{x_r/c} + e^{-x_r}/c]^2}; \quad \frac{dL}{d(LdL)_U} = \frac{[e^{x_r/c} + e^{-x_r}/c]^2}{4I_U} \quad [8]
\end{aligned}$$

$$\begin{aligned} & \text{Achromatisches Sehen mit relativ Leuchtintensitäten} \\ & \text{Mathematische Gleichungen mit Hyperfunktions-} \\ & \text{Funktionen} \\ F_{\text{Ch}}(x_r, c) &= b \tanh(x_r/c) = b \frac{x_r/c - e^{-x_r/c}}{e^{x_r/c} + e^{-x_r/c}}, \quad x_r/c > 0 [1] \\ \frac{\partial F_{\text{Ch}}(x_r, c)}{\partial x_r} &= \frac{4b}{c[e^{x_r/c} + e^{-x_r/c}]^2} = \frac{dx_r}{dx_r} \frac{4b}{c[e^{x_r/c} + e^{-x_r/c}]^2} \cdot \frac{1}{1/(1+e^{-x_r/c})} [5] \\ \frac{dL_d}{dx_u} &= \frac{4}{[e^{x_u/c} + e^{-x_u/c}]^2}, \quad \frac{dL_d}{dx_u} = \frac{4}{4L_u} [8] \\ \frac{dL_d}{dx_u} - 1 &= \frac{1}{x_u = 0}, \quad \frac{dL_d}{dx_u} - 1 = \frac{1}{x_u = 0} [9] \end{aligned}$$

### Achromatisches Sehen mit relativer Leuchtdichte Mathematische Hyperbel- und Potenzfunktionen

$$\begin{aligned} F_{\text{Ab}}(x_r, a) &= b \tanh(x_r/a) - b \frac{e^{x_r/a} - e^{-x_r/a}}{e^{x_r/a} + e^{-x_r/a}} \frac{x_r}{x_r + c_1} [1] \\ dF_{\text{Ab}}(x_r, a) &= \frac{4b}{dx_r} \frac{x_r = \ln(10)}{[e^{x_r/a} + e^{-x_r/a}]^2} \frac{dL_r}{dx_r = \ln(1/(10)) L_r} \\ \frac{L_r}{dL_r} &= \frac{4bm}{[e^{x_r/a} + e^{-x_r/a}]^2} \quad dL_r = \frac{[e^{x_r/a} + e^{-x_r/a}]^2 L_r}{4bm} \quad [7] \\ \frac{L_r}{dL_r} &= \frac{4bm}{[L_r^m + L_r^{-m}]^2} \quad dL_r = \frac{[L_r^m + L_r^{-m}]^2 L_r}{4bm} \quad [8] \end{aligned}$$

$$\begin{aligned} & \text{Achromatisches Sehen mit relativer Leuchtdichte} \\ & \text{Mathematische Hyperbel- und Potenzfunktionen} \\ & F_{\text{ab}}(x_r, a) = b \tanh(x_r/a) = b \frac{x_r/a - e^{-x_r/a}}{x_r/a + e^{-x_r/a}} = b \frac{x_r - e^{-x_r/a}}{x_r + e^{-x_r/a}} \quad [1] \\ & \frac{dF_{\text{ab}}(x_r, a)}{dx_r} = \frac{4b}{x_r^2 + e^{2x_r/a} - 4e^{-x_r/a}} \quad x_r = \ln(10) \quad [2] \\ & \frac{L}{dL} = \frac{4bm}{x_r^2 + e^{2x_r/a} - 4e^{-x_r/a}} \quad dL = \frac{[x_r/a + e^{-x_r/a}]^2 L}{4bm} \quad [7] \\ & \frac{L}{dL} = \frac{4bm}{L^2 + 2L^{-2m}} \quad dL = \frac{[L^{2m} + 2L^{-2m}] L}{4bm} \quad [8] \end{aligned}$$

$$\begin{aligned} & \text{Achromatisches Sehen mit relativer Leuchtdichte} \\ & \text{Mathematische Hyperbel- und Potenzfunktionen} \\ F_{ab}(x_r,a) &= b \tanh(x_r/a) = b \frac{x_r/a - e^{-x_r/a}}{x_r/a + e^{-x_r/a}}, \quad x_r \in [0, \infty] \\ \frac{dF_{ab}}{dx_r}(x_r,a) &= \frac{4b}{a[x_r/a + e^{-x_r/a}]^2}, \quad x_r = \ln(10) \cdot \frac{L_u}{L_s} \\ \frac{dL}{dL_d} &= \frac{4}{[e^{x_r/a} + e^{-x_r/a}]^2}; \quad \frac{dL}{dL_u} = \frac{4[e^{x_r/a} + e^{-x_r/a}]^2}{L_u} \quad [8] \\ \frac{dL}{d(L_u + L_d)} &= \frac{4}{[e^{x_r/a} + e^{-x_r/a}]^2}; \quad \frac{dL}{d(L_u + L_d)} = \frac{4[L_u + L_d]^2}{L_u} - \frac{4L_u}{L_u + L_d} \quad [9] \end{aligned}$$

