

Mathematikgleichungen der Hyperbelfunktionen
 Siehe: Papula, L., (2003), *Mathematische Formelsammlung*, Vieweg

$$F_{ab}(x_r/a) = b \frac{e^{x_r/a} - e^{-x_r/a}}{e^{x_r/a} + e^{-x_r/a}} = b \frac{u(x_r/a)}{v(x_r/a)} \quad [1]$$

$$F'_{ab}(x_r/a) = b \frac{u'(x_r/a) v(x_r/a) - u(x_r/a) v'(x_r/a)}{v^2(x_r/a)} \quad [2]$$

$$F'_{ab}(x_r/a) = b \frac{v^2(x_r/a) - u^2(x_r/a)}{a v^2(x_r/a)} \quad [3]$$

$$F'_{ab}(x_r/a) = \frac{4b}{a [e^{x_r/a} + e^{-x_r/a}]^2} \quad [4]$$

Mathematikgleichungen der Hyperbelfunktionen
 Siehe: Handbook of Mathematical functions, NBS, USA, Sec. 4.5

$$\sinh(x) = \frac{e^x - e^{-x}}{2} \quad [1], \quad \cosh(x) = \frac{e^x + e^{-x}}{2} \quad [2]$$

$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}} \quad [3]$$

$$\tanh(x/2) = \frac{\sinh(x)}{\cosh(x) + 1} = \frac{e^{x/2} - e^{-x/2}}{e^{x/2} + e^{-x/2} + 2} \quad [4]$$

$$\sinh^2(x) + \cosh^2(x) = 1 \quad [5]$$

Mathematikgleichungen der Hyperbelfunktionen
 Siehe: Handbook of Mathematical functions, NBS, USA, Sec. 4.5

$$F_{1b}(x) = b \tanh(x/a) = b \frac{e^{x/a} - e^{-x/a}}{e^{x/a} + e^{-x/a}} = b \frac{u(x)}{v(x)} \quad [1]$$

$$F'_{1b}(x) = b \frac{u'(x) v(x) - u(x) v'(x)}{v^2(x)} \quad [2]$$

$$F'_{1b}(x) = b \frac{v^2(x) - u^2(x)}{a v^2(x)} \quad [3]$$

$$F'_{1b}(x) = \frac{4b}{a [e^{x/a} + e^{-x/a}]^2} = \frac{b}{a \cosh^2(x/a)} \quad [4]$$

Mathematikgleichungen der Hyperbelfunktionen
 Siehe: Handbook of Mathematical functions, NBS, USA, Sec. 4.5

$$F_{ab}(x/a) = b \tanh(x/a) = b \frac{e^{x/a} - e^{-x/a}}{e^{x/a} + e^{-x/a}} = b \frac{u(x/a)}{v(x/a)} \quad [1]$$

$$F'_{ab}(x/a) = b \frac{u'(x/a) v(x/a) - u(x/a) v'(x/a)}{v^2(x/a)} \quad [2]$$

$$F'_{ab}(x/a) = b \frac{v^2(x/a) - u^2(x/a)}{a v^2(x/a)} \quad [3]$$

$$F'_{ab}(x/a) = \frac{4b}{a [e^{x/a} + e^{-x/a}]^2} = \frac{b}{a \cosh^2(x/a)} \quad [4]$$

Mathematikgleichungen der Hyperbelfunktionen
 Siehe: Handbook of Mathematical functions, NBS, USA, Sec. 4.5

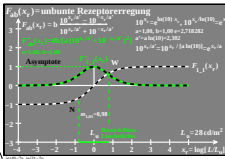
$$F_{ab}(x_r/a) = b \frac{e^{x_r/a} - e^{-x_r/a}}{e^{x_r/a} + e^{-x_r/a}} = b \frac{u(x_r/a)}{v(x_r/a)} \quad [1]$$

$$F'_{ab}(x_r/a) = b \frac{u'(x_r/a) v(x_r/a) - u(x_r/a) v'(x_r/a)}{v^2(x_r/a)} \quad [2]$$

$$F'_{ab}(x_r/a) = \frac{4b}{a [e^{x_r/a} + e^{-x_r/a}]^2} \quad [3]$$

$$x_r = \log(L/L_u) = \log(L_r) \quad [4]$$

$$e^{x_r/a} = 10^{x_r/a} = L_r / L_u \quad [5] \quad a' = a \ln(10) \quad [6]$$



Unbunt-Rezeptorerregungsfunktion
 $Q_{ab}(x_r/a)$ für $a=0.5$ und $b=1.0$
 mit $x_r = \log[L/L_u]$ (L = Testleuchtdichte)
 L_u = Umfeld-Leuchtdichte

$$Q_{ab}(x_r/a) = \frac{b}{\ln 2} \ln \left[\frac{1}{1 + \sqrt{2} e^{(x_r/a)}} \right] - b$$

Funktionswerte für $b=1$ und jedes $a>0$:

$x_r/a \rightarrow -\infty$	$= -1$	$x = \log L, u = \log L_u$
$Q_{a1}(x_r/a = 0)$	$= 0$	$x_r = \log[L/L_u]$
$Q_{a1}(x_r/a \rightarrow \infty)$	$= +1$	$x = x - u$

Mathematikgleichungen der Hyperbelfunktionen
 Siehe: Handbook of Mathematical functions, NBS, USA, Sec. 4.5

$$F_{ab}(x/a) = b \tanh(x/a) = b \frac{e^{x/a} - e^{-x/a}}{e^{x/a} + e^{-x/a}} = b \frac{u(x/a)}{v(x/a)} \quad [1]$$

$$F'_{ab}(x/a) = b \frac{u'(x/a) v(x/a) - u(x/a) v'(x/a)}{v^2(x/a)} \quad [2]$$

$$u'(x/a) = 1/a [e^{x/a} - e^{-x/a}] = (1/a) v(x/a) \quad [3]$$

$$v'(x/a) = 1/a [e^{x/a} + e^{-x/a}] = (1/a) u(x/a) \quad [4]$$

$$F'_{ab}(x/a) = b \frac{v^2(x/a) - u^2(x/a)}{a v^2(x/a)} \quad [5]$$

Mathematikgleichungen der Hyperbelfunktionen
 Siehe: Handbook of Mathematical functions, NBS, USA, Sec. 4.5

$$F_{ab}(x_r/a) = b \frac{e^{x_r/a} - e^{-x_r/a}}{e^{x_r/a} + e^{-x_r/a}} = b \frac{u(x_r/a)}{v(x_r/a)} \quad [1]$$

$$F'_{ab}(x_r/a) = \frac{dF_{ab}(x_r/a)}{d(x_r/a)} = \frac{4b}{a [e^{x_r/a} + e^{-x_r/a}]^2} \quad [2]$$

$$x_r = \log(L/L_u) = \log(L_r) \quad [3]$$

$$e^{x_r/a} = 10^{x_r/a} = L_r / L_u \quad [4] \quad a' = a \ln(10) \quad [5]$$

$$F'_{ab}(x_r/a) = \frac{4b}{a [L_r^2 + L_u^2 + 2 L_r L_u]^2} \quad [7]$$

TUB-Prüfvorlage ig4; Modell für normierte Erregungsfunktion $F_{ab}(x_r)$ und Ableitung $F'_{ab}(x_r)$
 Mathematische Berechnung der Ableitung $F'_{ab}(x_r)$, des Kontrastes $L/\Delta L$ und der Unterscheidung ΔL