

Mathematikgleichungen der Hyperbelfunktionen
 Siehe: Handbook of Mathematical functions, NBS, USA, Sec. 4.5

$$F_a(x_r) = a \frac{e^{x_r/a}}{e^{x_r/a} + e^{-x_r/a}} = a \frac{u(x_r, a)}{v(x_r, a)} \quad [1]$$

$$F'_a(x_r) = a \frac{u'(x_r, a) v(x_r, a) - v'(x_r, a) u(x_r, a)}{v^2(x_r, a)} \quad [2]$$

$$u'v = [(1/a) e^{x_r/a}] [e^{x_r/a} + e^{-x_r/a}] \quad [3]$$

$$v'u = [(1/a) e^{x_r/a} - (1/a) e^{-x_r/a}] [e^{x_r/a}] \quad [4]$$

$$u'v = [1/a] \{ [e^{x_r/a}]^2 + 1 \} \quad [5]$$

$$v'u = [1/a] \{ [e^{x_r/a}]^2 - 1 \} \quad [6]$$

$$u'(x_r, a) v(x_r, a) - v'(x_r, a) u(x_r, a) = 2/a \quad [7]$$

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$$F'_a(x_r) = 2 / [e^{x_r/a} + e^{-x_r/a}]^2 \quad [7]$$

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$$F_{ab}(x/a) = b \tanh(x/a) = b \frac{e^{x/a} - e^{-x/a}}{e^{x/a} + e^{-x/a}} = b \frac{u(x/a)}{v(x/a)} \quad [1]$$

$$F'_{ab}(x/a) = b \frac{u'(x/a) v(x/a) - u(x/a) v'(x/a)}{v^2(x/a)} \quad [2]$$

$$F'_{ab}(x/a) = b \frac{v^2(x/a) - u^2(x/a)}{a v^2(x/a)} \quad [3]$$

$$F'_{ab}(x/a) = \frac{4b}{a [e^{x/a} + e^{-x/a}]^2} = \frac{b}{a \cosh^2(x/a)} \quad [4]$$

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$$F_{abc}(x_r) = b \frac{e^{x_r/a} - e^{-x_r/c}}{e^{x_r/a} + e^{-x_r/c}} = b \frac{u(z, a, c)}{v(z, a, c)} \quad z=x_r \quad [1]$$

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$$u'v = [(1/a) e^{z/a} + (1/c) e^{-z/c}] [e^{z/a} + e^{-z/c}] \quad [3]$$

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$$u'(z, a, c) v(z, a, c) - v'(z, a, c) u(z, a, c) = 2 [1/a + 1/c] [e^{x_r/a} \cdot e^{-x_r/c}] \quad [5]$$

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$$F_c(x_r) = c \frac{-e^{-x_r/c}}{e^{x_r/c} + e^{-x_r/c}} = c \frac{u(x_r, c)}{v(x_r, c)} \quad [1]$$

$$F'_c(x_r) = c \frac{u'(x_r, c) v(x_r, c) - v'(x_r, c) u(x_r, c)}{v^2(x_r, c)} \quad [2]$$

$$u'v = [(1/c) e^{-x_r/c}] [e^{x_r/c} + e^{-x_r/c}] \quad [3]$$

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$$F_{ab}(x_r/a) = b \frac{e^{x_r/a} - e^{-x_r/a}}{e^{x_r/a} + e^{-x_r/a}} = b \frac{u(x_r/a)}{v(x_r/a)} \quad [1]$$

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$$F'_{ab}(x_r/a) = \frac{4b}{a [e^{x_r/a} + e^{-x_r/a}]^2} \quad [3]$$

$$x_r = \log(L/L_u) = \log(L_r) \quad [4]$$

$$e^{x_r/a} = 10^{x_r/a'} = L_r^{1/a'} \quad [5] \quad a' = a \ln(10) \quad [6]$$

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$$F'_{abc}(z) = b \frac{u'(z, a, c) v(z, a, c) - u(z, a, c) v'(z, a, c)}{v^2(z, a, c)} \quad [2]$$

$$u'(z, a, c) v(z, a, c) - v'(z, a, c) u(z, a, c) = 2 [1/a + 1/c] [e^{z/a} \cdot e^{-z/c}] \quad [3]$$

$$F'_{abc}(x_r) = 2 b [1/a + 1/c] \frac{e^{x_r/a} \cdot e^{-x_r/c}}{[e^{x_r/a} + e^{-x_r/c}]^2} \quad [4]$$

Achromatisches Sehen mit relativer Leuchtdichte
 Mathematische Hyperbel- und Potenzfunktionen

$$F_{ab}(x_r, a) = b \tanh(x_r/a) = b \frac{e^{x_r/a} - e^{-x_r/a}}{e^{x_r/a} + e^{-x_r/a}} \quad x_r = \log(L_r) \quad L_r = L/L_u \quad x_r < 0 \quad [1]$$

$$\frac{dF_{ab}(x_r, a)}{dx_r} = \frac{4b}{a [e^{x_r/a} + e^{-x_r/a}]^2} \quad x_r = \ln L_r / \ln(10) \quad dx_r/dL_r = 1/(\ln(10) L_r) \quad m = 1/(\ln(10) a) \quad [5]$$

$$\frac{L}{dL} = \frac{4bm}{[e^{x_r/a} + e^{-x_r/a}]^2} \quad dL = \frac{[e^{x_r/a} + e^{-x_r/a}]^2 L}{4bm} \quad [7]$$

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$$\frac{L/dL}{(L/dL)_u} = \frac{4}{[e^{x_r/a} + e^{-x_r/a}]^2}; \quad \frac{dL}{dL_u} = \frac{[e^{x_r/a} + e^{-x_r/a}]^2 L}{4 L_u} \quad [8]$$

$$\frac{L/dL}{(L/dL)_u} = \frac{4}{[L_r^m + L_r^{-m}]^2}; \quad \frac{dL}{dL_u} = \frac{[L_r^m + L_r^{-m}]^2 L}{4 L_u} \quad [9]$$

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TUB-Prüfvorlage igi5; Farbsehmodell für normierte Rezeptorerregung $F_{a/c/ab/cb}(x_r)$
 Ableitungen $F'_{a/c/ab/cb}(x_r)$ mit Hyperbelfunktionen $e^{(x_r/a)}$ und $e^{(x_r/c)}$ für Kontrast $L/\Delta L$ und ΔL