$$\begin{aligned} & \text{Achromatisches Sehen mit relativer Leuchtstärke} \\ & \text{Mathematische Hyperbel- und Potenzfunktionen} \\ F_{ab}(x_r, a) &= b \tanh(x_r/a) = b \frac{e^{x_r/a} - e^{-x_r/a}}{e^{x_r/a} + e^{-x_r/a}}, \quad x_r = \frac{-\log(L_r)}{\log(10)} \\ \frac{dF_{ab}}{dx_r}(x_r, a) &= \frac{4b}{a[e^{x_r/a} + e^{-x_r/a}]^2}, \quad x_r = \frac{-\log(L_r)}{\log(10)}, L_r = 10^{x_r} \\ \frac{L/dL}{dL/dL_u} &= \frac{4}{(dL/dL_u)^2}; \quad \frac{dL}{dL_u} = \frac{e^{x_r/a} + e^{-x_r/a}}{4L_u}, \quad [8] \\ \frac{L/dL}{dL/dL_u} &= \frac{4}{(dL/dL_u)^2}; \quad \frac{dL}{dL_u} = \frac{(L_u^{2m} + 2L_u^{-2m})}{4L_u}, \quad [9] \end{aligned}$$

chromatisches Sehen mit relativer Leuchtdichte  
Mathematische Hyperbel- und Potenzfunktionen

$$\begin{aligned} F_{\text{ch}}(x_r, c) &= b \tanh(x_r/c) = b \frac{e^{x_r/c} - e^{-x_r/c}}{e^{x_r/c} + e^{-x_r/c}} \stackrel{x_r = \ln(b/\ln(10))}{=} [1] \\ F_{\text{ch}}(x_r, c) &= \frac{4b}{c(e^{x_r/c} + e^{-x_r/c})^2} \stackrel{x_r = \ln(b/\ln(10))}{=} [10] \\ L &= \frac{4bn}{L(x_r/c + e^{-x_r/c})^2} \stackrel{dL = [e^{x_r/c} + e^{-x_r/c}]^2/L}{=} [7] \\ L &= \frac{4bn}{L[x_r^2 + e^{-2x_r/c}]} \stackrel{dL = [L^2 + Lx_r^2]/4bn}{=} [8] \end{aligned}$$

$$\begin{aligned}
& \text{chromatisches Sehen mit relativer Leuchtdichte} \\
& \text{mathematische Hyperbel- und Potenzfunktionen} \\
\text{ch}(x_r, c) &= b \tanh(x_r/c) - \frac{b}{c} \frac{e^{x_r/c} - e^{-x_r/c}}{e^{x_r/c} + e^{-x_r/c}} \frac{x_r - \ln(10)}{x_r + \ln(10)} \quad [1] \\
\text{ch}_r(x_r, c) &= \frac{4b}{c e^{[x_r/c + e^{-x_r/c}]^2}} \frac{x_r - \ln(10)}{x_r + \ln(10)} \quad [5] \\
\frac{d\text{ch}_r}{dx_r} &= \frac{4bn}{L} \frac{[e^{x_r/c} + e^{-x_r/c}]^2 - 2L}{[e^{x_r/c} + e^{-x_r/c}]^2} \quad [7] \\
\frac{d\text{ch}_r}{dx_r} &= \frac{4bn}{L} \frac{[L_{\text{2n}}^2 + 2L_{\text{T-2n}}^2] - 2L}{[L_{\text{2n}}^2 + 2L_{\text{T-2n}}^2]} \quad [8]
\end{aligned}$$

$$\begin{aligned} & \text{chromatisches Schema mit relativ Leuchtende} \\ & \text{Mathematische Hyperbel- und Potenzfunktionen} \\ & \text{bch}(x_r, c) = b \tanh(x_r/c) = \frac{e^{x_r/c} - e^{-x_r/c}}{e^{x_r/c} + e^{-x_r/c}} = \frac{e^{x_r/c}}{e^{x_r/c} + 1} \quad [1] \\ & F_{\text{bch}}(x_r, c) = \frac{4b}{dx_r} = \frac{4b}{c(e^{x_r/c} - e^{-x_r/c})^2} = \frac{4b}{dx_r \cdot x_r = 1/(10n(c))} \quad [5] \\ & \frac{dU/dL}{dU/dL} = \frac{4}{[e^{x_r/c} + e^{-x_r/c}]^2} = \frac{dU}{dx_r \cdot x_r = 1/(10n(c))} \quad [2] \\ & \frac{dU/dL}{dU/dL} = \frac{4}{L_r^2 + L_{r+1}^2} = \frac{dU}{dx_r \cdot x_r = 1/(10n(c))} \quad [3] \\ & \frac{dU/dL}{dU/dL} = \frac{4}{L_r^2 + 2 + L_{r+1}^2} = \frac{dU}{dx_r \cdot x_r = 1/(10n(c))} \quad [9] \end{aligned}$$